

Library of Philosophy.

EDITED BY J. H. MUIRHEAD, M.A.

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THE LIBRARY OF PHILOSOPHY is in the first instance a contribution to the History of Thought. While much has been done in England in tracing the course of evolution in nature, history, religion, and morality, comparatively little has been done in tracing the development of Thought upon these and kindred subjects, and yet "the evolution of opinion is part of the whole evolution."

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By the co-operation of different writers in carrying out this plan, it is hoped that a completeness and thoroughness of treatment otherwise unattainable will be secured. It is believed, also, that from writers mainly English and American fuller consideration of English Philosophy than it has hitherto received from the great German Histories of Philosophy may be looked for. In the departments of Ethics, Economics, and Politics, for instance, the contributions of English writers to the common stock of theoretic discussion have been especially valuable, and these subjects will accordingly have special prominence in this undertaking.

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J. H. MUIRHEAD,

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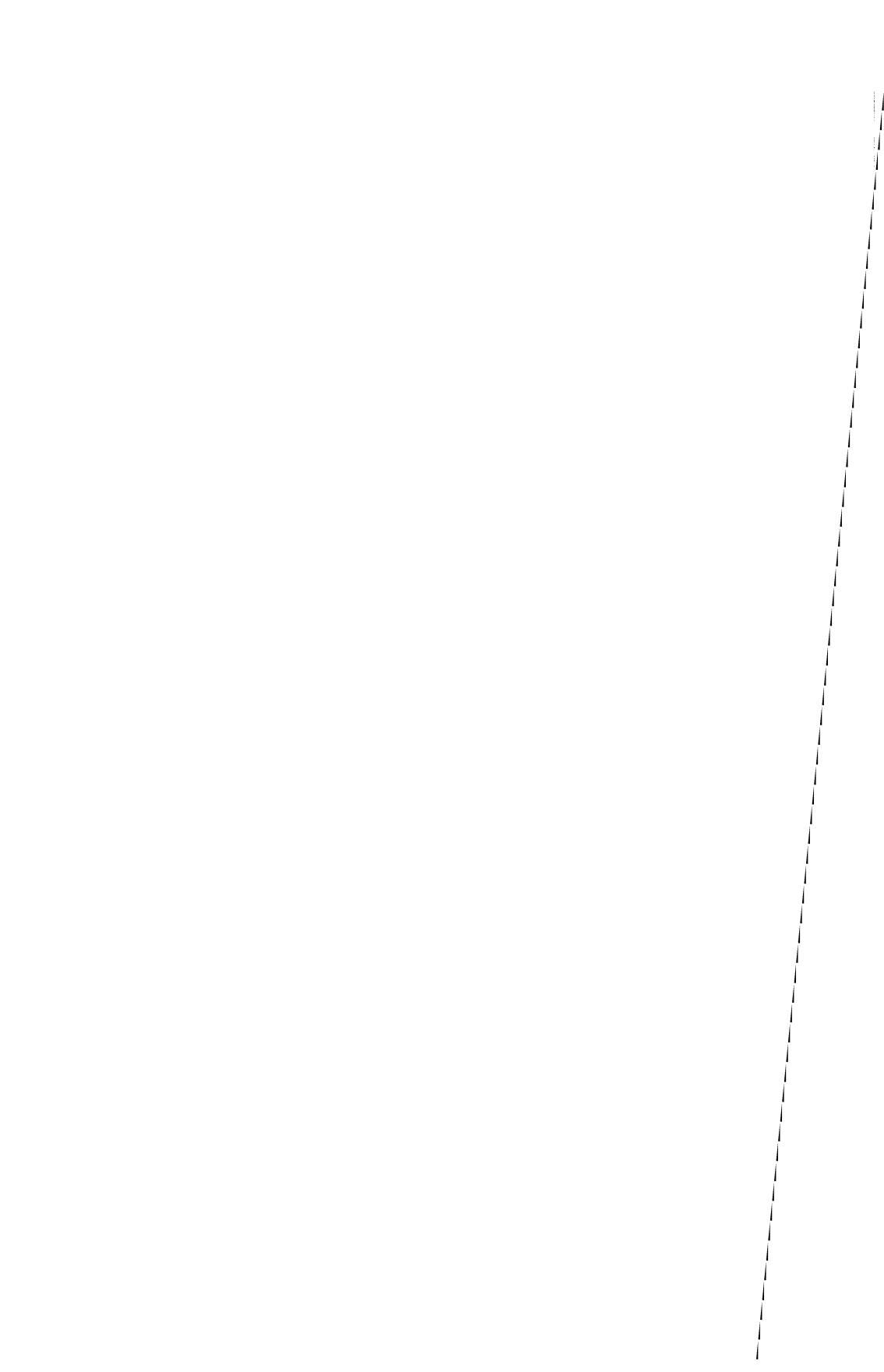
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LOGIC

BY

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VOL. I

THE JUDGMENT, CONCEPT, AND INFERENCE

SECOND EDITION, REVISED AND ENLARGED

TRANSLATED BY HELEN DENDY



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AUTHOR'S PREFACE TO THE ENGLISH TRANSLATION

WHEN I was requested to give my consent to an English translation of my *Logic*, I complied willingly, because in preparing and achieving this work an essential part of my studies had been devoted to English logicians from Francis Bacon down to Jevons, Bradley, and Venn ; thus I may hope that to English readers my book will not appear entirely as a foreigner.

Whoever has himself attempted to translate a philosophical treatise is acquainted with the difficulties of the task ; it is sometimes impossible to find in one's own language simple and current expressions which might exactly correspond to the terms of the original, and the translator is beset by the danger of either missing the precise meaning of the text, or of straining his own language and impairing easy understanding.

I think that Miss Dendy has done her best to overcome these difficulties and avoid this double danger. I have carefully revised her manuscript from beginning to end, and co-operated in correcting the proofs, so that I may safely assure the reader that the translation is completely free from misunderstandings, and that it represents everywhere as exactly as possible the original text. I feel obliged to express my hearty thanks to Miss Dendy for the trouble she has taken to give a really faithful and reliable translation.

C. SIGWART.

TÜBINGEN, *November*, 1894.

AUTHOR'S PREFACE TO FIRST GERMAN EDITION

THE following attempt to reconstruct logic from the point of view of methodology, thus bringing it into active relations with the scientific problems of the present day, must be justified by the success with which it is carried out ; this first volume contains the preliminary and fundamental parts of the doctrine, and in them I have adhered as closely as possible to the traditional form of the science. I would only ask that the small extent to which I have referred to previous and contemporary treatments of the subject as a whole, or to particular views on special points, may not be taken amiss. In a science which has been so much discussed it seemed to me that to commend or dispute even the most important previous doctrines would be an undue extension of the work ; and I further feared lest the course of investigation and exposition prescribed by my view of the subject would be confused if I made a rule of discussing at every point opinions which are to a large extent based upon presuppositions quite different from my own. I have therefore thought it right to confine myself to what was indispensable for the correct representation and justification of my own doctrine. I need hardly say that I have made extensive use of both ancient and modern authors. Three of the men whose works have been most frequently before me, and to whom I hoped here to express my obligation—Trendelenburg, Ueberweg and Mill—have died while the book was being planned and carried out ; I must also make special mention of the assistance I have received from Prantl's great work.

July, 1873.

AUTHOR'S PREFACE TO SECOND GERMAN EDITION

IN the fifteen years which have passed since this book first appeared the literature of logic has been enriched by a remarkable series of valuable books. Important works have appeared by Lotze, Schuppe, Wundt and Bradley—to name only the most eminent; and all start from the conception which has guided this attempt. That is, logic is grounded by them, not upon an effete tradition, but upon a new investigation of thought as it actually is in its psychological foundations, in its significance for knowledge and its actual operation in scientific methods. Particular points of logic, again, have received welcome elucidations from more special investigations, and amongst these Windelband's studies on the negative judgment, Meinong's treatment of relational-concepts, and Volkelt's acute and original treatise are most nearly akin to my views.

It has thus become incumbent upon me to test anew my own views by the conclusions arrived at by my fellow-workers, to find more accurate expression where misunderstanding might arise, and elsewhere to amplify and extend, or to fortify against adverse opinions. But for the reasons already stated in my first preface, I have been obliged to refrain from incorporating more extensively in the work the considerations which determined me to adhere to my conclusions, or from mentioning in detail all the critical observations which have been so abundantly bestowed. Where these criticisms were really applicable to what I had said, I have gratefully made use of them; where they were due merely to misunderstandings, I was loth to weary the reader with unfruitful discussions. In the same way I have been forced to abstain from enriching the work by introducing more freely investigations not included in its original design, even though I might agree with them; the subject is so inexhaustible that completeness is not attainable, and I would rather sacrifice the appearance of completeness in treatment than obscure the clearness of the design.

TÜBINGEN, *October*, 1888.

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GENERAL INTRODUCTION

§ 1.

IF we consider the nature of our Thought, we find that an important part of it is engaged in the attempt to arrive at propositions which are *certain* and *universally valid*, but that it frequently fails to do this when left to its natural development. Hence arises the problem of ascertaining the conditions under which this object can be attained, and of determining in accordance with those conditions the rules to be followed in its attainment. The solution of this problem would place us in possession of a technical science of Thought, directing us how to arrive at certain and universally valid propositions. Such a science we call Logic.

1. To determine what Thinking in general is, how it differs from other psychical activities, in what relations it stands to these, and what are its different varieties, is primarily the business of Psychology. It is true there is no generally accepted Psychology to which we can refer, but for our present investigation it will be enough to refer to our ordinary use of language. By Thinking in its widest sense we always mean an Ideational Activity; *i.e.* an activity which in itself does not include that inner subjective excitement which we call Feeling, nor yet give rise to an immediate effect upon ourselves or others as in willing and acting. All that it signifies is that something is present to consciousness as its object. It must however be further distinguished from Perception and Intuition; these express immediate reference to an object which is given to the subjective activity independently of itself,

while Thinking denotes a *purely inner activity* of ideation, thus seeming to be a spontaneous action proceeding from the energy of the subject alone. Its products—thoughts—are therefore distinguished, as merely subjective, ideal images, from the objects which are looked upon as real in Perception and Intuition. In this sense language gives the name of Thinking to Recollection—bethinking oneself of something, and to Imagination, picturing something in Thought, as well as to reflection and consideration. But when Perception and Thought refer to the same object, as in knowledge of the external world, we distinguish between the spontaneous selection, combination and elaboration of the elements given to Perception, and the immediate presentation of those elements. It is the former which we regard as the factor belonging to Thought.

2. If for the present we take Thinking to mean all that we mean by it in our ordinary use of language, then it is certain that with the development of conscious life Thought arises *involuntarily* and of necessity. As soon as the individual begins to reflect upon his inner activity he finds that he is already engaged in various kinds of Thought; he can have no immediate knowledge of its beginning nor of its development out of simpler and more primitive activities. Only by means of a difficult psychological analysis of Thought, as we find it at work, can we discover its particular factors and the faculties which give rise to it, and thus form some idea of the laws of its unconscious growth.

Moreover the involuntary production of thoughts continues throughout our whole life. It is absolutely impossible, when conscious and awake, to check the inner activity which is incessantly excited by the most varied motives to form a constant succession of ideas which it combines in different ways, and thus, without any intention on our part, maintains an inner world of thoughts present before us.

3. But besides this involuntary Thought, there is a *voluntary*

action, a will to think, which is led by definite interests and aims to endeavour to rule the formerly involuntary course of the thoughts and to direct them towards a given end. From amongst the thoughts which arise involuntarily this activity makes its selection, losing sight of some altogether and retaining and developing others by means of attention, thus seeking for thoughts and pursuing them. We need not enter here upon the question as to whether the production of thoughts is ever really voluntary in a direct way, or whether we can only indirectly bring about the conditions necessary to the involuntary production of what is desired. The result is practically the same in both cases ; that is, thoughts which satisfy a definite interest are produced by the influence of the will.¹

But this interest is of two kinds. From one point of view the voluntary activity which we direct towards our Thought falls under the general law that the pleasant is sought after, the unpleasant avoided. Now we may regard Thought as pleasant in a double sense. Firstly, in so far as every natural activity gives a feeling of satisfaction within certain limits of intensity ; secondly, in so far as the manifold content of our Thought affects us pleasantly or unpleasantly.

Taking this question of pleasure alone, we find within us both a general tendency to excite Thought, or let it be excited, as an entertainment and escape from tedium ; and also a tendency to direct it in such a way that what we think may be pleasing to us. When we linger over pleasant recollections and seek to intensify them, when we make projects and build castles in the air, when we strive to chase away disagreeable recollections or to dispel fear and anxiety, then it is this motive which determines the influence of the will upon our Thought.

¹ Cf. Windelband, über Denken und Nachdenken (*Präludien*, p. 176 sq.). His treatment contains much that is true and valuable on certain points, although he does not seem to me to have rightly determined the relation between "unconscious" and conscious willing.

The satisfaction which arises in such cases is of a completely individual character; the particular subject refers only to itself, to its own peculiar nature and circumstances. As a rule, therefore, Thought of this kind varies with each individual, nor could we wish it otherwise.

4. But this desire of obtaining immediate pleasure from Thought is the subordinate interest of the two. By far the most important part of our mental activity, both in extent and in value, follows more serious aims.

In the first place, our *wants* and the necessities of life take Thought into their service and set before it aims which are consciously grasped and pursued. Our existence and our well-being depend upon conscious action and upon our influencing things around us to serve our purposes. This action does not accomplish its purpose with the ease and certainty of instinct; it calls for attentive and careful observation of the nature of things and of their relations to us, as well as for manifold calculations and considerations as to the way in which they may serve for a means to the satisfaction of our wants. Human Thought attains its end—the preservation of our well-being—only when its knowledge of things enables it to predict the future; when, therefore, anticipation coincides with the actual course of events as partly conditioned by our interference.

But the desire for knowledge which makes itself felt everywhere seeks a right understanding of things and their relations even beyond the sphere of our practical needs. Solely for the sake of knowing it urges Thought to penetrate into the nature of things, and thus to reproduce in the totality of our subjective knowledge a faithful and complete picture of the objective world. Satisfaction of the desire for knowledge must then include this aim of practical Thought also. The immediate purpose which excites Thought and determines its direction is *knowledge of that which is*.

5. But the purposes towards which Thought is directed are far from being exhausted by this one interest—the desire for knowledge. It is found to be just as active in a direction which cannot be included in the idea of knowledge of that which is. We find ourselves as a matter of fact under the dominion of certain *laws* according to which we estimate the *worth* of human deeds, and to which we are willing to submit our wills and actions. It matters not for our present investigation whence these laws have their origin, nor by what motive we are led to recognise them as valid for us. Enough that we are constantly endeavouring to observe the rules of propriety, of morality, of justice and of duty, and are incessantly called upon to decide what we ought to do and how we ought to act if we are to avoid conflict with the principles, which we acknowledge, and maintain honour and conscience unsullied. No material result informs us whether or not our Thought has attained its end, by proving that our calculations accorded with the nature of things. Even the result at which we aim consists solely in thoughts, and the actual results also are the thoughts which accuse or absolve us, the recognition or non-recognition by others and by ourselves of the conformity of the particular action with the general rule.

6. Let us consider more closely this, the most important part of our practical Thought, as well as of our estimation of practical relations. Before the Court of Conscience the only evidence we have as to whether the Thought which guides our action has attained its purpose or not, is the inner consciousness of the *necessity of our Thought*; the *certainty* that the given mode of action follows inevitably from the general rule; the *self-evidence* which satisfies us that it was right and good to act so under the circumstances because the general principles of justice and morality demanded it. Nor have we any external confirmation that we have attained our end except the assent of others, who, starting

from the same presuppositions, affirm the same consequences to be necessary.

In speaking of the *necessity* of our Thought we must guard against a confusion of meaning. From a psychological point of view everything which the individual thinks may be looked upon as necessary, *i.e.* as an activity which results in accordance with general laws from whatever conditions may be present. That just this, and not something else, is thought, is the necessary consequence of the particular individual's range of ideas, disposition and character, and the stimulus of the moment. But besides this necessity of psychological causality there is another which springs entirely from the *contents* and *object* of Thought itself ; which is, therefore, grounded, not upon the variable subjective states of the individual, but upon the nature of the object thought of, and which may so far be called *objective*.

It is in this consciousness then of its objective necessity and universal validity that our Thought finds satisfaction, and we shall find upon examination that the same characteristics express the aim of our Thought in its search for knowledge of that which is. Here, again, we can only define the end towards which our intentional Thought strives by saying that it aims at satisfaction in the consciousness of its necessity and universal validity.

It is true that any one unbiassed by critical reflexion is impelled by a psychological necessity to objectify his sensations and the thoughts relating to them, and to picture to himself a world to which he ascribes an existence independent of his subjective activity. When his desire for knowledge bestirs itself he does not hesitate to attempt to know this objective world, and to frame his thoughts in such a way that they may conform to that which is. Nevertheless it is a question of dispute whether this purpose can be achieved. We cannot refute the critical assertion that immediately, and in the first instance, all our knowledge is only something for us, consisting in a system of ideas. That there is

an Existent corresponding to this thought of ours and in accordance with it, is either a blind belief, or the certainty must be grounded upon a refutation of the doubt it dispels—upon the proof that doubt is impossible. That is, it depends, on the one hand, upon our finding that the assumption of an Existent involves us in no unthinkable contradiction; on the other hand, upon the nature of our ideas being such as to force us to assume this Existent. In both cases the proof rests upon a necessity in Thought. It may be counted amongst the surest results of the analysis of our knowledge that every assumption of an external world is mediated by Thought, and in some way or another derived by unconscious mental processes from the subjective facts of sensation. Thus, except by Thought, we have no means of ascertaining whether we have really achieved our purpose of knowing the Existent; the possibility of comparing our knowledge with things as they exist apart from our knowledge is for ever closed to us. At the best we must be satisfied to find agreement and absence of contradiction amongst the thoughts which presuppose an Existent; just as we are fully satisfied in the region of outward action when our ideas and movements, together with their consequences, are in harmony amongst themselves and with the ideas of others.

Granted that there is a knowable Existent, then a knowledge of it is impossible unless there is some relation between the Existent and our subjective action which is governed by laws, and by virtue of which the thoughts necessitated by what is given in consciousness correspond to the Existent. Further, if we grant a knowable, external Existence, then it is the same for all knowing and thinking subjects, and all who know the Existent must think alike in reference to the same object. Thus Thought which knows the Existent is of necessity a universally valid Thought.

If, on the contrary, we deny the possibility of knowing anything as it is in itself—if the Existent is only a thought of our production

—it still remains true that the ideas to which we attribute objectivity are those which we produce with a consciousness of necessity. The fact that we regard anything as existing implies that all other thinking creatures of like nature with ourselves (even when only hypothetically assumed) would also be forced with the same necessity to regard it as existing.

We may then unhesitatingly say that if all we can attain to is but necessary and universally valid Thought, then knowledge of the Existent is included therein, and when we think with the object of knowing our immediate aim is nothing more than this necessary and universally valid Thought. It is in this conception also that the essence of "Truth" consists. When we speak of mathematical, actual, or moral truths, the common characteristic of all which we call true is that it is thought necessarily and with universal validity.

7. This view of the task undertaken by the Thought of which Logic treats avoids the difficulties which beset every system of Logic calling itself a theory of knowledge. The difficulty of such a system is that it must begin by showing whether and how far knowledge is possible at all; and in doing this, it not only passes into the disputed regions of metaphysics, but it presupposes in its proofs and refutations that necessity and universal validity of Thought from which the conviction of its objectivity is to proceed. In the same way we avoid the one-sidedness into which the Logic which professes to be a theory of knowledge is apt to fall—that of treating only of the thought which aids us in our knowledge of the purely theoretical, and forgetting the Thought which serves to guide our action. Nevertheless the psychical activities are the same in both cases, and the two aims must be classed together.

8. By thus including all Thought which pursues the common aim of certainty of its own necessity and universal validity we can also complete its psychological limitations. All such Thought results finally in *judgments*, which are expressed inwardly and outwardly

as *propositions*. Every practical consideration of ends and means terminates in judgments ; all knowledge consists in judgments ; every conviction ends in judgments. All other functions come into consideration only as being the condition of, and preparing the way for the judgment. Again, the judgment can be an object of scientific investigation only in so far as it is expressed in a proposition ; only through the proposition can it become an object of general consideration, and only as a proposition can it aspire to universal validity.

9. Now we know from the fact that error and dispute exist that our Thought frequently misses its aim in the judgments which it actually forms. Sometimes these judgments are relinquished by the individual thinker himself under the conviction that they are invalid, *i.e.* that it is necessary to think otherwise. Sometimes, again, other thinkers refuse to accept them : they dispute their necessity, affirm them to be mere conjecture, or even deny their possibility inasmuch as they find it necessary to judge differently concerning the same matter.

Since then actual Thought can and does miss its aim, we have need of a discipline which shall teach us to avoid error and dispute, and to conduct Thought in such a manner that the judgments may be *true*—that is, necessary and certain—that is, accompanied by a consciousness of their necessity, and therefore universally valid.

Reference to this aim distinguishes the logical from the psychological treatment of Thought. The latter is concerned with the knowledge of Thought as it actually is, hence it seeks the laws according to which, under certain conditions, a certain thought appears in just one way and no other. Its task is to explain all actual Thought according to the general laws of psychical activity, and as arising from the particular conditions of the individual instance, thus dealing with all Thought alike, whether erroneous and disputable, or true and generally accepted. The antithesis

of true and false is no more a psychological one than is the antithesis of good and bad in human action.

The logical treatment, on the contrary, presupposes the desire to think the truth; it has no meaning except for those who are conscious of this desire and in that region of Thought which is governed by it. Starting with this aim before it, and investigating the conditions of its attainment, Logic proposes, on the one hand, to set forth those Criteria of true Thought which are due to the demand for necessity and universal validity; on the other hand, to direct us how to conduct the mental operations in such a way that the end may be attained. In one aspect then Logic is a critical discipline having reference to Thought which has already taken place; in the other, it is a technical discipline. But as criticism is of value only as a means to the end, the principal task of Logic and that in which it properly consists, is to be a technical discipline or Art.

§ 2.

Logic as the Art of Thought cannot undertake to instruct us how, beginning at any given time, we may attain to Thought which is *entirely and absolutely true*. It must restrict itself to showing what are the *general Conditions* which, from the nature of our Thought, every proposition must satisfy in order to be necessary and universally valid, and under what conditions and according to what rules we can pass from given premises in a manner which shall be necessary and universally valid. It declines to give any judgment as to the necessity and universal validity of the premises from which we start at any time. Thus observance of logical rules ensures merely the *formal correctness* of the procedure and not the *material truth* of the results. In this sense our doctrine is of necessity *formal Logic*.

I. Instruction in an art which claims to ensure success for the activity to which it gives rules presupposes that this activity is

completely free and voluntary. This implies, first, that I am able to bring about the conditions necessary to the activity whenever I choose; secondly, that I only need to be conscious of the aim and of the rules of its attainment in order to fulfil every particular operation in accordance with those rules, and so as to attain the end in view. If there is to be an Art ensuring the production of necessary and universally valid Thought, by means of which we may know the Truth, we must assume that all the conditions lie within our grasp, and that at any given time we are completely free to control our Thought in accordance with its rules.

This it was which Descartes had in view when he designed his "*Methodus recte utendi ratione et veritatem in scientiis investigandi*." It was to put an end, once and for all, to all possibility of error, to exclude all doubt, and to produce a series of thoughts which—starting from one necessarily true and certain proposition and proceeding by infallible steps—should contain none but absolutely true propositions. His assumption was that, though we may not be free to choose what ideas shall be present to us, still judgment is a completely free and voluntary act, inasmuch as we can withdraw our assent to every proposition which we do not recognise with full conviction to be true and certain. Hence it is possible, by means of a thorough-going scepticism, to free oneself from every presupposition involving danger of error, and to make an entirely fresh start in the activity of Thought. In the same way he assumed that the chief conditions of this activity—concepts and principles—are innate in us, and that they are thus dependent upon our self-consciousness alone.

Now even if this latter assumption were as true as it is disputed, the method would at best find complete application only in the region of *a priori* knowledge, and only for those capable of making and carrying out the resolution to free themselves from all presuppositions. But it is absolutely impossible voluntarily to break the continuity between past and present Thought, and to begin com-

pletely *ab ovo*. Voluntary Thought grows out of an involuntary production of thoughts by which it is continually fed, and we could make no progress unless there were a supply of these thoughts forthcoming and a language representing them. We need only turn to Descartes himself to see how, in spite of the best intentions, elements of past Thought will force their way into the newly started series. Nor is it true that we can voluntarily refrain from any judgment while we have no choice as to whether the ideas to which it relates shall be present or not. One reason for this is that a part of the presuppositions we bring with us are judgments which inevitably involve others; another, that our judgments concerning the relations of ideas are predetermined by the nature of those ideas, and we are not at liberty to choose whether we will affirm or deny.

There can then be no method whatever by which we may begin Thought entirely anew. All that we can look for is a method of carrying it on from already existing data, which must always form the starting point for future Thought, even when acknowledged to be uncertain.

2. The necessity of confining Logic to 'the regulation of the procedure of Thought makes itself felt more especially with reference to the Thought which aims at empirical knowledge of the world. Such knowledge presupposes correct perceptions; and in making observations which are to serve some end, we are dependent not only upon the Thought which accompanies them, but also upon the conditions of sensation and the relation between our senses and the object. The art of correct observation is only partially included in the art of right thinking; it depends also upon the acuteness and training of the organs of sense, upon mechanical adroitness, and upon the art of bringing the object into the most favourable relations with our organs of sense and of eliminating errors of observation. It must be guided in its different expedients by the varying nature of the objects with which

it deals, each class of these requiring its particular technology. If we were to suspend Thought and Judgment in the region of empirical knowledge until we could start from absolutely certain and necessary premises, empirical science would be altogether impossible. We should be forced to leave in suspense, together with the validity and exactness of our perceptions, not merely the whole question as to the reality of the material world, but also the possibility of universally valid laws of phenomena.

Moreover, the history of the development of our knowledge shows that the discovery of truth has often been indirectly attained by starting from erroneous and uncertain premises. In the course of scientific research dispute is frequently settled by pursuing false propositions to their conclusions. Every apagogical proof is an example of this procedure.

Finally, a large part of the Thought which aims at universal validity is connected with premises which derive their validity from a Will, and which are so far purely arbitrary. To insist that Logic should establish the material truth of all propositions would be to exclude from logical consideration all practical jurisprudence.

3. Thus all that an Art of correct Thought can achieve, and all that it can propose to do, is to instruct us how we may proceed in Thought from given data in such a way that each step shall be accompanied by the consciousness of necessity and universal validity. It does not teach us *what* we must think—to do that would be to comprehend all science. It teaches only that *if* we think this we must also think that, no matter how our data—the supply of involuntary ideas and of particular observations and general propositions—may be constituted in other respects.

It must be understood that by “proceeding” we mean a progress in any direction—from grounds to consequences, as well as from consequences to grounds, from general to particular, as well as the

reverse. Thus the art must be applicable to all problems whatever which present themselves to our Thought.

4. This then is the sense in which we take Logic to be a *formal* science ; for the sake of the generality and practical fulfilment of our task we cannot undertake to construct Thought without involving any presuppositions, nor do we extend our logical investigations to the validity of the data from which Thought actually starts at any time, but only to the correctness of its procedure from those data. But by calling Logic formal, we do not mean that it should make the vain attempt to regard Thought in general as a merely formal activity, which may be considered apart from all matter of Thought, and which is unaffected by differences of matter. Nor do we mean that logical investigation should entirely abstract from and ignore the general nature of the matter and presuppositions of actual Thought. Of Thought developing entirely from itself in the particular individual we have no knowledge ; we know it only under the general relations and conditions, and with the general purposes of human Thought. Hence we can abstract neither from the particular manner in which our Thought receives material and contents from sensations and forms them into ideas of things ; nor yet from the way in which its history has been determined by the community of mankind. We abstract only from the particular nature of the occasions which at any given time give rise to a series of mental processes.

§ 3.

The possibility of determining the criteria and rules of necessary and universally valid procedure in Logic depends upon our ability to distinguish objectively necessary Thought from that which is not necessary, and this we find in the *immediate consciousness* of evident truth which accompanies necessary Thought. Experience of this consciousness and belief in its trustworthiness form a *Postulate*, beyond which we cannot go.

1. When we ask whether and how it is possible to solve the problem as we have stated it, the question resolves itself into the difficulty of finding an infallible sign by which we may distinguish objectively necessary and universally valid Thought from that which differs according to the person thinking, and thus, as we saw, misses its aim. And here there is, in the last instance, no answer but an appeal to our subjective experience of necessity, to the inward feeling of certainty by which some of our Thought is accompanied, to the consciousness that, starting from the given premises, we cannot think otherwise than we do think. Belief in the truth of this feeling and in its trustworthiness is the last anchorage of all certainty; for the man who does not acknowledge it there is no knowledge—nothing but accidental opinion.

2. Certainty of the universal validity of our Thought rests ultimately upon the consciousness of necessity, not the consciousness of necessity upon certainty of universal validity. When we assume a reason common to all men we are convinced that what we think with a consciousness of inevitable necessity others will also think in the same way. Our assumption that other people are bound by the same laws as ourselves, may indeed be strengthened by empirical confirmation when all agree; but this cannot supply the place of the immediate feeling of necessity, much less give rise to it. Habit, also, to which the empiricists appeal, and coincidence between experience and our calculations, affect only the validity of the data from which we start; they can neither produce nor modify the specific characteristic of mental necessity. Here, then, we have the fundamental fact upon which every logical system must rest. Logic can do nothing but bring to our knowledge the conditions under which this subjective feeling of necessity arises, and express them in a general form. It may be said that if this be so, then Logic is an empirical science, and this is true in the same sense in which it is true that mathematics is an empirical science. Mathematics also starts from facts

of consciousness and the necessity by which they are accompanied, but both are distinguished from merely empirical science in that they find in their facts the necessity which is wanting to accidental experience, and make this the basis of the certainty of their propositions.

§ 4.

By stating the problem in this way we find the course of our investigation determined for us. First we must consider the nature of the function for which rules are to be found then we must lay down the conditions and laws of its normal action; finally, we must try to find rules for the process by which natural Thought working with given presuppositions and expedients may pass from an imperfect into a perfect state. Thus our investigation falls into three parts; *analytical*, *normative* and *technical*.

1. Since it has been shown—§ 1. 8—that the activity in which all intentional Thought fulfils its purpose is *Judgment*, we must begin by a right understanding of the nature of this function with the correctness of which we are concerned, and by recognising what elements and presuppositions are involved in it. This is all the more necessary because the judgment takes the same form whether the Thought expressed by it be successful and universally valid or fail to attain its purpose. Only in so far as Thought takes shape in judgments and finds, or seeks to find, in them its final expression can we have truth and error, certainty and doubt, agreement and dispute. It is the same function which is in the one case correctly carried out, in the other incorrectly; and not until we know in what it consists can we give the rules for its correct operation.

This knowledge can only be gained by an analysis of actual judgment, by considering what it is that we do when we judge, and what other functions, if any, are presupposed in Judgment. We must also consider the process by which Judgment is formed from these other functions, and the general principles by which

this process is naturally governed. Meanwhile we must assume for the present that we know what mental acts are included under the name of Judgment ; language will serve us here for a guide, and the immediate object of our investigation may be defined as all propositions which claim to be true and to be believed or acknowledged as valid.

Of the propositions enumerated by Grammar, we will for the present put aside all which, like Imperatives or Optatives,¹

¹ No doubt the Imperative also includes the statement that the speaker wills the act which he commands ; while the Optative states that he wishes what he says. But the statement is here contained in the *fact of speech*, not in what is said ; it is just as true that in every utterance of the form *A is B* the mere fact of speech involves the statement that the speaker thinks and believes what he says. All speech alike is accompanied by such statements concerning the subjective state of the speaker, which are implied in the fact of his speech and are valid on condition of his veracity ; we cannot therefore found any distinction amongst propositions on these. The Imperative "Silence !" is of course a way of varying the expression of the statement "it is my will that you should be silent," but its purpose is not direct communication of this fact, but the determination of the will of the person spoken to, it does not call for belief but obedience. I have spoken of it as containing an intransferable element because the person addressed does not repeat the act of will of the person giving the command in the way in which he appropriates the thought of the speaker when he believes a statement.

No essential difference occurs in this primary and usual meaning of the Imperative as the expression of a particular individual will when it takes the form of a general law. When the legislator addresses an Imperative to the citizens of the State or members of the religious community his relation to them is that of one individual to another. His purpose in speaking is not to communicate a truth which they are to believe, but to announce a command which they are to obey. It makes no difference whether the source of the command be an actual individual or a collective body, whether the assumed motive of obedience be submission to personal authority or an impersonal constitution—the import of what is said is not the communication of a truth but the summons to do this, to leave that undone.

The form "Thou shalt" again, which appears in laws such as those of the Decalogue, has originally no other significance. Shall is the correlative of will. In delivering the command of master to servant we say "thou shalt do so and so" ; it contained originally no more than the simple Imperative, the revelation of a law to one whom I hold to be subordinate to the will of another, either of a third person or myself.

But this form does now contain a twofold meaning not to be found in the simple Imperative. "Shall" (sollen=shall+ought) may also have the force of a proper Predicate in a statement meant to be true ; it signifies duty, obligation, and is thus a modal Predicate (see below § 6. 3 d) expressing the relation between the subjective individual will and an authoritative power, or an objective law. The original Imperative here passes over into the significance of a Predicate expressing the obligatory relation of a command to the will to which it applies ; and if we assume a legal or moral constitution, the statement that I am under an obligation may be true or false. In "may" again we also find two meanings. "You may" is originally the expression of momentary per-

contain an element peculiar to the individual and intransferable to others. All those again which while they refer to an assertion, do not present it as true, may for the time be disregarded; such are questions, or propositions which express merely an opinion or a subjective view. How far the latter come into consideration as preliminary to the judgment, further investigation alone can show.

But all propositions of actual statement or declaration, no matter to what they refer, must be investigated by us. In this we follow Aristotle,¹ and reject the distinction of the so-called logical judgment from other statements, according to which Logic would deal only with the subsumption of the particular under the general, and exclude the mere communication of facts. But such communications also aim at truth, and make claim to be believed; error and dispute arise in reference to them and hence they, as well as judgments of subsumption, must have the conditions of their validity investigated.² The desire to limit Logic to judg-

mission by which I liberate the will of another, disclaiming the determination of his action or inaction, and declining to be a check upon his will. It expresses merely the subjective fact that I have no wish to prohibit. The real import of the proposition is practical, and to this extent "you may" is akin to an Imperative. On the other hand "may" may also be the Predicate of an actual statement expressing the fact that no authoritative prohibition is opposed to an action, that the existing order of things permits it.

Finally the same ambiguity extends to propositions which bear the grammatical form of a simple statement. That portion of the penal code which says that whoever acts in a certain way will incur a certain penalty, is not meant, like the formulæ of a natural law, as a communication of what actually happens, it conveys a command. But when the law is depicted as taking effect the same proposition contains an actual statement; it states what regularly happens within a given state. Cf. here Zitelmann, *Irrthum und Rechtsgeschäft*, p. 222, sq. Bierling, *zur Kritik der juristischen Grundbegriffe*, ii., 259, sq.

Thus the mere grammatical form is no infallible indication that we have to do with a statement. A statement is nothing more than a proposition which is meant to be true, and of which we can ask whether it be true or false.

¹ Aristotle constantly mentions as the characteristic which distinguishes the judgment, the ἀπόφανσις, from other forms of speech, the fact that we can attribute truth or falsehood to it. *De interpr.* 4 (λόγος) ἀποφαντικὸς οὐ πᾶς, ἀλλ' ἐν ᾧ τὸ ἀληθεύειν ἢ ψεύδεσθαι ὑπάρχει. Again, *De anima*, iii. 6.

² Cf. on the other hand Ulrici, *Comp. der Logik*, ed. 2, § 72, p. 266, 267. Hegel, who calls the judgment the determination of the concept by itself, says first (*Logik*, Werke, iv. 69): "A proposition has indeed Subject and Predicate in the grammatical sense, but is not therefore a judgment. For this it is necessary that the Predicate should

ments of subsumption could have arisen only under the scholastic view of the nature of science, the view that Definition alone has any scientific value. Where it is recognised that particular facts are the basis and test of a great part of our knowledge, Logic must also deal with those judgments which express particular facts.

It also forms a part of our plan of investigation to take up the analysis of the judgment at the point where it takes shape in the natural course of Thought without skill or reflection on the part of the thinker.

2. Our investigation of what takes place in Judgment completed, we may then proceed to enquire as to the conditions which a judgment must fulfil in order to be perfect and answer fully to its purpose. In this way we may set before us an ideal, agreement with which will be the end and object of our Thought. That is, we begin by asking that Thought shall be necessary and universal, and apply this test to the function of judgment after discovering all its conditions and factors; there then result certain laws to which judgment must be submitted, and hence certain criteria by which we may distinguish between perfect and imperfect judgment. These laws, so far as they fall within the scope of a logical treatment in our view, may be summed up in two principal points. First, the elements of the judgment must be completely determined, *i.e.* fixed in concepts; secondly, the act of judgment itself must proceed from its data in such a way as to be necessary. Thus

stand to the Subject in the relation of conceptual determinations, that is, as general to special or particular. 'Aristotle died in the 73rd year of his age, in the 4th year of the 115th Olympiad,' is a mere proposition, not a judgment." But he adds decisively, "It would have something of the nature of the judgment only if one of the circumstances, the time of death or the age of the philosopher had been called in question and there had been *some ground* for affirming the given number. . . . Thus the communication, 'my friend N has died' is a proposition, and would be a judgment only supposing it to be questioned whether he were really or merely apparently dead." So that Hegel also agrees in calling every proposition a judgment, in so far as we can ask *whether it is true and what are its grounds*. Cf. also Franz Kern (*die deutsche Satzlehre*, 1883, p. 189), who errs only in directing his remarks against Logic instead of against a one-sided logical theory.

this part of our treatment will contain the doctrine of concepts and inferences as subject matter of the normal laws for the formation of perfect judgments.

3. But knowledge of the nature of ideally perfect Thought does not necessarily enable us to attain to this ideal state, nor even show us the way which leads to the goal. Hence we have also to consider how, starting from a given state, with the means placed at our disposal by Nature, and subject to the conditions imposed upon human thought, we may reach logical perfection. We are concerned here, therefore, with the methods of obtaining right conceptions and hypotheses which will serve towards the formation of judgments and inferences. This is the domain of the Art in its narrower sense, of the properly technical instruction to which the two preceding parts form a necessary preliminary. The most important place in this technical instruction is taken by the Theory of Induction as the doctrine of the method by which concepts and general propositions may be obtained from particular perceptions.

4. We believe that by thus apprehending our task and arranging our investigation, the different views which have made their appearance as Logic has been elaborated, are combined and justice done to each. One view has assigned to Logic the task of expounding the natural forms of Thought and the natural laws which it necessarily follows, and we also recognise the necessity of stating the natural laws to which all Judgment is subject, and of discovering the principles applicable to it as a conscious function of a particular kind. But we deny that this completes the task of Logic, which aims at being not the Physics but the Ethics of Thought. Others, again, have defined Logic as the doctrine of the normal laws of human Thought or Knowing, and here again we recognise this regulative character as an essential feature. But we deny that any knowledge of these laws can be attained that is not founded upon the study of the natural forces and functions to

be regulated by them, and we deny also that a mere code of normal laws can by itself be productive or fulfil the purpose for the sake of which alone it is worth while to construct a logic. Much rather do we hold it necessary that Methodology, which is generally made to take a subordinate place, should be regarded as the special, final and chief aim of our science. And since this Methodology must have for its principal object the growth of science from the natural data of knowledge, we hope to satisfy also to some extent those who endeavour to avoid the barrenness and abstract character of the formal scholastic Logic by making it include a Theory of Knowledge ; nevertheless we exclude all questions relating to the metaphysical significance of the processes of Thought, and keep strictly to the prescribed limits within which we regard Thought as a subjective function. We do not extend our claims upon it so far as to demand a knowledge of Being, but limit them to the sphere of that necessity and universal validity which, even in ordinary language, are always and everywhere regarded as the distinguishing and essential characteristics of what is logical.

PART I

THE NATURE OF JUDGMENT AND ITS
PRESUPPOSITIONS

LOGIC

PART I

INTRODUCTION

§ 5.

THE Proposition, in which something is stated about something, is the verbal expression of the Judgment. This originates as an active movement of Thought, and always presupposes that there are present to the person forming and uttering the judgment, two ideas—the Subject and Predicate ideas, which can at first be only superficially distinguished by saying that the Subject is that of which something is stated, the Predicate that which is stated.

1. The judgment which we find in the form of a spoken assertion appears in the first instance as a completed whole, a finished product of mental activity. As such it is reproducible in memory, capable of entering into new combinations, transferable by communication with others, and such that it can be perpetuated in writing to all time. But this objectivity and independent existence, which leads us to speak of it as stating, connecting or disconnecting, is apparent only, and the expressions merely figurative; rightly speaking, the judgment as such has real existence only in active judging, in that mental act of a thinking individual which takes place at a given moment, and the only possible way in which we can perpetuate the judgment as an active process in thought is by consciously repeating it. The judgment itself never attains to objective existence, but only its material sign, the spoken or written proposition, which, by being externally present for others and recognisable, announces that a certain mental act has taken place in active Thought.

Now the proposition, as this outward sign, presents two aspects for our consideration, which must be carefully distinguished from the beginning.

On the one hand it refers to its source, the mental processes of the person who gives utterance to it, and in so doing reveals his thoughts. On the other hand it addresses itself to the hearer by whom it is to be understood; he is called upon to interpret the outward signs, and out of them to reconstruct the thought expressed by the speaker. But the functions of the person understanding the spoken words are not the same as those of the speaker; although if we assume complete understanding, the final result in the mind of the hearer must coincide with that from which the speaker started. When I express a perception in the words "the castle is on fire," my starting point is the image of the burning castle. In this image I recognise the familiar form of the building, and the flames rising from it, and I describe what I saw by first separating these two elements, and then uniting them in the proposition. The person who hears my proposition must begin by uniting the ideas which are excited in him by the two words, and which are at first separate; only when this is done does he obtain the idea from which the speaker set out.

Grammar and Hermeneutics, which start from the spoken or written word, naturally tend mainly to the standpoint of the hearer; they direct their attention to those functions which are active in understanding, and treat of them in the order in which they occur in the hearer. But in psychological analysis, where our aim is to investigate the nature of thought in judging, the other side—the action of the speaker—is of primary importance; and this the more so because communication to others is not necessarily the object of all Thought which clothes itself in words.

For us, then, the investigation of the nature of the judgment consists in a consideration of the mental act which takes place when we are actively engaged in judging, and to which we proceed to give expression in words. And since every repetition of a judgment (whether mental or spoken) presupposes its original production, the cases with which we must deal are those in which our Thought produces a new judgment and provides it with its verbal expression (as, *e.g.*, whenever we give utterance to a new observation).

2. The process which goes on when I form and utter a judgment may be described for the present by saying that I state something about something.¹ There are always two elements present, the one is that which is stated, τὸ κατηγορούμενον, the Predicate; the other one is that about which

¹ λόγος καταφατικός ἢ ἀποφατικός τινὸς κατὰ τινός. *Aristotle's Anal.*, pr. I. I. We shall consider later on (§ 12) the view that judgments do not all have two elements.

the statement is made, or to which it refers, τὸ ὑποκείμενον, the Subject. But this description is merely superficial, and derived from language. Statement is an activity of the organs of speech, and the question arises: what is the mental process of Thought when we "state something about something?"

3. If we start from the spoken proposition we must begin by making a distinction. There are some propositions in which we mean only the words as such to be Subject or Predicate, the words as these particular combinations of sounds. Possibly remarks are made about them which are purely grammatical and have no reference to their meaning (Samiel is a Hebrew word, Contra is a preposition); or it may be that the proposition refers to the meaning of a word or name, (Oxide is a compound of oxygen; Alexandros is another name for Paris; Jagsthausen is a village and castle on the Jagst¹). We may for the present put aside these merely etymological and hermeneutic statements, and there then remain for our investigation only those propositions in which the words appear as signs of ideas, and are assumed to be understood by both speaker and hearer, *i.e.*, to be connected with the same definite ideas, so that the statement is not concerned with the words themselves, but with the ideas connected with the words.

4. Here then, if the statement is to have any meaning, each element, Subject as well as Predicate, must be some idea immediately present to my consciousness. At first sight, and speaking generally, the Subject-idea appears to be that one which is first present to me. Any object whatever which I can retain independently in consciousness is qualified to become by itself the subject of a judgment; it may be the immediate intuition of a particular thing, or an abstract idea, an object or an event. To it is added as second in consciousness, the Predicate-idea. It is essential that this should be taken from those of our ideas which are already known to us and named by words which are understood. It must be an idea which has been already received into consciousness, which is connected with a word enabling us to retain and reproduce it, and which is distinguished from all other ideas. I cannot say "this is blue, this is red," unless I am already familiar with the ideas blue and red, and can reproduce them with the

¹ It has been objected that the real meaning of this phrase is: That which is called Jagsthausen is a village, etc., hence that the proposition does not refer to a mere name. But for anyone as yet ignorant of the significance of the word, the only possible import of the proposition is to give him the meaning of a hitherto unknown word; he cannot connect any idea with the subject.

word. Judgment is possible only where a number of such distinct ideas are retained and easily called into consciousness. Hence conscious judgment presupposes that these ideas are already formed.

Now, it is true that some Thought is already involved in the process by which these ideas are formed. Whatever we may think of the functions by which we arrive at ideas of definite objects, or at any ideas which can serve for Predicates, it cannot be denied that it is necessary to distinguish between different sensations, to comprehend a plurality as a whole, to refer this whole as a unity to its manifold content, and to retain in memory the product so gained ; and these are all of them acts which we can only think of as analogous to conscious mental acts of the nature of judgment. But this activity which results in distinct and independent ideas, precedes our conscious and intentional Thought, and is governed by laws of which we are unconscious. If we begin to reflect upon it we are conscious only of the results of these processes in the form of ideas already possessed of names. The processes themselves must have been originally guided by a psychological necessity, since their course has been essentially the same in all human beings ; or else they have become so much a matter of habit, and attained to such mechanical facility, that they take place with unconscious certainty even within the conscious life. We must further presuppose the first beginnings and appropriation of language, since conscious and voluntary Thought takes place almost entirely through its aid. Properly speaking, then, that Thought by which ideas are first produced does not come within the scope of our enquiry, nor yet the investigation of the origin of language, and the appropriation of it by the individual, although we may be obliged to touch upon these questions as our analysis proceeds. Still it is necessary to take a survey of those ideas which are capable of entering as elements—either as Subject or Predicate—into our judgments, and to determine the relation between the mental fact and its verbal expression.

CHAPTER I

IDEAS AS ELEMENTS OF THE JUDGMENT, AND THEIR RELATION TO WORDS

§ 6.

IF we examine the contents of our Thought, that which can enter into our judgments as Subject or Predicate, or as part of Subject or Predicate, we find that it consists of:—

I.—THINGS, their Attributes and Activities, and Modifications of these.

II.—RELATIONS OF THINGS and of their Attributes and Activities.

These may be Spatial and Temporal, Logical, Causal or Modal.

1. Language itself in its classification of words seems to give us the clue to the different kinds of objects of Thought; a clue which was certainly made use of by Aristotle in his enumeration of the Categories as the highest classes of that which is thought and that which is. But the clue is not an infallible one. It is the peculiarity of the structure of language that as it develops its different forms vary in their functions. A special form is not coined for every new kind of idea; but as in the organic kingdom morphologically equivalent organs can discharge functions which are essentially different physiologically, so it is also with classes of words, such as Substantives, Verbs, Adjectives. Different kinds of words do not necessarily correspond to differences of meaning in such a way that we can be guided entirely by these external characteristics. While still keeping sight of the indications given by language, we must take a general survey of the nature of the objects of Thought, which alone will show us how far differences in forms of speech correspond to differences of mental content.

2. The most universal possession of humanity consists in that series of ideas which constitutes the world of the Existent, and though as a matter of fact this series is generally accompanied by language, we are forced to think of it as possible to every individual, even without the co-operation of language. To this series belongs, besides the idea of self, the idea of our

whole empirically known surroundings, as well as the idea of everything which we think of as existing in the same way as ourselves and the objects of immediate perception.

Ideas of particular Things form the material from which this world is built up, and the words by which they are denoted are concrete Substantives. These Things we think of as supporting Attributes which find their expression in Adjectives, and as developing with the course of time Activities and falling into conditions which are expressed by Verbs.¹

This distinction between the ideas of Things and the ideas of Attributes which inhere in them, and Activities in which they are engaged, must be regarded as a fundamental fact of Thought. That part of our judgment which is conscious and can be guided by reflection—and here alone can there be any logic—presupposes both this distinction and the necessity which obliges us to refer the different kinds of ideas to each other, and to regard every thinkable and independent object as the unity of a Thing with its Attributes and Activities. No doubt the impressions from which we

¹ The generality of the process by which the affections of sense are referred to Things, is in no way prejudiced by the fact that such reference may be uncertain in particular cases, and that a thing perceived in a certain phenomenon may be apprehended in various ways. Night, shadow, rainbow, wind, etc., are originally things in the fullest sense of the word—concrete and particular. It is scientific reflexion which first deprives them of this solidity, and causes us to look upon them as merely the results of certain relations amongst things. Hence, also, we avoid using the expression “substance” in this connection; for it is an expression which implies that a scientific reflexion, and a criticism of those ideas which arise immediately and naturally, have already taken place. Not everything which the ordinary consciousness—led by the analogy of its own mental processes—apprehends as a thing, is for that reason substance in the strict sense, or will permit of the conscious application of this category.

We need not enter here upon the difficult question as to whether the present verbal form, in all its applications, is founded upon one definite common conception, nor what that conception may be. To me it appears certain that according to the original distinction between noun and verb, the expressing of temporal action (in its widest sense) belongs to the latter; and the thought of movement and change which proceeds from, and arises in, the Thing, and which can, moreover, extend its influence to other things, forms the nucleus of that group of ideas which the verb serves to denote. In proportion as our imagination looks upon things as alive, even persisting states, such as lying, standing, remaining, etc., seem to be reflective actions—the result of a resistance to change which is active and almost volitional. Or at least it is looked upon, as is evident in the Greek *ἔσθηνκα, κάθημαι*, as the result of an activity. It seems to me, then, more correct to regard the concept of action as original, and to subordinate to it the concept of state, than to reverse the relation, as Wundt does. That according to our present treatment many verbs seem to have the value of Adjectival Predicates, in no way alters the original distinction. The same state of a Thing may be regarded either as its inactive Attribute, or as its continuous Activity—as, for example, at rest and resting, *ruber* and *rubeo*, silent and holding one's peace, etc.

gain the idea of light and that of the light-giving object, or the ideas of hardness and coldness, and those of hard and cold Things, are the same in each case: but we cannot in conscious Thought force ourselves back to a point prior to the distinction, any more than we can talk in the roots from which the forms of Verb and Noun have developed. What the linguistic forms of Substantive, Verb and Adjective imply is just this unity in difference. Every Verb implies a Subject, every Adjective a Substantive, and only when they are thus supplemented can Thought come to rest in a relatively complete act, having attained a whole which is thinkable independently and by itself. In this process the part of the Substantive is to denote more especially the unity, a unity, however, which tends constantly to develop itself into its elements; the Adjective and Verb exhibit these elements as abstracted from, but always striving back to unity. Thus wherever we find our ideas denoted by various forms of speech such as Substantives, Adjectives and Verbs, there Thought has been busy, distinguishing and unifying according to the categories of Thing, Attribute and Activity; our way of speaking is governed by our habit of bringing everything under these categories. Only in a few onomatopoëtic words at the most—such as pop, splash—can we reproduce an impression which has not yet been seized upon by categorizing Thought.

The antithesis between Verb and Substantive claims priority, both actually and etymologically. Even if it were true that roots in their original signification are of the nature of verbs, and that names were first given to events, changes and movements, this would prove no more than that quick movement and activity exercise the strongest stimulus, and excite the accompanying sound more easily, not that the idea of activity in general preceded the idea of the agent. Movement, the fundamental intuition upon which all ideas of external activity are grounded, cannot be perceived without a localization of the moving object and its background, and a comparison which presupposes that stationary images have been seen and retained by memory.¹ It is just in movement that the identity of the agent in its activity, as the distinction of the permanent Thing from the temporary event, is most easily grasped; it is more difficult in beginning and ceasing—in change of attributes. For the attribute expressed by the adjective when it has a purely sensuous signification—*e.g.* colour—is not distinct from the idea of the Subject, it is permanent like it, and what we

¹ This agrees with what Steinthal says, *Abriss der Sprachwiss.*, I, 396 sq.

perceive of the thing is just its Attribute. It is the plurality of Attributes, their different combinations in different Things, and their changeableness in one continuously intuited object, which first impels us to disengage them from one another, and makes us able to think of each independently; and it is the repetition of action which first impels us to express its permanent ground by an Adjective. Thus there arise the two classes of Adjectives; those which partake more of the nature of Nouns, and those which approach more nearly to Verbs.

While the ideas of Thing and Activity are so bound together that an action or attribute must always be the action or attribute of some Thing, and a Thing must always be thought of as having a definite Attribute and Activity, we are nevertheless enabled by the distinction to think of an Attribute or Activity by itself, and disengage it from reference to any particular Thing. In this way they are thought of as abstract, *i.e.*, withheld in artificial isolation from the unity towards which they naturally tend. In this abstraction not only are they cut off from unity with definite Things, they are also raised to universality; *i.e.*, it becomes possible to refer them to and find them in any number of particular instances. These two processes—that of resolving a given complex idea into the different elements of Attributes and Activities, and that of forming from these elements abstract and general ideas—are mutually dependent, or rather we should say that they are one and the same process, and that their result presents different aspects. In my intuition of a stone as a round, white Thing, the ideas of round shape and white colour disengage themselves from this particular combination, and thus it becomes possible for them to enter into or be recognised in any other combination.

By the distinction of Attributes and Activities from Things we are enabled to think of the same Attributes and the same Activity as belonging to different Things; and a basis is also given for the comparison of similar Activities and Attributes in different Things, and the recognition of differences between them, which we think of as variations in degree or in mode. And just as Things are distinguished by their Activities and Attributes, so the similar Activities and Attributes of particular Things are distinguished by degrees and modes which we may comprehend under the name of Modifications. Here we have a new distinction and a new unity which finds expression in language in the relation in which Adverbs stand to Adjectives and Verbs. The etymological form of the Adverb also announces it to be a dependent element which must be united to the idea

of an Attribute or Activity; it is incomprehensible except in connection with such an idea as its more accurate determination.

So far as Abstract ideas can be retained independently, and form a centre to which other ideas may attach, language gives them a substantival form; they are Abstract Substantives, and their meaning consists in ideas of Attributes and Activities. This form implies that they are comparable with Things, in so far as their relations to Adjectives and Verbs are similar to those of Concrete Substantives. But they are not therefore really Things, and the unity between them and their determinations, as expressed in Adjectives or Verbs, is not that unity of inherence or action through which they themselves, as abstract, indicate a something which supports them. Except where Relations are involved, only the differentiation of a common element, the modification of the Attribute or Activity, can be thought of in connection with that Attribute and referred to it in a manner analogous to that in which it is itself referred to a Thing. The chief point which the two relations have in common is that in both there is contained a unification in the sense that where we have a substantival idea, its determinations and the distinguishing characteristics which it offers for comparison both occupy an independent position in consciousness, and at the same time maintain their unity with it. (The ball is round—the ball moves—the movement is rapid—the rapidity increases).

The one characteristic common to the ideas of Things and their Attributes and Activities which we have been considering is, that in all there is an immediately intuitable element, which is determined by the function of one or more of our senses, or by inner perception. This intuitable content never constitutes by itself the whole of the idea; it is seized upon by Thought which gives it form, and it is then retained as the idea of the Attribute or Activity of a Thing to which it is referred as to a permanent unity. This unity is as much a part of our idea as the element of sense intuition; but while the categories of Thing, Attribute and Activity are always the same, the product of sense-intuition, or of imitative imagination, constitutes the real essence of the idea, and gives to it its distinguishing content.

3. It is this element which distinguishes the ideas of Things and their Attributes and Activities from the second main class—*IDEAS OF RELATION*. In these the ideas of Things are always presupposed; while on the other hand their content is the result of a relating Activity, and induces them from the first with a generality which makes it impossible to excite the idea of the particular by means of the corresponding words.

(a) The relations which are first and most easily comprehended, because already implicitly contained in our intuition of Things and their Activities, are those of TIME and SPACE. Right and left, above and below, before and after, are ideas which, as consciously distinguished elements of our ideal world, are due entirely to a subjective Activity, passing to and fro between Things which are already intuited in spatial and temporal extension; their content consists in the consciousness of the determination of this Activity as it passes through Space and Time, and is therefore independent of any particular reference. It is true that these relations are all implicitly contained in our ideas of Things as spatially extended and enduring in time—in the manifold which extends before us in spatial and temporal order, but they do not as yet come into consciousness independently. We have an idea of a spatial object in which there is right and left, above and below, and our intuition moves through space in these various directions in order to grasp the spatial image as a unity; but this does not necessarily imply that we are conscious of the movement and of its different directions. At first we are conscious of the result only, of the definite figure and its situation with reference to others. Relational words have no meaning for us until this Activity of movement to and fro itself comes into consciousness, until we distinguish one direction from another, the further reaching movement of sight or hand from the shorter: they presuppose a spontaneous movement of Thought supervening upon the immediately given material, and being thus disengaged from any particular sensuous affection they possess a generality peculiar to themselves. “Movement” must always be thought of in the last instance as movement of something, however faint the sensuous image of that something may be: but “direction” presupposes only that we ourselves describe a line in space and are conscious of its determinations. These relations are expressed by Adverbs of Space and Time, and these, when used to denote relations thought of as connected with the objects to which they refer, become prepositions or case-suffixes, or blend as prefixes with Adjectives and Verbs; in other words, again, such as “follow,” “fall,” a spatial or temporal relation is blended with the meaning of the word, and finds no separate expression.

The relation of Whole and Part is also derived from spatial relations. Intuition begins by a process of limitation and distinction which disengages what we perceive as a single Thing from the surroundings by which it is accompanied in immediate sensation. It is in this way that we obtain images of men and animals, their power of movement compelling us to

distinguish them from their background, and for this reason also, because their shape facilitates complete limitation and distinction, we regard trees and stones as unities. But within this original unity there appear new differences, and we can describe new boundaries, and thus there arise subordinate spatial unities within the circumference of the first. Their relatively independent power of movement causes the limbs of the human or animal body to appear as such unities, and when we break a stone the separate pieces are perceived as distinct by the same intuition to which it is still present in its original shape. Now, when we thus break up a whole, what we have at first is merely a plurality of new unities, new things to fix limitations to. The fact that we have an idea of the head as well as of the whole body, of the finger as well as of the hand, does not necessarily make us think of the head as part of the body, or of the finger as part of the hand. Immediate intuition or reproduction may even proceed to complete the idea of the head by that of the body, the idea of the finger by that of the hand; but not until we are conscious of the relation between the subordinate unity and the higher, not until we reunite what we have separated, and refer the two processes to each other, does the head appear as part of the body, the finger as part of the hand. No doubt the idea of the relation, of belonging to a whole, is more intimately connected with the idea of such things as the limbs of the body, which we always perceive as parts, and never as isolated wholes (hand, arm, limb, etc.) than it is with objects which may be indifferently presented as parts or as independent wholes (*e.g.* flower as a whole, blossom as a part).

This relational idea is thus presupposed in every idea of magnitude. *A* is large in comparison with *B* if *B* is a part of *A*, or can be regarded (by means of contact, superposition, etc.) as a part of *A*. All comparison of magnitudes and all true measurement is grounded upon the observation or establishment of the relation of parts to a whole, and the first principle, "the whole is greater than the part," is really an interpretation of the idea "great." (Such words as great and high first appear as absolute predicates or Attributes after we are familiar with a definite standard of measurement.)

Again, the idea of the whole as a Thing with Attributes and Activities is not without influence upon the idea of the parts. The parts do not merely stand side by side in external juxtaposition, the whole does not merely surround them as a frame; a causal relation is implied, the whole *embraces* the parts, holds them together and *has* them. More of this hereafter.

The same differentiation takes place in the region of Time. The word breaks up into syllables, the melody into single phrases; and here also the ideas of temporal magnitudes, of longer and shorter, are developed in proportion as the temporal relations obtain an independent position in consciousness.

(b) These ideas have their origin in the relating Activity which moves in Space and Time, and their content consists in the intuitive consciousness of the movement through Space and Time. But their completion requires also the co-operation of functions of relating Thought, which gives rise to other ideas of relation resulting from distinction and comparison. The idea of Difference is not something which is given. In our consciousness of several distinct objects it is certainly presupposed that we distinguish them; but at first it is the result only of the function which comes into consciousness, that is the co-existence of several objects, each of which is independently perceived. The idea of Difference, of likeness or unlikeness, does not develop until distinction takes place consciously, and is accompanied by reflection. The idea of Identity does not presuppose merely that the same object was present for some time, or repeatedly; it arises first through negation of difference in the contents of two or more ideas which succeed each other in time, and it is this Activity which forms its content. Only in so far as it presents conditions and ground for this Activity can identity be ascribed to an object. Difference, Identity and Similarity are never to be regarded as mere abstractions from the intuited content, which can give nothing beyond itself; they are mental processes of which we have become conscious, and it is this that gives them their content. Number also has its origin in such mental processes, and arises when similar contents are spatially or temporally differentiated, and the distinguishable repetition of the same intuition comes into consciousness as such, while each step of the repetition is retained in memory and comprehended in a new unity with the preceding series. The fact that I see three things, and that the impression made by them differs from the impression made by two and one, is not sufficient to give me the idea *three*. The difference is not perceived by me as one of number until I count: *i.e.* consciously perform the act of proceeding from one unity to another.

(c) The third main class consists of the CAUSAL RELATIONS. These all contain, in infinitely varied modifications, the idea of Efficient Action, that is, of action which is directed upon some other thing (transitive verbs). Any explanation of the origin of the causal conception, and even the

question of its exact meaning, is reserved for later enquiry ; nevertheless we must indicate the position occupied by it in the totality of our ideas. This is not easy inasmuch as the close connection between efficiency and action impels us to regard the latter as a relation because the former is one, and in so doing we are in danger of representing the position of the agent towards his action as mere relation, and the action of the Subject as other than himself, an independent something of his creation. We are all the more liable to regard the position of a Thing towards its varying actions as a relation, inasmuch as the identity of a Thing throughout its changes cannot be thought except by a unifying synthesis, so that as a matter of fact we are forced to distinguish between the changes and the Thing. The impossibility of drawing any fixed line of distinction seems to find confirmation in two ways. When a man walks he moves his legs ; that which from one point of view is mere action, appears from another as an effect upon his limbs, which are relatively independent things ; and it is the same in all cases where there is room for doubt as to what is a single thing, and what must be regarded as a complex of different things. Even the inactive relation of the whole to its parts appears as an influence exerted by the whole upon the parts, or by the parts upon the whole. The whole has—*i.e.* holds—the parts, binds them into unity by its influence, the parts “form” the whole. Again, in view of the fact that what we ordinarily look upon as Attribute, such as colour, smell, etc., has been resolved by the progress of knowledge into an effect upon our sense-organs, much can be said for the proposition that efficacy and Attribute pass over into each other, and that Substance is causal in its Attributes. Thus Inherence and Causality become merely different ways of regarding one and the same relation.

But these considerations merely show the difficulty of deciding where the determinations of Attribute, Action and Efficiency can be applied with objective validity ; they do not obliterate the distinction between the concepts of Attribute, Action and Efficiency as distinct elements in our ideas. We may come to think that what we used to regard as an Attribute inhering in a Thing, such as colour, does not really inhere in the Thing, but is its effect upon our sensibility ; nevertheless it takes effect because of some attribute, which we must conclude to be there, even though it is not directly cognizable through the senses—the effect, that is, is due to the structure of its surface and its power of reflecting and absorbing light. And before it can *take effect* it must, before all, *be active*, it must enter upon a change of state, or some sort of movement. The fact still remains

that if we think of a definite Thing we must think of it as possessing Attributes which inhere in it, and constitute its particular nature, and can be predicated of it without reference to anything else. It is the same with Action. Our world would be a chaos, presenting no clear distinctions for our recognition, unless we think of it as a plurality of particular individual Things, each having its definite nature, and active in so far as it asserts this nature in Time, or changes and alters it, moves or grows. Of course in this action it is both determined by other Things which affect it, and affects other Things and determines their action, but that is not the question here. Efficiency cannot even be expressed without distinguishing it from Action. The antithesis is that between *causa immanens* and *causa transiens*. That which proceeds from the former is inseparable from the idea of the Subject, a mode of its being; that which proceeds from the latter cannot be thought except in relation to something else. We cannot therefore obliterate the distinction that the idea of Efficiency belongs to the ideas of Relations between different Things, while that of Action constitutes an inherent portion of the idea of the particular Thing by itself, and involves only the Relations of space and time without which no particular Thing can ever be thought. It is for this reason also that the idea of efficiency is never intuitable, since the passing over of Causality from one thing to another is an addition of the Thought which connects; only the Action itself, the change in the Things which are entering into Relation with each other, can be intuited.

The many forms in which this Relation finds expression can only be briefly indicated. It is most accurately and appropriately denoted by transitive verbs; but when these are regarded as proceeding from a permanent ground, adjectives are developed which indicate that a Thing is capable of producing an effect, is constantly doing so, and always prepared to do so. And when the idea of efficiency is joined in Thought to the Thing itself, and the Thing takes its name from the effect which it produces, then there arise the numerous substantives which denote Things according to their Causal Relation alone. From this there naturally results an incongruity between the substantival form, which indicates permanence and independent existence, and the fortuitousness and change of the Relation, and the possibility arises of confusing between what is true only of the Relation, and what is true of the thing. This applies to the expression "cause" itself. A Thing is a Cause only in so far as it takes effect, and at the moment in which it takes effect; but on the other hand

we give the name of Cause to a Thing having a permanent existence. Now with reference to the Relation it is quite correct to say that where there is no effect there is no Cause ; but it is incorrect to extend the proposition to Things which might become causes under certain circumstances, or are causes from some other point of view. This is true again in reference to the Relation of subject and object in the famous proposition "no object without a subject." If I mean by the word Object merely the Relation by virtue of which the name of object is conferred upon a Thing only when it is actually presented, then the proposition becomes merely a truism. But I may mean by Object everything existing externally to myself, or, indeed, everything which is other than my Thought, because under certain circumstances it may be presented to me ; and then it is not true that all the things ever presented to me vanish so soon as the subject withdraws and the Relation ceases. If it were so I must needs vanish myself when I fall asleep. We do not hesitate to say "I have been asleep," but "I" denotes a subject which is conscious of itself, consciousness disappears in sleep, hence "*I*" cannot sleep if by "*I*" is meant the subject in so far as it is conscious of itself. According to the theory "no object without a subject" I must cease to be during sleep. "Cavalry on foot" is an absurd contradiction, if cavalry means only men on horseback ; but if I mean men who serve in a cavalry regiment, then it is a matter of course that they sometimes walk. The proposition "no object without a subject" is true in the same sense that the proposition "a rider cannot walk" is true.

(*d*) The relation between ourselves as the subjects of psychical activity, and the objects of our subjective action—of our intuition and thought as well as of our desire and will—cannot be compared with any other. An object or definite content of Thought or Will, contains in itself as such all the categories we have been considering ; it is Thing, Attribute, Activity or Efficacy. But under which category shall we class seeing, hearing, intuiting, thinking, willing, etc., when we consider these functions in reference to their objects, and not merely as manifestations of activity on the part of the subject ? Do seeing, hearing, and thinking, belong to the Causal Relations ? They are not mere action, for they refer to something other than the subject ; but neither are they efficient, for they neither produce nor alter a Thing. Only that which never exists but in Thought (such as imaginations and fancies) comes under the Causal Relation of production and creation, and this only in so far as we may be justified

in regarding a thought or a dream as a "Thing." But what we can think of as in any way existing is not produced by our Thought; it makes no difference to the real Thing when we think about it; nevertheless we regard it as an object of Thought, and as standing in relation to it. By an extension of the Kantian phraseology we may call this class of Relations MODAL; it will contain all the relations which exist between objects and ourselves when we think of them, and desire, wish for, and estimate them according to their value for us. Thus it will include not only all the verbs which express an ideational activity with reference to an object, but also adjectives and adverbs, such as "true" and "false," which express the relation between my idea and the Thing to which it refers, or "beautiful" and "good," which express the reference of a thing to a standard of value, and therefore express an attribute belonging to the Thing as such only indirectly and when this standard is absolutely fixed. Finally it includes such substantives as "sign," or "purpose."

§ 7.

Every object of which we form an idea is thought of in one of two ways. We may think of it as having particular existence (*i.e.*, existing as a particular Thing, or as an Attribute, activity or relation of particular things) or as subject to the conditions of particular existence (as with the images produced by imagination). Or we may think of it without attending to its particular existence, and as general in the sense that as mentally represented it may enter into the thought of any number of things or instances. This mental representation finds its expression in the Word as such.

But when words are appropriated from existing language and employed to express the natural thought of the individual, their meanings differ with the individual and are liable to great transformations.

1. The ideas which have preceded the act of judgment in a judging subject are generally indicated by means of the words that denote them. Now there is no doubt that what language aims at is that a word should have the same meaning for every one; but in actual life this is far from being the case. It is more true to say that words have different meanings for different people, and for the same person at different times.¹ Hence in the analysis of actual judgment we must never start by assuming that a word has a generally accepted meaning without further investigation;

¹ Cf. Paul, *Principien der Sprachgeschichte*, ed. 2, p. 83.

we must always regard the word as being merely the sign of the idea present to the individual who is making the judgment.

2. The relation between the verbal expression and the idea which it denotes is not always the same. Some words (such as nouns and verbs) are connected with a definite ideal content, which constitutes their meaning as understood by the individual. Others—such as pronouns and demonstratives—denote nothing definite by themselves or by the mere sound of the word; all they serve for is to express a reference to the thinking and speaking Subject (or to that of which he speaks), and they do not therefore become the sign of a definite idea until this reference is understood by means of actual intuition. I and you, this and that, here and there, do not express by their sound the idea of a definite person, thing or place, although they are used to denote definite things or places. As the circumstances in which, and the people by whom they are used differ, they denote different objects, and these objects must be supplied from some other source.

3. But words which are significant in themselves are all of them, so far as understood, and in their primary and immediate meaning, no more than signs of ideas which are present to the mind, and can be reproduced by memory. Whether a word be a proper name or quite general in meaning it can never be of use until it has become capable of calling into consciousness a definite ideal-content, by the mere sound alone, and without help from present intuition. On the other hand we cannot have certain and firm possession of an idea, or use it in thought, unless we have a word by which to denote it; when the word is wanting to an idea we always feel it as a want, and have a difficulty in grasping the idea in its individuality and difference from others, in reproducing it with any certainty, and in guarding it from confusion. As a matter of fact the process of psychical development always is aided by language and greatly influenced by it, and every ideal-content, as it becomes a new mental possession, seeks a word to denote it. This is why we are always so anxious to know names, and are content to be answered by a name we have never heard before, in answer to the question "what is that?" Here we easily lend ourselves to the delusion of thinking that the learning of names adds to our knowledge of things, though we really gain nothing directly from knowing that this plant is called *Aristolochia* and that one *Clematis*. Still we have gained a means of recurring more easily to these things, of fixing them in our memory, and of afterwards extending our knowledge. Hence all progress

in knowledge is accompanied by a change and extension of scientific terminology.

4. Let us now consider the nature of the ideas which accompany our words. It is most important here to remember that the thought we are dealing with is that which takes place in the particular individual in the natural course of his mental development; and the idea as it is connected by the individual with one and the same word passes through stages of development which cannot be explained either by philology, which aims merely at determining the generally accepted meaning of the word, or by the ordinary logical treatment of the word.

5. Words are generally held to be the signs of CONCEPTS. But a Concept in the logical sense is a work of art, produced by a conscious elaboration of our ideas in which its characteristics are analysed and its definition fixed, and it is the work of logic to help us to attain to the ideal state in which words represent such Concepts. As a matter of fact most of our words only approximate to this state, and when we are treating of the beginning of judgment in the first appropriation of the simplest elements of language, it can only lead to confusion if we call every mental counterpart to a word a Concept, without further discrimination. If we did this we should have to follow Herbart's example, and give a much wider meaning than the ordinary one to the expression "Concept."

6. We seem able here to distinguish a twofold relation between the idea and the word. Some of our ideas—those, that is, which rest upon immediate intuition—form themselves up to a certain point without the help of language; and these ideas, developing independently in every individual, are the conditions under which speech first becomes possible. Here speech is added to an image which is already formed. But others amongst our ideas, *e.g.*, all those belonging to the region of the non-sensuous, are aroused in us by tradition, and the formation of these is occasioned and determined by the range of the Thought of the Community as expressed in the language we hear. The word comes first, and becomes only gradually richer and more definite in meaning in proportion as the individual enters into the Thought of the Community. The contrast between the two cases is, however, merely apparent. We never understand a word except by connecting it with our own ideas, and its contents consist in just those elements which the individual has really consciously grasped and retained. Even the immediate sense-intuition of the child is guided at an early stage by language; and on the other hand,

the most abstract terms are no more than empty sounds until their Content is independently reproduced in Thought. It is always a discovery when a thought of our own is found to agree with the meaning of a word as accepted in common language, and all our explanation of words aims at bringing about the conditions under which the ideas corresponding to them must be produced according to psychological laws. The real difference is merely that in the natural development the sensuous ideas come first and are all formed in more or less the same way, while the number of elements involved in the higher and more abstract ideas increases the variety of ways in which they are formed, and the individual differences of the products become more difficult to expound. But the general process by which idea and word become wedded together for the individual is essentially the same throughout. We connect the word with an ideal-content which must at one time or another have been self-produced, and it then passes through a series of developments in the process of which this content becomes enriched and modified.

7. Look how the child obtains the almost exclusively sensuous ideas, which belong to its first words, and enable it to make its first judgments. It always starts from the particular intuition of a thing or an event which is named to it; words are first understood in reference to particular cases. But in proportion as it is little practised in apprehension and scantily prepared by a previous store of ideas, the intuitive image which enters into memory, and is afterwards reproduced with the word, falls short of being a faithful and exhaustive copy of the thing presented to sense, and of containing all which might have been perceived in the object. Even what the man—unless a practised observer—actually sees in the presented object, what he receives in intuition and stores in his memory, is generally quite inadequate to the object itself. Hence when we first begin to speak the traces which remain of the particular object seen can be no more than a rough and faded copy of the thing, in which, as in a hasty sketch, only the more prominent features appear. Generally speaking then, we can have no idea of what image the child really connects at any time with the words which it hears. When an intuition occurs similar to those already retained in memory none of the conditions are present which would enable it to perceive a difference between the present and former objects; the fusion takes place immediately and is expressed by the fact that the new object is called by the name already learned. Children will give the same name to things only remotely similar so long as they agree in all or some

of those characteristics which have been securely apprehended, and this makes it possible for them to get on with few words. Here we may find the explanation, both of the wit which so often surprises us in child-language, and of the numerous confusions into which, as we think, they fall. Progress, for them, does not consist in subsuming something new under familiar ideas, but in learning to apprehend more completely and distinguish more exactly.¹

8. We may say, then, that in the earlier stages of the Thought of the individual the meaning of every word is connected with a particular intuition, and there is here no difference whatever between particular and general ideas. The memory image which remains from our first imperfect apprehension of an object is not retained by the mind as a fixed impression. Its reproduction is always the result of a new activity, and instead of speaking of images and ideas as of solid things lying in the memory as in a store-house, we ought really to speak of having gained habits and capabilities of reproducing ideas, such as do not exclude changes of more or less importance in every act of reproduction, and hence in the product itself. How often when seeing a familiar object—a house, or landscape—after an absence of some time, we find that its actual appearance is quite different from our recollection of it. This uncertainty of the memory-image, and the general law appropriately expressed by Beneke as the “attraction of the similar,” are all that is needed to unify it with a series of new images, and so to give it the function of a general idea. The process of continually naming new things—to confine ourselves for the present to substantives—serves on the one hand to fix the prominent features common to all, and on the other to keep the image fluid and shifting, so that now one, now another, feature can come to the fore and determine new associations. It is for this reason that in the natural course of Thought all words have a tendency to widen their sphere; their boundaries are indefinite, and always ready to open and admit new ideas of the same kind. This extension is continually favoured by the fact that in new objects we always notice and apprehend most easily those characteristics which coincide with an outline already familiar to us; we cover things, as it were, with our ready made images, and so conceal any new and distinctive features they may possess.

But side by side with this process there runs another. As we gain

¹ Cf. the very appropriate remarks of Steinthal, *Abriss der Sprachwissenschaft*, i., p. 148 sq., 401 sq., and Paul, *Princ. d. Sprachgeschichte*, ed. 2, p. 75 sq.

practice in apprehension we observe not only the more striking features, but also those which are less prominent. In this way our images become more defined and richer in content, and in proportion as their application to new objects diminishes, their number and our power of distinguishing between them increases. But in distinction of this kind wholes are compared with wholes: we do not begin by taking account of the differences in detail, and consciously sorting out the characteristics which agree and differ. We constantly and unhesitatingly distinguish between persons whom we know and do not know, without being conscious exactly *how* they differ. It is an unanalysed total impression, as immediate in its nature as feeling itself, which makes us recognise the familiar as such, and judge that the unfamiliar is not known.

It is not so much frequency of observation as interest, which determines the individual's attention and the accuracy of his apprehension. The images of objects which rejoice or terrify him, or which are connected with his needs and inclinations, impress themselves upon his memory in all their details; but where objects are indifferent to him, he does not trouble himself with accurate apprehension, and hence they leave only faded impressions which contain only the more striking characteristics and can fuse with similar impressions over a wide range.

In this way we may explain how it happens that of the images stored in his mind and connected with his words, some are well defined and rich in content, others less defined and more liable to change in meaning. He may give special names to the hen which lays eggs for him, the sparrow which is troublesome in his garden, the stork which builds upon his roof; all other varieties are "birds," and he cares nothing about the different kinds. Nor is he conscious that the idea "bird" includes in its indefiniteness all the specific kinds; we may hear others than children say "that is not a bird, it is a hen." Where there is no interest in distinguishing things the less defined and poorer image, which is derived only from the two principal characteristics of form and flight, is sufficient, and this extends to flying beetles and butterflies.

The history of language shows a similar development. Its roots are very general in meaning, not because the most general features were fixed upon from the first by a comprehensive process of abstraction, but because few distinctions were made, and only those phenomena retained and named which were easily apprehended and especially prominent. Particular things are generally named after some one of these phenomena; the

river from its movement, the cock from its crowing, and so on. Then as different aspects are apprehended in the things they receive new names, and there arise the numerous synonyms which cause them to be classified in different series of similar phenomena. Further specialization does not take place until, as language develops, derivatives arise, and words originally synonymous are applied to different specific classes of things and events; the more general meaning continuing side by side with the special. Contrary to the common doctrine of the formation of general ideas, general precedes special in the individual as in language, as certainly as the incomplete and indefinite idea precedes the complete idea in which careful distinction is presupposed.

A similar process takes place with regard to ideas of qualities and activities. Here, again, apprehension is at first of the most general kind, and deals only with considerable and easily distinguished characteristics. With the child—as with language—the first ideas of colours are few and uncertain in their differentiation, it is only gradually that sight is practised to distinguish differences which were formerly indiscriminately classed together; in movement it is the most familiar forms which are apprehended and unconditionally extended to all similar forms, only at a later stage are the manifold differences observed and denoted. How many kinds of movement are denoted by such a word as “going” or “running.”

9. Assuming that the idea connected with a word arises originally in this way from the intuition of a particular object, the incomplete and shifting image of which constitutes the first meaning of the word, we can now see in what sense Generality attaches to such an idea.

That any idea may be general, *i.e.*, applicable to any number of particular ideas, is involved in its nature as reproducible, and in no way depends upon its having been formed from a number of such particular ideas. As soon as it has disengaged itself from the original intuition with its spatial and temporal connections, and is a mental image which can be freely reproduced, it is capable also of fusing with a number of fresh intuitions or ideas, and of appearing as their Predicate in a judgment. If we confine our attention to the contents of the idea this kind of generality belongs not only to images such as those of the sun or the moon, but also to the images of particular persons. As often as the sun rises in the sky, or the moon becomes visible we have a new particular intuition, which is unified with the idea left by a previous intuition. The recognition of the material identity of all these suns and moons follows later, and, where continuity

of intuition is absent, by no means of necessity. In a similar way the reflection of a person in a mirror, or his portrait, is immediately identified with the memory-image, and here again our knowledge that these are merely images, and that the name belongs properly to one individual, comes later, and checks our attempt to treat the idea as general, in the widest sense ; so far, indeed, as concerns the idea itself, it is a matter of chance that it is not really general.

Thus it is not the special content of the idea, nor yet its origin, which determines whether or not it is general in the ordinary sense, but the fact that the idea is actually applied to a plurality of particular intuitions assumed to be copies of a real plurality of things, we being conscious of this plurality as such—the Singular must extend to a Plural.

10. This plurality is at first merely numerical. When a number of like or indistinguishably similar things present themselves contemporaneously or successively in intuition, not only is each one identified with the memory-image, but the fact that the content of the idea is the same each time makes it necessary to *count* in order to reconcile the external spatial or temporal difference with the sameness of the image. It is here that the antithesis between singularity and plurality first appears.

11. As a rule, however, it is not this numerical generality to which we refer when we say that words are general in their signification. What we mean is that their generality is such that they include *different* objects, which are distinguishable and are actually distinguished by their contents. In this sense the idea tree is said to be general with respect to oak, beech, fir, etc., the idea colour with respect to red, blue, and green.

Here we must be careful to distinguish between the generality of the *idea* and the generality of the *word*. So far as the actual meaning of words for the individual is derived from particular images, the very indefiniteness of an idea renders it applicable to things which differ, not merely spatially and temporally, but also in their content. A visible thing may be represented by copies of every degree of resemblance, from the few lines with which schoolboys draw horses and men in their books, to the perfect photograph, and in the same way we may have a series of ideas, representing the same object with a gradually increasing definiteness, all continuing to exist side by side. The more indefinite the idea, the easier it is of application. But until we become conscious of the difference between the various objects to which an idea once formed is constantly being applied, that idea is of the same kind as the idea of the sun, or an idea having

merely numerical generality. When the word grass brings to mind only a group of green, narrow, pointed leaves, and we disregard the differences between particular kinds of grass, then we merely see grass and all is grass alike; but as each apprehension becomes more definite and we notice distinctions between things which at first sight all coincided with a given idea, then, while the common name still remains, new ones are formed for the more fully determined ideas. In course of time these crowd out the more indefinite idea, which becomes too vague to be made vivid. The botanist has no ideal image corresponding to the word grass or tree; for him there arises a rivalry between differently determined forms which would be set down as alike by a less practised apprehension—a rivalry like that in the field of vision when different images are presented to the two eyes. To such ideas there is nothing in common but the word. The word is general in its signification in so far as it comprehends different things, and denotes a number of distinguishable images by elements which are alike in all. It is here that we are first called upon to realize the common element in difference; *i.e.* to form by abstraction the concept in the ordinary sense of the word.

The same process goes on with the more fully determined ideas. Here also, as apprehension becomes more acute and memory for small differences more faithful, the originally single image resolves itself into many. But language cannot keep pace in its derivatives, combinations and qualitative determinations with this process of specialization; memory also fails to be equally retentive of all particulars and imagination to lend an equal vividness to all images. So that for every word there finally remains a group of distinguishable ideas, all of which it serves to denote. They do not, however, all bear the same relation to the word; one image, more definite than the others, is specially connected with it, as the centre of the group round which the others gather. In a neighbourhood where fir trees grow the image of the fir is that which a man primarily connects with the word tree; any other varieties he may know are less distinct and fall into the background. The word "red" is most immediately connected with an especially striking impression, easily distinguished from all others; as it becomes extended in application to further modifications of the colour it ceases to denote anything definite, and calls to mind now this shade, now that, amongst a number which occur to us as equally possible. The word has become general by losing its definite meaning and reproducing a series of shades which is not at first distinctly limited.

Each of these shades is a general idea, in so far as it is itself applicable to a plurality of particular intuitions, but their names (blood-red, rose-red, etc.) serve to remind us of the original process by which words derive their meanings from particular intuitions.

12. Essentially different from this natural development of the relations between word and idea is another process which is conditioned by the fact that in naming we are always under the influence of a language which is already there. The language in use hinders combinations which would naturally take place, and forces upon us others which would not. To declare the common element in all the things which language denotes by one word is a very different matter from declaring what things any given individual subsumes under the idea which he considers they resemble. Many words are for the individual mere homonyms, the essential similarity of the ideas which first caused them to be named alike escapes his notice ; and in the same way it is language which first makes us conscious of many similarities which would never have been discovered by one man carrying on his comparisons independently. On the other hand the language in use prohibits and destroys many similarities, and forces upon us distinctions which might never have occurred to the individual thinker. In the latter case the idea is forced into fuller determination, but in the former it is impossible to say how many really disconnected ideas may correspond to one and the same word. In Etymology we aim rightly at tracing the most remote connections, but our task then is quite different from what it is when we endeavour to realise the actual process of thought in the individual.

For the individual the meaning of a word is determined, not by its etymology, but by the thought of those objects to which it is applied in ordinary language. We never think, until we have been taught, that there may be something common to the cock on the barrel, the cocking of the trigger, and the cock in the poultry yard, which led to their all being called by the same name ; for us the three meanings are completely disconnected, the words have become mere homonyms. In the same way the original meaning of the words by which we denote most of the psychical activities is completely lost to us ; no one feels now the figurative, metaphorical force of words such as concept, judgment, conclusion.

13. If the words which we use are only signs of a definite ideal content, which in its detachment from present intuition has gained an independent existence in that it is capable of mental reproduction at will,

then it follows that by themselves, by their mere sound, they can never denote the particular as such, and as it is presented to intuition. In order that a word may be understood to apply to a definite particular object we have to make use of special expedients, such as possessive or demonstrative adjectives or gestures; or else we must be able to assume that though the reference to a definite particular thing is unspoken, it will be correctly carried out by the hearer. But when a particular thing is denoted by means of a word it is always because it is recognised as agreeing with the general idea expressed by the word; I can denote the thing lying before me as this book, or my book, only because the general meaning of the word book is applicable to it.¹

It is true that one class of words denotes particular things as such. This is either because the thing corresponding to the idea is actually to be found only once in the world, as in the case of sun and moon, sky and earth, or because a name has been given by express agreement to the particular thing as such, on purpose to distinguish it from other similar objects, as with the proper names of people, towns, mountains, etc. Where the meaning of these is still recognisable it may be traced back to general words, as in Mont Blanc, Neustadt, Erlenbach; but this meaning, which explains the giving of the name, is in most cases forgotten, and the idea aroused by the now meaningless name is only that of a definite particular object. Nor can they fulfil the function of comprehensible words even in this way unless the object has been intuited and remembered. The import of the name is related to the momentary intuition of the person or mountain, as the general word to the particular thing; it cannot be applied to anything present to sense unless the identity of the present intuition with the mental image is recognised. The proper name is distinguished from the general word only by the consciousness which accompanies it that the corresponding reality is a particular thing and always the same in fact.

Finally, there are also certain relational words of general content, which apply only to a single thing; reference to a single object forms part of their meaning, as with all true superlatives, or ordinal numbers. In so far as only their context can tell us *what* is compared or counted they are like demonstrative adjectives, which also can express their particular

¹ In reference to this Mill's analysis is superficial (*Logic*, bk. I., ch. 2), when he classifies the adjectives, white, heavy, and even the demonstrative adjective "this," as names of things. Cf. also Paul, *Princ. d. Sprachg.*, ed. 2, p. 66 sq.

object only by means of a Relation. The first of January 1871 is a single day, but it is *definite* only if we assume a definite system of enumeration; it differs for the Russian calendar and for ours, and the meaning of the expression depends further upon the idea of a series of days and years which is nothing more than an object of Thought.

§ 8.

The peculiar function of words makes it indispensable to the completion of the judgment that the predicate should be expressed in them; the subject, if not general, can dispense with the verbal expression.

1. It follows from what has been said of the nature of words that we must be careful to distinguish whether a word signifies merely the ideal content immediately denoted, or whether it is being used to denote a definite particular thing which is not indicated as such by the meaning, but which represents that meaning and can therefore be named by the word.

Upon this distinction depends the essential difference of the way in which words are related to the subject and predicate of a judgment. When, that is, a statement does not refer to the content of the subject term as such—as in definition—but to a definite particular thing, it is quite unnecessary that we should denote, or even be able to denote, the subject idea by a significant word. We may merely use a demonstrative—"this is ice," "this is red," "that is falling"; and we may substitute for this demonstrative a mere gesture, or even give utterance to the predicate without anything further,—the mental process will still be a judgment in which a statement is made about something.

We see this most clearly in the judgments in which all human judgment begins, those in which definite objects of sense-intuition are recognised and named. The child makes a judgment when it names the animals in its picture-book by pointing to them and pronouncing their names, and the exclamations elicited by a startling sight (father! fire! the cranes of Ibycus!) are proper judgments; it is only the verbal expression which is incomplete, not the mental process.¹

¹ Herbart, *Psychologie* (sämmtliche Werke), iv. 169. The first step is the sight of the thing; the idea immediately given by this arouses the previous idea, which fuses with it. The immediate perception gives the subject, it is the fusion which would be denoted by the copula; the place of the predicate is taken by the previous idea, which is aroused and fuses with the perception.

Paul (*L.c.*, p. 104) takes sentences in which what is said is the subject for both speaker and hearer, the situation being the predicate: "Some one sees, for example, that a child

2. On the other hand it is essential to the completion of the judgment that the predicate should be spoken. It is true there may be cases where (*e.g.*) an object is recognised for which the approximate word is wanting, and where, therefore, the mental process can find no expression; but for this very reason we regard the process as incomplete—as an immature production, and consider that judgment alone to be complete in which the predicate appears, denoted by a word. It is indeed essential to the predicate that the idea belonging to it should be none other than the meaning of the word, the ideal content which is connected with the word as such and has thus become ours. It is no matter whether this idea is general in the ordinary sense, or whether it is the idea of a particular thing. “This is Socrates” is as much a judgment as “Socrates is a man”; “to-day is the first of January” as much as “to-day is cold”; although neither Socrates nor the first of January are general ideas in content.¹ It is enough that they are ideas at all—ideas which can be reproduced by and with the spoken word.

is in danger; then he merely calls to the person in charge of it—‘the child.’ This only indicates the object to which attention is to be directed, that is the logical subject; that which the person addressed sees when she obeys this direction of her attention gives her the predicate.” But here, I think, we must distinguish between two things. The exclamation is, in intention, an imperative, not a statement; and it can only be understood as an imperative. For the judgment really passed by the person speaking—“the child is in danger”—finds no place in the words of the exclamation; at the most it makes itself felt in the anxious tone of voice. The object is to direct attention to the thing named. With this view it is simply named; thus the full expression of the thought would run thus, “take care of the child.” It is a distinction similar to that between the cry of alarm, “Fire!” and the word of command, “Fire.” The former is a judgment, of which fire is the predicate; the latter is an imperative. “Fire” is the object, not the subject, of the incomplete imperative; and the case is the same with “child” in the preceding example. By merely exclaiming “the child” all that I can communicate to any one as the object of his and my belief and assent, is that what I see or think of is the child. But then the word is predicate. In like manner the exclamation, “the rascal,” or “a Daniel, a second Daniel,” contains the judgment that the person thought of is a rascal, or a second Daniel; and on this is based the indignation or joy which is manifest in the tone of the exclamation.

¹ Volkelt holds (*Erfahrung und Denken*, 319) that in propositions such as “This is my father, this is the moon,” the predicate indicates the common characteristics of that which I denote as my father, etc., hence not the individual as such. It is certainly true that the relation between a proper name which is used as a predicate and its subject (according to p. 42) is similar to the relation between a general idea and an idea comprehended under it. It is similar, that is, in so far as the proper name (particularly in the case of changeable things) does not indicate a momentary state, but that which is *identical* through all states, or that which might with less accuracy be called the common element. It does not, however, follow from this that it is not the individual as such which is meant.

CHAPTER II

SIMPLE JUDGMENTS

BY a SIMPLE JUDGMENT we mean one in which the subject may be regarded as a single idea ; it does not include a plurality of independent objects, and may therefore be represented as singular ; the statement made concerning this subject takes place in one act of judgment. Two classes must be distinguished of these simple judgments : those in which the subject is thought of as having a particular existence (this is white)—NARRATIVE JUDGMENTS ; and those of which the subject idea is the general meaning of a word, nothing being stated about any definite particular thing—these are EXPLICATIVE JUDGMENTS.

I. NARRATIVE JUDGMENTS.

§ 9.

The simplest and most elementary form of judgment is that in which we name particular objects of intuition. The subject-idea is given immediately to intuition as a unity, the predicate-idea is mentally reproduced with its appropriate word, and the act of judging consists in the thought by which the two ideas are consciously unified (*σύνθεσις νοημάτων ὡς περ ἐν ὄντων*, Aristot., *de anima*, III. 6. 430a, 27).

1. It is easy to explain the inward process which corresponds to such a sentence as "this is Socrates," "this is snow," "this is blood"; or to the abbreviated exclamations "fire," "the stork," when these appear as the expression of immediate recognition. The object before us awakens an idea left by some former impression, and connected with the word, and the two are unified. That which I am looking at is, according to its content, one with that which I have in my idea ; I am conscious of this unity, and it is this consciousness to which I give utterance in the proposition. By this the judgment is distinguished from the kindred processes. In the first place it is distinguished from what has been called

unconscious fusion—we will not here discuss whether the expression is appropriate and descriptive of an actual process. Here the new image is supposed to unite at once with previous ideas in such a way that the product of the union is merely a repetition of the previous idea—at the most more vivid; there is no distinguishing nor holding apart of new and old, of that which is present and that which is remembered. With reference to this Herbart is right in pointing out that a judgment as a conscious act is possible only when such fusion is delayed and the two ideas are held in suspense, and that it therefore has its most characteristic form when a question or a doubt has intervened. As a rule, however, it is true that attention is chiefly claimed by the present, and only its utterance reveals that the idea already in our possession has become active, particularly in the case of the mere exclamation which accompanies recognition.¹

In the second place, the judgment is distinct from the merely involuntary reproduction of a previous image which might co-exist with the first without being unified with it. It might happen, for instance, that on seeing a fire I should remember other occasions of fire, but that each image being accompanied by its distinguishing circumstances, they should be prevented from combining into one, and being thus retained in their

¹ From the Denominative Judgments described above, in which the object presented is (according to his view) compared with others previously known and called by their name, Stumpf (*Tonpsychologie*, vol. i., p. 5) distinguishes yet another kind of judgment—the Judgment of Habit. It often happens entirely as a matter of habit that a phenomenon when seen or heard reproduces in consciousness both the appropriate name and the judgment “*x* is red, *x* is the note A”; “in which case the previously perceived object does not enter into consciousness at all, much less is compared with the present object.” I cannot however find sufficient reason for this distinction. On the one hand it is not, generally speaking, the case that in denominative judgments as I understand them—a present object is compared with previous objects in the sense that the latter are present to thought as differentiated particulars and their name transferred to the new object. That which is reproduced by the present object is only the general idea connected with the word, and there is no need of any express comparison in order to be aware of its coincidence with what is present. On the other hand it is clearly too much to say that what was previously perceived “does not enter into consciousness at all”—how else should a recognition take place? Only so much is true: that it is not necessarily as *differentiated* that it comes into consciousness. The process goes on so rapidly that I am not aware of each particular step; when I meet an acquaintance the memory-image which is necessary before I can recognise him pales in contrast with my present perception, but it must have taken effect in consciousness. Thus it becomes impossible to draw a line between judgments of habit and those which are not of habit; all that we can allow is that when we are dealing with familiar ideas which are well known and often applied, the process, which is always essentially the same, takes place more rapidly.

individuality present merely a number of similar images. The union can only take place when no such hindrance occurs either because all the accompanying circumstances are the same, or because the content of the idea is already isolated and raised into generality.

2. When this simplest and most immediate form of judgment—knowing in its original sense of recognising—takes place, it is presupposed that both ideas are undivided wholes, not consciously resolved into their particular elements. Immediate unification is thus distinguished from the other case in which a series of intermediate acts of thought is necessary in order that subject and predicate may be unified. When “snow” or “blood” denotes a scientific concept, the distinguishing characteristics of which are present in memory, we do not form a judgment at first sight, but investigate the object in order to ascertain whether all the characteristics of the concept are appropriate to it. It is only upon the ground of a process of inference that we place the object under the concept; *i.e.*, that we attribute to it the whole complex of qualities universally accepted as contained in the term snow or blood. This judgment, then, is manifoldly mediated; in it is repeated several times the process which takes place at once in the coincidence of two images through the unanalysable act by which they are brought together. Between these two extremes there lies a whole series of ideas which may be connected with the predicate terms, and corresponding to it a graduated series of mediation of the judgment. But what the judgment always states is, that the idea of the predicate agrees with that of the subject in such a way that the predicate, as a whole, is one with the subject.

It might also be held that there is a process of inference in the numerous cases where the predicate contains more than can be offered by the first intuition which gives rise to the judgment. When a child sees an apple and names it, then the predicate-idea includes also its taste and its quality as edible; and in the judgment “this is an apple” we might suspect an inference from the visible image to the presence of the other attributes. But previous experiences have made the association of the other attributes with the visible image so firm that there is no conscious distinction between them; the visible image at once awakens recollection of the other attributes, and it is with this enriched intuition that the predicate-idea is connected. The child does not make the inference “this looks like an apple, therefore we can eat it”; but the sight of it gives rise to desire, and both together reproduce the idea “apple” and lead to its being named. In such cases

also, then, we still have the simple coincidence between the present intuition and the remembered idea, and they are to be distinguished from cases in which further attributes only occur to us in consequence of the name.

3. Complete coincidence between a present and a reproduced image does not take place only when we recognise one and the same object as such; when, that is, we may add to the judgment which identifies the ideas, a consciousness of the material identity of the things which is not itself included in the judgment. It occurs also whenever there is no consciousness of difference between subject- and predicate-ideas, so that what is apprehended and consciously noticed in the object exactly coincides with the predicate-idea. This will always be the case where particular phenomena of the same sort are only distinguishable by special attention (this is snow—this is a sheep—this is a poplar); or, again, when the apprehension of an object is determined by a preconceived idea, and that part of it which comes into consciousness is exhausted by the predicate-idea; here the predicate-idea itself is not absolutely fixed, but is often unconsciously affected by the subject which happens to be present.

4. To these must be added other cases, in which we are conscious of difference, but are not led by it to an express judgment. Such are the judgments which merely make a comparison or note a similarity, and which frequently—as in fanciful or humorous comparisons—take exactly the same external form as denominative judgments: most of the metaphors of language also depend upon this process. Such, again, are the judgments in which the subject-idea is richer and more fully determined than the predicate-idea, but only that part of it is noticeable which coincides with the predicate-idea; judgments, that is, in which the predicate is a less definite and more general idea than the subject, which we know is not exhausted by it. This is particularly evident when I do not know the special name of the idea coinciding with my object, and am therefore obliged to be content with its more general name (this is a bird, a tree, a fluid); or when the special name is not so familiar to me as the more frequently used general name; for in the natural course of thought the predicate-idea which most easily connects itself with every image is that which is most like to it and most fully determined. Not until scientific thought begins is there any interest in subsuming under the most general ideas; ordinary thought, which is concerned with the particular, clings to the most concrete ideas at its command. (For logical purposes ideas which are expressed in

language by the more exact attributival determinations of a substantive, such as "black horse," "round leaf"—are just as much one as those which can be expressed by one word. Their comprehension into one whole has already taken place when they appear as predicate.)

5. In naming we naturally think first of the ideal content as a unity, but in the process of thought the predicate-idea becomes connected with the idea of *plurality* wherever we have the numerical generality of many vaguely remembered individuals, or wherever the meaning of a word consists in a number of slightly different ideas. When a word denotes a sharply defined individual image, this is accompanied by a number of individual images amongst which the new object ranks itself as another (this finds expression in the form "this is a tree"); when the meaning of the word has not this individual determination the generality of the predicate appears in our consciousness of other ideas which are nearly akin to the one which happens to be before us ("this is paper," "this is wine"; here the words paper and wine stand for a number of slightly different ideas). So far Herbart is right in saying (*Introduction to Philosophy*, I. 92) that the concept which serves as predicate always loses part of its meaning in that capacity, only so much of it enters into thought as can be connected with the subject which it determines. Of all the many ideas comprehended by the word, that one is pre-eminent which coincides with the subject.

6. These denominative judgments¹ have always preceded, when the definite object concerning which the judgment is being made is denoted, not merely by a demonstrative, but by a significant word. "This flower is a rose" includes a twofold denominative judgment; first, there is the naming by the less definite word flower, which has taken place previously, and of which only the result appears in the verbal expression of the subject; then the more exact naming in which the judgment itself consists.

¹ I choose this expression in order that I may have one denotation to cover statements which elsewhere are sometimes treated as judgments of subsumption (when the predicate is a more general idea), sometimes as judgments of identity (when the predicate completely coincides with the subject). In the simplest cases there is no clear line between the two; in both the process is essentially the same, *i.e.* we are conscious that what is presented is in its totality one with a previously known idea. Schuppe (*Erk. Logik*, p. 375 sq.), in his thorough and appropriate treatment of the subject, calls this process *pure identification*; but since it is seldom a question of absolute identity between the ideas of subject and predicate, I should prefer to avoid the expression. In the same way I avoid using the expression *subsumption*; for this, when applied in its strict sense, does not represent the general idea which is connected with the term in its popular use, but a logically determined generic concept under which we place a particular thing or a specific concept.

7. The habit of referring attributes and processes to things is so strong, that there are comparatively few instances in which denominative judgments about them do not also include a judgment of attribute or activity. Still our power of abstraction enables us by means of "that" and "this" to denote even the mere attribute or activity as such. "This is not walking, but running"—"this is dark-blue, not black,"—it is not things which are spoken of here, but colours and activities apart from things. There is however always the tendency to pass from the attribute or activity to the thing. Cf. § 11.

§ 10.

When the predicate of a judgment concerning a specified particular thing is a verb or an adjective, then the judgment contains a twofold synthesis.

1. The synthesis which unifies the thing and its activity, the thing and its attributes, in the subject-idea itself; 2. the synthesis which unifies the activity or attribute contained in the thought of the subject with the activity or attribute denoted by the predicate-word, that is, names it by the predicate word.

1. In uttering a judgment, such as: "this cloud is red"—"the stove is hot"—"the iron glows"—"the horse runs," we first express the unity of a subject with its activity or attribute, this being indicated by the form of the words; we then name the attribute or activity which we have perceived by unifying it with the general idea "red,"—"hot"—"glow" or "run." That which is given to perception is the red cloud, the hot stove, the glowing iron, the running horse; but we analyse the originally undivided whole of perception by distinguishing and separating the attribute and activity from the idea of the subject. We have recognised by its shape and position that what we see is a cloud, and this recognition is expressed in calling it by the definite subject-word cloud; we are struck by its colour, and hence this is easily disengaged from the whole. It is this colour which we call red, and ascribe to the cloud as its attribute. We recognise the thing which is running to be a horse; as presented to us it is in movement, it is running, but we distinguish this incident from the subject which under other circumstances is known to us as standing still. It is this particular kind of movement which we express as running. We have thus distinguished two constituent parts in the total-image, the thing and its activity; in each of them we find again an idea already familiar to us, and by uniting these two elements in our statement we express just what we

have seen as a unity of a thing with its attribute or activity. Thus the judgment presupposes an analysis, whilst its own work is to synthesize the different elements.¹

By this twofold synthesis the judgments which state attributes and activities are distinguished from the simple denominative judgments; in the latter it is the subject of an undivided whole which is unified with the predicate.

We have spoken of the generality of substantives when treating of the ideas of things; and what we then said applies also to the relation between the generality of the predicate-idea and that element in the subject-idea which corresponds to it. Such a relation may vary from the case in which we are conscious of a complete coincidence (as, *e.g.*, with sharply defined colours, "this lichen is sulphur-yellow"), down to cases in which the predicate-word is too indefinite to denote the fully determined attribute or activity of the subject, and can only be made to agree with the idea by means of further determination with adverbs and other distinguishing words.

2. The view we here take is opposed to that which would force even such judgments as these into the conception of a simple subsumption of the subject under the more general predicate. But the predicate which expresses an attribute is general only with reference to the attribute of the subject, never to the subject itself. The predicate which expresses an activity is general only with reference to its activity. We must distinguish attributes and activities in the subject before we can apply adjectival or verbal predicates to them. Simple naming is the answer to the question, "What is this?" But if we are to answer with an adjective or verb, the question must be, "What is the quality of this? What does it do?" Such a judgment presupposes, therefore, that we have distinguished between the action or attribute and the thing.

§ II.

In judgments which express the attribute or action of a thing, the movement of thought is not always the same; that which first comes into consciousness is sometimes the thing (the grammatical subject), sometimes the attribute or activity (the grammatical predicate). In the former case the attribute or activity is first distinguished as a part of the total idea, and then named; in the latter it is perceived first by itself and named, and then referred to a thing.

¹ Cf. Wundt, *Logik*, I. 136 sq. Also my treatment of the question in the *Vierteljahrsschr. für wiss. Philos.*, 1880, iv. 458 sq.

This last act—reference to a thing—is, under certain conditions, omitted, and in this omission we find the explanation of the so-called impersonals.

Strictly speaking, impersonal propositions are those only from which all thought of a subject-thing is excluded, not those in which a subject-thing is implied, but only vaguely indicated and expressed.¹

1. When the statement which ascribes an attribute or activity to a thing starts from immediate perception, two things may happen. I may perceive from the first the thing with its action, state and attribute, so that I analyse this total idea, and form my judgment out of it :—the leaf is faded, the iron glows, the balloon rises ; or I may perceive at first only that element which is expressed by the adjective or verb, a colour, a flash of light, a movement ; and then it is not until afterwards, by a second act, that I recognise the definite subject of the attribute or activity, and can name it : “there runs—a hare” ; “there flies—a dead leaf” ; “there sparkles—the Rhine,” etc

In the latter case the synthesis which first takes place is that which *names* the given phenomenon of shining, sparkling, movement ; the reference of the attribute or action to its thing is not added until afterwards. In such cases language will also naturally begin with that which is first present in consciousness, with the adjective or verb. The Hebrew custom of putting the predicate first is the immediate expression of a mode of thought which moves pre-eminently in sense-perception ; and in proportion as particular languages continue to be the immediate and unartificial expression of the active movement of ideas, they have retained their freedom of beginning either with the predicate or with the subject. French, which determines the position of the word entirely according to its grammatical character, is furthest removed from this original flexibility.²

¹ Cf. with this § F. Miklosich, *Subjectlose Sätze*, 2nd ed., Vienna, 1883 ; W. Schuppe in the *Zeitschrift für Völkerpsychologie und Sprachwissenschaft.*, vol. xvi. 3, 1886 ; my essay, *die Impersonalien*, Freiburg, 1888, and Steinthal's review, *Zeitsch. für Völkerps.*, xviii. 170.

² It might be maintained that we must always regard that which first enters into consciousness as the logical subject, because it is the given point with which a further element is connected. But it would be a doubtful proceeding to ground the distinction between subject and predicate only upon the chance priority of the elements of the judgment as they might happen to succeed each other in the individual, instead of upon the contents of the ideas themselves. The relation between the ideas which we denote by verbs and adjectives on the one hand, and by substantives on the other, necessarily involves the thought that the objective foundation and presupposition of that which is expressed in the verbal form is that which the substantive denotes. When we apprehend motion, etc., we are led by its similarity to other cases to think of it as something depen-

2. These two acts, the naming of a perceived attribute or activity, and the reference of it to its thing, are sometimes still more widely and obviously distinct. All that is immediately perceived is an impression, which is denoted from its resemblance to previous cases, by an adjective or verb ; the thing belonging to it being added in thought only through association and on the ground of former experience. This takes place especially with sensations of hearing and smelling. That I can predicate a sound or a smell of a visible and tangible object is due to an act of combination by means of which the sensation of the ear or the nose is referred to the object which makes itself known at the same time to the eye and the hand. We cannot here investigate how this combination comes about ; but in all ordinary cases it is so much a matter of habit, the visible signs of the origin of the sound are so familiar (as with crying and talking, the knocking of a hammer and the stamping of the foot) that we think the sound is immediately perceived by us as the activity of a definite visible thing. But when a noise strikes the ear without our being able to see the thing which produces it, this thing must be added in thought ; our judgment does not appear as the result of the analysis of a given complex, as when I say, "the leaf is yellow," but as the result of a synthesis by which the thought of a thing belonging to it is added to the sound which was given alone. In very many cases this association is perfectly easy and certain, and we are scarcely conscious of it. When I hear my dog barking at the door, the familiar idea of the dog is there as soon as the sound is heard ; my idea of it represents it in the act of barking, and my judgment, "the dog is barking," may be regarded as the analysis of the idea of the barking dog which has thus been put together by association. But it is different if the association is not certain, if I hear unusual or imperfectly defined noises, such as the cry of an unknown animal in the wood. Now there intervenes the question, "What is that crying?" and I am unable to put together any definite image. I am certain, from the analogy of my former experience, that the sound proceeds from a thing, but I cannot get any definite idea ; the synthesis referring the sound to a thing remains uncompleted, and the

dent, something which presupposes and demands a reference to a thing. By our choice of the adjective or verbal form, we indicate that there is a subject, of which the verb and adjective are to be regarded as determinations. Grammar is right, therefore, in retaining the substantive as subject, even when the concept of the verb is the first in the psychological series of which we are clearly conscious ; it contradicts the fundamental presuppositions of our thought, to predicate a thing of an attribute or activity. How far this rule is subject to apparent exceptions will be discussed hereafter.

thing can only be denoted by a "something" which is quite indefinite.

It accords with this that we are apt to look upon the sounds which we hear as independent objects, and to abstract from the things producing them. Having a longer or shorter duration in time, and definite limits, they are apprehended as distinct phenomena; and substantives like thunder, shot, whistle, call, etc., waver between being abstract nouns, which point to a thing, and concrete subjects which denote independent objects, and of which, in their turn, verbs may be predicated:—a call resounds, *e.g.*, where there is no reference to the person who calls. The same thing occurs in the region of the other senses. Coldness and warmth are on the one hand names of the attributes of a thing; on the other they appear as independent existences, the question of the subject to which they belong remaining in the background. Here, again, the synthesis by which thought adds a thing to every sensation (which can at first be expressed only by an adjective or a verb) is not complete—or, at least does not find complete expression.

3. By recognising the twofold synthesis in all propositions which ascribe actions or attributes to a subject-thing, we obtain the key to the solution of the difficult and much-discussed question as to the logical nature of the so-called Impersonals; more correctly speaking, of impersonal propositions.

Amongst the statements which contain a predicate—whether a simple verb or the verb "to be" combined with an adjective or substantive, without any expressed or definitely denoted subject, it is important to distinguish two classes; those which are really impersonals, and those which are only apparently so. Real impersonals are those in which the thought of the thing to which the predicate would apply is entirely wanting, in which we cannot even ask what the thing is. The others are propositions in which a subject, though not named, is nevertheless implied, although the idea may be indefinite and denoted only by the neuter pronoun or the terminal inflection. When we say in German "*mich hungert*," "*mich dürestet*" there is no room for the question what "*hungert mich*"; any more than a substantive can be added as subject to "*pu-det*," or "*pœnitet*." But when I say "*it is beginning*," "*there it goes*," "*it is over*," "*it is finished*," I always mean something definite, a series of events either expected or going on: a play, a piece of music, or a battle; and I assume that the person who hears me has his attention

directed toward the same thing, so that any more accurate denotation is unnecessary. Here "it" is a real pronoun, which is only chosen for the sake of brevity, because the express denotation of what I mean is superfluous, or perhaps owing to the nature of the thing meant, too circumstantial. In the same way when I say, "it is slippery, dusty, wet," I mean the roads; it would be difficult to name a definite subject in words, because of the indefinite extension of that which is slippery or wet; on the other hand, the subject is already indicated definitely enough by the nature of the predicates belonging to them. When we say "it is shady, it is full," we can only mean a certain space; when we say "it thaws"—snow and ice.

It is true that an imperceptible transition from one class to the other often takes place; and we cannot tell from the mere grammatical form whether or not the pronoun "it" or the personal ending of old languages still indicates a subject-thing to which the predicate applies. This explains why the two classes of so-called impersonals are often confused, although in extreme cases they are definitely distinguished. It explains also why it has been thought that for all impersonal forms we must be able to find a subject in the sense of a thing to which the predicate belongs as its attribute or activity, though all that could be finally found to serve as such a subject was an indefinite idea of the totality of being, an idea which no one really thinks of in speaking of a particular perception.

When real impersonal propositions serve to express something which is accessible to immediate outer perception,—it thunders, it lightens,—then we start from a simple sense-impression to which neither perception itself nor memory supplies a subject. When, for instance, I see a rocket rise or hear a carriage rattle over the pavement, the action immediately added to the sound or sight which was given alone is *naming*, the unification of the present impression with a familiar idea. This naming may take place by means of uninflected, onomatopoëtic words, which merely reiterate the special characteristics of the impression; or by substantives (that is thunder, that is lightning) which, wavering between concrete and abstract, leave it undetermined in what direction thought will proceed to follow out the event. But language, guided by experience of similar cases, offers verbs for the temporal event, and the present perception is expressed by means of the customary inflections, the justification of this being that the personal ending of the third person was without doubt originally a demonstrative adjective. If a substantive were added, it would interpret what was thus indicated, and would determine it more exactly as the

thundering thing. But if this reference which the verb implies cannot be actually carried out, then only the impression itself remains as the subject of the statement, and the termination can indicate nothing but the present impression. The reference to a subject-thing which is contained in the pronoun of modern languages is then an empty customary form; we cannot ask "what lightens?" and answer "it" in the sense of a thing, however indefinite. All that the impersonal proposition can do is to name the phenomenon present at the moment; the subject is nothing more than the particular flash of light itself.

This limitation becomes very clear in cases where we know quite well what the thing is which flashes or makes a noise, but omit any expression of it in language as a matter of course, because we are interested merely in what is seen or heard. "Es läutet," "es pfeift," "es klopft," we say in German, when we are quite certain as to the nature of the cause of the sound. What we are interested in is the signal we have heard and its meaning; there is no object in saying who makes the signal. In the same way "es brennt" lays stress upon the fact that a fire has broken out; of course something is burning, but the tacit subject of the verb is not this something burning, it is merely the fire itself as perceived by us.

Nor is there any doubt that this limitation of the statement to the state which is perceived or felt, applies also to the numerous impersonals which express subjective states of feeling—"Mich hungert, dürstet, mir ist heiss, mir schwindelt, ekelt, graut," etc. permit of absolutely no reference of these verbs to a subject of which they are the activity. Nothing is given but the present feeling itself, and this contains no reference to a thing giving rise to it.

Again, we find also statements which express in the passive form, and without any reference to an agent, an activity which has been perceived: "es wird gespielt," etc. Here again, all that we do is to name the event perceived without proceeding further to the denotation of the subject in which it takes place. For further examples I may refer to my above-mentioned treatise.

The way is already prepared for this separation of the synthesis of naming from the synthesis which refers the named phenomenon to a subject-thing, by the distinction between the verbal forms of substantive, adjective and verb. From verbs and adjectives we can abstract substantives which enable that which generally appears as dependent upon a thing to be thought of by itself; we can make use of infinitives (I hear talking, ringing,

etc.), which are quite impersonal; and in the same way a statement is possible of which the logical subject is nothing more than the event or state perceived at the moment.

These propositions are "without a subject" only in the narrower sense that a subject-thing is wanting; they are no exception to the general nature of the proposition which expresses a judgment. They contain the synthesis of a known general idea with a present phenomenon; and it is this phenomenon which is the subject, and which is indicated by the personal ending with its originally demonstrative significance.

But just because they name something which is present, such propositions also contain an implicit statement of the actuality of the event named; for it is always immediately assumed that the particular object of perception is something actual. This does not, however, make them existential judgments in the ordinary sense; when we say "it lightens!" we do not mean to predicate actuality of the lightning, but to predicate lightening of something actual. The naming of the present impression is the fundamental act, without which the proposition, as the expression of present perception, could never come about. When we say "it lightens, it thunders," we must have seen a flash in the sky and recognised it as lightning, we must have had a sensation of sound and called it thunder; but all that we directly say is that what we have seen is lightning, and what we have heard, thunder. It is true that the hearer goes through the same process as with an existential judgment. He first receives the general idea of lightning from the word, and by its inflexion is called upon to think of the flash of lightning as particular and present; to this general word he must add in thought the corresponding particular phenomenon. So far those who start with the completed proposition and explain it grammatically are justified in emphasizing this view, according to which the judgment declares the actuality of the lightning. In some cases of propositions derived from the original form this declaration of existence comes to the fore for the person speaking as well; when, for instance, his information is based upon memory or hearsay, and also in the future tense and in general propositions. "It often rains in the Alps" means—"raining often occurs"; it is the two-sidedness of the original form which makes this application of it possible.

§ 12.

Those judgments which predicate a relation of a definite particular thing contain a manifold synthesis. The connection which is brought about by

the relational idea itself takes the place of the unity of the thing and its attribute or activity upon which the judgments treated of in § 10 are grounded. Every relational idea presupposes at least two points of reference which are thought of as independent ; these it connects by an act of mental reference, though not coalescing with either. Thus a judgment stating a definite relation between given things, names the given relation by a general relational idea, and at the same time unifies the necessary points of reference with definite objects.

Logically considered Existential Judgments come under the same point of view as Relational Judgments ; what they express in the first instance is the relation in which an object of thought stands to me, the subject thinking and intuiting it, but the meaning of their predicate extends beyond this mere relation.

1. Judgments which state relations (A is equal to B , different from B , greater than B , to the right of B , to the left of B , earlier, later than B , etc.) contain a synthesis of another kind from that contained by statements which ascribe attributes and activities to a subject. These predicates remain distinct from the subject-idea ; no inward unification can take place between them. Their predicates never belong to the subject when thought of alone as this particular definite thing ; whether they are affirmed or denied of the subject nothing is changed in the idea of the subject itself. For my thought the sun is just the same sun whether it is to the right or the left of me, whether it is visible or invisible ; the different predicates do not affect the idea of the sun itself, as when I say, "the sun is pale," "the sun is blood-red," "the sun moves," "the sun stands still." The predicates considered hitherto, whether predicates of denominative judgments, or attributes and activities, form a part of the subject-idea. But in order to state a relational predicate I must pass beyond the idea of the subject, and must first place it in relation to something else, and then realize what kind of relation that is.

The peculiarity of relational ideas consists just in this—that they presuppose at least *two objects*, which are in the first place thought of as separate and independent of each other ; our ideas of these must be already there before a relation can be stated of them. Thus the unity which binds together the elements of relational judgments is of quite another kind from the unity of the constituent parts of a particular object which can be thought of alone ; it is contained only in the relational idea itself, and in this therefore lies the ground of the peculiar synthesis now before us.

It follows further from the nature of the connection between these relational ideas and the points of reference presupposed by them, that every relation between two objects *A* and *B* can be apprehended and expressed in two ways, according as we pass from *A* to *B*, or from *B* to *A*. The reference is always *mutual*, but the relations are of different kinds; they may be the same, whatever the direction, or they may be the opposite of each other—*A* beside *B*, *B* beside *A*, *A* equal to *B*, *B* equal to *A*; or *A* upon *B*, *B* below *A*, *A* greater than *B*, *B* smaller than *A*. It depends upon the movement of thought in connecting *A* and *B* by the relation how a given reference shall be apprehended or expressed. Thus, every relational judgment implies in its nature a second equivalent one, every relational concept has its correlative concept.

2. If we look for the psychological ground of the synthesis brought about by relations, we shall find it most easily in spatial relations, as these are immediately intuitable. It belongs to the nature of our idea of spatial things that we can never perceive them apart from their surroundings, and from these we have to separate them as particular things. In intuition itself, before any conscious reflection takes place, we combine the particular parts of all that is perceptible around us into a spatial image; and thus every particular thing appears to us as comprehended in a larger spatial whole. We are able to isolate a particular object—this tree, this house—and attend to it alone, and the fact that a great number of particular things are movable favours this isolation by enabling us to think of them as disengaged from any given surroundings. But wherever we may perceive them they always stand amongst others in one continuous space. As soon as we pass beyond the intuition of the particular thing, we find others already there occupying a definite position. We distinguish and become conscious of the directions in which we thus pass beyond one Thing to combine it with others—directions which originally all have reference to our own standpoint, such as right and left, before and behind, above and below; and in so doing we analyse the complex picture presented to us, and express it by means of general relational concepts which represent the particular kind of unity in which the spatial whole contains its parts.

When I say “the house is on the road,” the starting-point of my judgment is a complex image of the house with its surroundings. I notice first the building, and call it a house; I look further and notice its neighbourhood, and call what I see here a road. The position in which the

two parts of my image stand to each other is immediate contiguity, and I denote it by the preposition "on," which is used to name this kind of spatial co-existence. In the same way the propositions, "The stork is in its nest," "The dog is under the table," presuppose the same analysis of a given complex image into its parts, and their particular kind of spatial co-existence. It is the preposition, containing as it does the relational idea and applying it to what is given, which expresses unification into a whole, and which is needed by the hearer to enable him to unify in a definite way the parts presented to him. In order then that the judgment may be expressed, we must have a threefold naming of the particular parts, besides the unity contained in the thought expressed by the relational word. One of these namings preceded the judgment and appears in the word by which we denote the subject; the other syntheses are expressed by the judgment itself.

3. Now these various syntheses may be combined in various ways, and take place in different orders. This is mainly because we are accustomed to connect with every object the thought of relations in which it may possibly stand. When two objects, *A* and *B*, are presented to me side by side, I may look at *A* first; but since every object stands in spatial proximity to others, the idea of something next to it presents itself, and I proceed to make this second point of reference clear. On the other hand I may start from *B*, connect the relation first with it, and then attribute this relation to the second point *A* as subject. Finally, I may look at both together and determine their relation to each other. These different ways of proceeding are expressed as *A* next to *B*; next to *B*, *A*; *A* and *B* next to each other. This is most obvious in those spatial relations which start from myself as point of reference. A judgment such as "Socrates is here" proceeds from an intuition which comprehends Socrates and myself in one and the same space. Now in every intuitable idea of a space, the place which I occupy and a space surrounding it is posited, and this idea, which always accompanies me and which is expressed by "here," is added to the intuition now given and unified with it. But the space surrounding me must have something in it; it is the general possibility of something else, and in the present instance this something else is Socrates. Socrates occupies the vacant position of the "here." Thus our natural way of describing such relations as these, in which I am first conscious of my own position as point of reference, is to place the local predicate first. (To the right is *A*, to the left *B*, before *C*, behind *D*.)

On the other hand, it may happen that what we first observe is one of the objects—it is recognised as Socrates. But with this idea, as with that of every other spatial thing, we can at once connect the idea of a surrounding, of the proximity of other things. Socrates is somewhere; and this indefinite reference is now unified with the definite one; the space surrounding him is unified with my space, with “here.” In this case also, then, a judgment such as “Socrates is here” may take place in two ways: on the one hand, in answer to the question, “Who is here?” on the other, in answer to the question, “Where is Socrates?”

There are many predicates, primarily expressive of states and movements, with which relational ideas may be connected to determine them more exactly. The dog stands, sits, lies, denotes, in the first place, different positions of its body, which have reference to it alone. But the verbs themselves contain a further reference to that upon which it stands or sits, or its definite place, and the relational idea connects itself with the predicate as its more exact determination. In other verbs, such as “follow” and “fall,” a relation to something else is part of their meaning; hence in such statements the synthesis of the thing with its state of activity is also combined with the relation.

What is true of space-relations applies equally to time-relations. Here again, from the nature of our apprehension, every particular object appears to us in temporal connection with others, and as a member of a temporal series running parallel to other temporal series.

4. The relational ideas denoted by like, different, similar, etc., are less intuitable. Here the reference to something else is not given with the intuition itself, but is added by our thought, which can, at will, bring together the most remote objects for its purposes of comparison. Two things which are alike or different do not, apart from my reflection, form a single whole, which can be resolved into its constituent parts; the unity in which they are placed by the relational judgment arises from my consciousness of mental activities having reference to the content of the objects of thought. The immediate and evident certainty with which we apprehend and recognise likeness, difference and similarity in the simplest cases, is apt to make us look upon these determinations as if they were something sense-given, and to overlook the particular functions through which we become conscious of them—functions which always presuppose that several objects are presented and compared as to their nature. Here also the relational ideas, when taken alone, are quite without significance; it

would be meaningless to say A is like, A is different. Like and different are really predicates only when combined with a definite point of reference.

It is for this reason also that mathematical equations, of the form " A and B are equal," cannot be looked upon as judgments which state the same predicate of two subjects, such as " A and B are ten feet long"; for they cannot be resolved into the two judgments, " A is equal," and " B is equal." If we start from both subjects then the full expression is, " A and B are equal to *each other*," and here we have the two judgments, " A is equal to B ," " B is equal to A "; thus the real predicates are "equal to B ," "equal to A ."

A mathematical object again naturally calls forth the question, "What is equal to it?" and has a reference beyond itself to something else. In the same way comparisons between larger and smaller things are always forcing themselves upon us; then $A > B$ or $B < A$ according as one or the other magnitude is first noted.

5. The causal relations, which are expressed by propositions containing transitive verbs and their objects, are difficult to analyse because of the close connection between "action" and "efficiency." Let us start again with a definite intuition, which is to be narrated by the judgment, *e.g.* the intuition of a bull butting against a tree. Then at any moment that which is immediately given together with the idea of the subject is its action, which we can think of by itself as a definite form of movement. Butting, striking, hurling, grasping, etc., all contain the idea of definite forms of movement, which can be thought of without any reference to a particular object, and can therefore be referred to the subject simply as its action. But the judgment, "the bull butts," does not completely exhaust the image, in which the bull is accompanied by the tree; that which happens must somehow be expressed as a relation between the two. This may be done by merely determining the general form of the movement by its direction, as, for instance, by adverbs of direction (the bull "butts *against* the tree"—local significance of the case and the preposition). So far the only relation contained in the judgment is that necessitated by the spatial nature of the movement in which the action consists, if it is to be expressed as determined in the particular case. By specifying a definite object we merely determine the predicate-idea more exactly; the idea itself is not a purely relational predicate, it only includes an action which is made more complete by a relational idea.

But if we look to the result upon the object of the activity of the subject, the shaking and bruising of the tree, then to this extent the *causal* relation appears. This result is no longer confined to the action of the subject in itself, it extends to that which takes place in the object; the effect, as such, lies outside of the efficient agent. The general idea denoted by "butting" no longer contains merely a form of movement which needs a subject, but a movement which has the effect of shaking or crushing something else. When we look upon what takes place as conforming to the idea of butting in this sense, we must determine the idea more exactly by reference to a definite object, and this gives us the first two syntheses; the reference to the subject is the third.

In the case of verbs, whose nature it is to signify an effect, production, annihilation, destruction, etc., the causal relation is to be found in the meaning of the word itself; it is the generalization of definite effects upon various particular objects, and it necessitates a something which is produced or destroyed. "To cause," and "to cause something," are equivalent expressions; the idea of an object, which is affected by the activity expressed in the verb, and with which the given object is partially unified, is more or less definitely connected with the verb itself. The meaning of the word includes also the idea of the second point of reference, the source of the effect, and with this the subject is identified. The meaning is just the same whether I say, I eat, I eat something, or I eat food; whether named or not, the two points of reference are given with the verb. The point to notice is that now the process is reversed—the idea of action is included in that of causation, and the syntheses brought about by the relation are accompanied by the syntheses in the category of action. We may see how these syntheses may succeed each other from the questions, "Who causes *B*? What does *A* cause? What is *A* doing?"

6. The nature of this relation shows itself in the interchangeableness of the active and passive forms by which the same event can be expressed. When I say, "The stone is thrown," the event is not expressed as it first appears to us, *i.e.* as action of the stone (the stone flies). Instead of this first immediate statement there is substituted the more distant relation, which denotes this action as the effect of something else, and with the idea of which is connected definitely or indefinitely the thought of the whence of this effect. Thus predicate-ideas which are denoted by passive verbs cannot be subsumed under that form of unification which has for its ground the category of action; they are neither more nor less than

relational predicates, although they include an action having reference to the subject alone.

It is true that these simple and different fundamental relations are frequently concealed by infinitely varied forms and disguises of verbal expression. The forms of language as ordinarily understood, are far from coinciding exactly with the distinctions of the idea. "To suffer" is itself an active verb, and we generally forget that as such it represents the subject in the activity of endurance or of feeling pain; as a rule we think of it only in opposition to efficacy, as the relation to some efficient agent.

7. The predicate "to be," in the so-called Existential Propositions, must also be regarded as a relation—that is a modal relation—though it occupies a unique position.

In the first place, there can be no doubt that in their external form these propositions exhibit exactly the same structure as any other propositions, whatever their verb. "Being" is stated of that which is denoted by the subject-term, and thought of under the subject-term, and a definite unity is brought about between the subject and the general concept of Being. In these judgments, therefore, as much as in any others, a synthesis of distinguishable thoughts takes place. Nor can the question, "does *A* exist?" be understood except as expressing the doubt whether actual existence can be declared of the *A* we are thinking of, whether it is in accordance with truth to connect the thought of existence with it.¹

¹ Brentano (*Psychologie vom empirischen Standpunkte*, vol. I, 1874, p. 266 sq.) disputes the ordinary doctrine that a connection or separation of two elements takes place in every judgment. According to him, the essential characteristic of judgment is acknowledgment or rejection, which refers to the object of an idea; in acknowledgment and rejection consciousness is related to an object quite otherwise than when framing an idea of it. But in acknowledging and rejecting we are concerned sometimes with connections between ideas, sometimes with isolated objects. In the proposition "*A* is," the object of our acknowledgment is not the connection between a characteristic "existence" and *A* but *A* itself.

It is undoubtedly true that judgment does not consist merely in a subjective connecting of ideas, and we shall deal more fully with this below (§ 14). But that there is a kind of judgment which contains no connection of ideas at all, that in addition to judgments having two terms there are to be found judgments having one only, and that these judgments of one term are none other than existential propositions—this I cannot allow. When I form the idea of an "object" *A*, I am first conscious of it as an idea, as something thought; its primary relation to me is that of being the object of which I form an idea. In so far as I actually have an idea of it, I cannot reject it; and should I wish to acknowledge it, then all I could acknowledge would be that I actually have an idea of it. This "acknowledgment," however, would not be the statement of its existence. The very point in question is whether, in addition to my having an idea of it, it is further implied that the object forms a part of the actual world surrounding me, can be perceived

About the meaning of the predicate also there can be no doubt, if we start from its popular meaning as it appears before critical and philosophical reflection; though indeed we cannot define the concept of Being, nor derive it from other concepts, but can only contrast it with its opposite. "Being" is opposed to that which is merely an idea, a thought, an imagination. That which "is" is not merely produced by my mental activity but is independent of it, and remains the same whether I am thinking of it or not. Being belongs to it in the same sense as to myself; it is opposed to me, as I think of it, as something independent of my thought, something which is not made by me, but only recognised in its independent existence. But although the chief meaning is this independence of the existent, the judgment also includes an open or concealed reference to myself, as the subject which is thinking the existent and which can be actually affected by it.

The attempt to derive the thought of being from anything else is as vain as the attempt to find an explanation of self-consciousness in the unconscious. It is included in self-consciousness from the beginning; as often as we say "I" it is present in thought, although not expressly emphasized. In the same way it is from the first inseparable from the objects of our intuition and thought, for we never find ourselves conscious without being surrounded by a world of objects which *are*, in just the same way that we are. We know ourselves only together with something else which is, and as opposed to other things which are not ourselves.

No occasion presents itself at first for disengaging the idea of Being from this original connection with the consciousness of ourselves and of the objects before us, or for expressly declaring Being of ourselves and of the external world. This is because the thought never arises that it is

by me, and can take effect upon myself and other things. If I wish to maintain the existence of the object, I must connect this last thought with the mere idea. When I begin a proposition "the tower of Babel—" these words are in the first place a sign that I have the idea of the tower of Babel as it is aroused by the story in Genesis, and this same idea is excited within any one who hears me. So far the idea is simply there, and as such it can neither be rejected nor does it call for any acknowledgment. But now the question arises, "What is the meaning of this idea?" If I complete the proposition by saying "the tower of Babel exists," then I pass beyond the mere idea and state that the thing denoted by the words may be perceived in some place. If I say it does not exist, then what I have rejected is not the idea of the tower of Babel, but the thought that it is the idea of a visible and tangible thing. Thus what is acknowledged or rejected by me is not the idea of the tower of Babel, but the thought that the given idea corresponds to an actual thing—that is, a connection. On this whole question cf. my *die Impersonalien*, p. 50 sq.

possible for me not to be, or that the whole external world might not be ; it is quite superfluous to assert that I am, when neither I myself nor anyone else doubts it. Only in an advanced state of reflection can we attain to an express consciousness of our own being ; at first my being is indistinguishably included in the immediate consciousness of myself ; the only question is, in what state or activity do I find myself.

Immediate sense-perception does for external things what this immediate self-consciousness does for myself. If we reflect, and consider what it is that induces us to acknowledge the being of particular external things, we find that it is sensation. That which we touch and see is there, and when we try to realize what we mean by Being, we find that we connect with it the idea of being perceived, and of the possibility of being perceived, the capability of taking effect upon the sense-organs of a feeling subject. But being perceived is not Being itself, it is only its sign and consequence ; Being does not begin with being perceived, nor does it cease when being perceived ceases. That which is perceptible must be in order that it may be perceived, and the perception of a thing is only the most direct and irrefutable proof that it exists.

When we attribute Being to insensible or supersensible things, as in the ontological proof of the existence of God, or in the conception of things *per se*, we have always a difficulty in freeing ourselves entirely from the spatial ideas which accompany the thought of being in the world of sense. We say that "*there is a God*," and when we try to impart greater reality to the thought, we can only do so by including in it an effect upon a perceptible world, in and through which there is an effect upon us by means of which the non-sensuous reveals itself and may be known. But this efficacy is not the origin of the thought "Being," it is merely a consequence of it, and hence the ground of our knowledge that the efficient agent is.

Here we see clearly the peculiar difficulty which this concept of Being brings with it. On the one hand, it cannot be spoken of at all without presupposing a relation to me, the person who thinks of it. I have an idea of the object because it has entered into some relation to me ; that it is, is my thought. But by this very thought I set aside this mere relativity and declare that the existent is, apart from its reference to me or any other thinking creature ; that its being does not wholly consist in the relation of being thought as an object of my consciousness. The Herbartian formula of absolute position in its double meaning combines these two aspects, with-

out however solving the difficulty ; but it has at least the merit of having made clear what it is that our natural thought really means when, careless of difficulties, it predicates Being.

If we wish to analyse and understand existential propositions, we must start from this meaning which they have in common use, before they have been critically approached ; and then the question arises : what do we think when we say *A* exists, and in what sense do we declare the unity of the subject and predicate ?

In order to answer this question, we must first see what is the general presupposition which gives rise to the judgment "*A* exists," as a judgment concerning a particular thing in the ordinary course of our thought. Evidently this presupposition is, that some doubt has been or may be raised as to the existence of the subject ; and this is only possible when that which the subject-word originally signifies is merely an *idea*, something which appears in consciousness either in the form of memory or as occasioned by communication from others. Concerning that which is immediately present, I cannot ask whether it exists ; certainty of its existence is given by the way in which it is intuited. But I learn from my experience of the passing and vanishing of things which I have previously seen in definite places, and from my experience of being deceived by others, that not everything of which I form an idea is also to be found in actual perception. Thus I am driven to distinguish between the intuition of that which is present and the mere idea, to which there is no corresponding intuition present. When I have lost anything, when I fail to find what I formerly possessed or knew, then, though I have the image of the thing in my memory, the present intuition is wanting ; it is not there, not forthcoming, not to be found. It is only the idea of the thing which is present in my consciousness as subject idea, and to this I seek the corresponding perception. Only in reference to such an idea is it possible to raise the question as to its existence, and the meaning of the question is whether that of which I have an idea still forms a part of the perceptible world.

Thus in every existential judgment the subject-word becomes a sign of something which is merely an idea, from the very fact that the thought of its existence is withdrawn from it in order that it may for the first time be expressly attributed to it. This happens as soon as I have found what was wanting, *i.e.* as soon as I have experienced the corresponding perception, or have convinced myself through some process of inference or through information from other people that it is still perceptible somewhere. Thus

all existential judgments within the sphere of the empirical world rest upon the distinction between the merely inner idea (of memory or of fancy) and present perception ; and what they assert is the identity of that which is perceived and the mere idea which is named as subject.

This is especially clear when my idea of the object whose existence is in question has only been communicated to me from other people. They create in me the idea of a Hercules or a Theseus, of the Tower of Babel, or of the magnetic mountain ; thus the question arises whether these have existed, whether therefore the ideas connected with the words are ideas of real beings or are mere imaginations, whether the information is based upon perception or fiction.

This throws light again upon the proposition so strongly emphasized by Kant : that the predicate "to be" adds nothing whatever to the *content of the idea as such*. Whether I say "*A* is" or "*A* is not," my thought of *A* is just the same ; the meaning of the statement itself demands that neither more nor less shall be present in the actual world than just the *A* thought by me. Thus "being" forms no part of the subject-idea, no "real predicate," as Kant says ; it merely expresses the relation of the *A* which is thought to my faculty of knowledge. The synthesis, then, which is primarily contained in the existential judgment referring to the empirical, is the identity between an object as idea and an object as intuited ; it is due to the fact that I may be conscious of the same contents in two forms, in the form of the mere idea and in the form of intuition. The thought of "being" is immediately connected with the intuited object.

So far existential judgments reverse the process of denominative judgments. In the latter the object is given as intuited, and is therefore thought of from the first as actual ; to it is added an idea which was known before, and the agreement of the two is declared in the denominative judgment. In the existential judgment it is the mere idea which precedes, and it is then said to agree with a particular object of intuition.

But while this relation—the agreement of the idea of a thing with a possible perception—is its chief significance, the meaning of the predicate "to exist" extends further. That which exists does not stand in this relation to me alone, but to everything else which is ; it occupies space between other objects, it exists at a definite time before and after other things, and it stands in causal relations to the rest of the world ; hence even of the perceptible we may assert an existence which is merely inferred. Herbart finds in the concept of Being complete unconditionedness and

absence of relations, and against this Lotze rightly shows that what we think of in the concept of Being is just this fact of standing in relations.

From this point of view the ever-present thought of an actual world around me is presupposed in every existential judgment; such a judgment merely supplies a definite subject for some place in this totality of the existent. It is always assumed that there is something external to me; the question is whether the object of my thought is to be found in that which is given, or whether something actual falls under a certain concept.

This last train of our thought leads to those statements in which the expression of being stands first, and which are thus akin to the impersonals. To some extent also they assume the outer form of impersonal propositions — *ἔστι*, there is, *es gibt* — these phrases first indicate something which exists, which is, is there, is offered to us by the world which is presented, in order to denote it more definitely afterwards. This form of the existential proposition, then, is the natural one, when the question we are dealing with is not whether a thing is present which we think of as definite and particular (perhaps because we knew it before) but whether a thing exists which falls under a given concept and can only be denoted as “an *A*.”¹

§ 13.

Those judgments concerning particulars in which the subjects are abstract nouns and the predicates, adjectival or verbal, cannot be brought under the categories of thing, attribute and activity. The first synthesis upon which they are grounded is to be found either in the unity of the attribute or activity with its modification, or in that view of a thing which attributes a predicate to it merely because of some given attribute, activity or relation.

1. The first, and, to an undeveloped thought, the natural way of regarding perceived events, is to refer them to concrete things, and to express everything which is and happens as attribute, activity and relation of particular things; there are few propositions in Homer of which the subjects are not particular persons or things. It is only when thought comes to distinguish more accurately and compare more extensively that we have any occasion to employ the attributes, activities or relations of particular things as independent subjects of a statement. This may be done either with the intention of distinguishing and determining more

¹ Cf. my *Impersonalien*, p. 65 sq.

exactly an event or attribute, or in order to refer a causal relation to one definite element of a thing.

2. In judgments such as "this red is bright," "the gait of this animal is a hop," there is presupposed the judgment of attribute or activity which breaks up the presentation into a thing and its determinations. The synthesis of the judgment consists partly in the synthesis of the attribute or activity with its modification, partly in the naming of the latter (cf. § 6, 2, p. 32 sq.).

3. Before an attribute or activity can be the subject of a causal relation the general idea of an efficient agent which is originally connected with a thing as cause must have been more exactly determined by a process of comparison; so that a thing is regarded as taking effect only by virtue of one of its attributes, or only in so far as it is engaged in a certain activity. When we say that friction warms, and weight is oppressive, the true subjects belonging to the verbs are the body by which the friction is produced and the heavy thing; these alone are capable of causing an effect. But by comparison our thought distinguishes that element in the body in virtue of which it gives rise to the effect, and expresses it by an abstract noun, because in this way the event can be presented as the expression of a general law.

4. Relational ideas also—such as distance, difference, etc.—can in the same way appear as the subjects of adjectives or verbs which express an effect. When the attraction of two bodies is diminished by their distance the words in which the change is stated attribute an efficacy to a spatial relation as if it were a substantial cause. But there is no need to prove that this is a mere abbreviation.

We know from general laws, which include both the fact of the efficacy and the conditions of its modifications, that a certain result will necessarily follow upon the change in distance, and this result is stated as the effect of the change itself. The higher the abstractions in which our thought and knowledge move the more unsuited to them do the original meanings of words and constructions become. Without our perceiving it, language, chiefly by the aid of abstract nouns, abbreviates and leaves unexpressed that which mental habits lead us to understand as a matter of course.

The complex relations of scientific laws, according to which the particular event depends upon many conditions, are forced into simple forms of expression in such a way that the efficient cause itself appears insignificant in comparison with the varying circumstances under which it

takes effect. The original idea of efficacy is attenuated into the uniform interdependence of different movements, and this is adequately expressed by a mathematical formula, but can only be represented in words by the aid of figures of speech and metaphors which we have ceased to feel as such.

§ 14.

The unification of different ideas does not exhaust the nature of judgment ; every complete judgment as such includes also the consciousness of the objective validity of this unification.

But objective validity does not, as might be supposed, rest immediately upon the fact that the subjective connection corresponds to the relations of the corresponding existent ; it rests upon the necessity of the unification.

This necessity has its root in the principle of agreement, and this again presupposes stability in our ideas ; but these logical principles can give not assurance of the real identity of things.

1. Those definitions of the judgment which limit it to the merely subjective connection of ideas or concepts, overlook the fact that the import of a statement is never merely to assert the subjective fact that *I*, at the moment, make such a connection. On the contrary, the judgment, by the form it assumes, claims that the connection is true of the objects themselves, and that just for this reason it must be acknowledged by every one. It is this which distinguishes the judgment from the merely subjective combinations of ingenious and witty comparisons which assume the outward form of the proposition, but do not aim, like the judgment, at making an objectively valid statement. In the same way the judgment differs from mere conjecture, opinions and probabilities.¹

2. But objective validity has more than one meaning. In the first place we must distinguish a verbal, nominal validity, from a material, real validity. When I say "this is red" the first question which may be raised is, whether what I call red is what every one else calls red. Then the objective validity, which may be denied of my judgment, refers to the general use of language, which is opposed as an objective rule, or general law, to subjective caprice. All verbal disputes turn upon the question of this validity ; such disputes are possible partly because the subjective meaning of words for the individual differs from that which is generally recognised, partly

¹ Ueberweg, for instance, § 67, gives a correct definition from this point of view, when he says : the judgment is our consciousness concerning the objective validity of a subjective connection of ideas.

because language is not absolutely fixed even in its general use, and because the boundaries of particular words are fluctuating.

3. But when the nominal correctness which is implied¹ in every judgment, inasmuch as it is spoken and meant to be understood, is present, when the speaker connects the same ideas with his words which every one else connects with them, then the point to observe is that the connection of ideas is stated as objectively valid, and the spoken proposition as true, so that every one is called upon to believe it and to judge in the same way about the same subject.

To determine the meaning of this material validity is not so simple as might appear when it is said that the same connection must exist between the corresponding objective elements as between the elements of the judgment; or, that what is thought must actually happen. For it is the peculiarity of our thought, when judging, that its processes are *incongruent* with the existent to which they refer. Let us confine ourselves to the judgments so far considered: those which ascribe attributes and activities to particular things, or denote things by a name. In the first place we find that there is nothing existent which agrees with the predicate-idea in the same sense as there is something which agrees with the subject-idea; for the predicate-idea, as such, is from its nature general and does not refer directly to any particular thing which is thought of as having a particular existence. All words (with the exception of proper names) are the immediate signs of ideas which, though formed from intuitions of the

¹ Marty has made the following objections to the view that every judgment contains the implicit statement that its terms are correctly used (*Vierteljahrsschr. für wiss. Phil.*, 1884, VIII., 1 p. 85). "When, in good faith, I state the proposition, 'This is snow,' it is, no doubt, presupposed that I believe what I call snow to be called snow by every one else. But we cannot say that this judgment concerning language is *implicitly contained in the statement*." But suppose I say to some one, "That is crimson," and he replies, "No, it is scarlet," does he mean to say that I am wrong as to the colour itself, and see a different colour from that which the object really has? or does he not rather mean that I am mistaken in my denotation, that I am calling crimson what, in the ordinary use of language, is called scarlet? Thus the judgment, "That is crimson," contains the statement that I not only see the colour rightly, but also give it its right name; for to this latter part only did the "No" apply. Marty goes on to say that there is no need for the judgment as to language (the agreement of my use of language with the ordinary use) to be present to consciousness in any form when I utter the proposition in question. "It is enough that it has previously been there, and that a habit of speech, which can now act by itself, has been formed on the *ground of our confident assumption of it*." Thus, according to Marty himself, my judgment includes a *confident assumption*; all that he maintains is that it need not always come into express consciousness. But what does this mean, if not that it is implicitly contained in the statement?

existent, do not represent it as particular, nor as it exists in particular given cases. Again, the judgment presupposes the mental separation of subject and predicate; it takes place in the recognition of the unity of two ideal elements which, for our consciousness, had formerly a divided existence. There is no such separation in the existent to which our judgment refers. The thing exists only with its attribute, the attribute only with the thing; the two form an undivided unity. In the same way a body exists only as at rest or as moved, its state is not separable from it in reality. Thus the general and the particular, predicate and subject, with their preliminary separation and act of unification, find no counterpart whatever in the existent; we cannot say that the connection between the elements of the judgment corresponds to a connection between analogous objective elements. Only when, by the act of judging itself, we do away with the subjective separation between subject and predicate, and thus think of the two as united, do we revert to the existent, which itself remains an undivided unity and never undergoes an actual separation corresponding to our mere distinction. There has been no "*distinctio realis*" corresponding to the "*distinctio rationis*." If, then, it is the essential characteristic of judgment to be a function of a merely subjective kind, its objective validity must have some other meaning than that of the agreement between the connection in the judgment and an objective connection; a meaning which can only be understood when we take into consideration the peculiar nature of our predicate-ideas.

4. The simplest judgments are those which are merely denominative-judgments, declaring the immediate coincidence of images, unmediated by subsumptive inferences. Presupposing nominal correctness, it is necessary to the validity of such a judgment, as ordinarily understood, that intuition and idea should coincide—a relation which is purely mental. It is also necessary that the subjective intuition, which claims to be the copy of an objective thing, should really correspond to this; *i.e.*, that the subjective image which is present should be that which, according to the general laws of our sensual intuition, would necessarily be aroused in everybody by the same object. The judgment, "This is snow," is objectively valid when that which is seen coincides with the idea called "snow" by every one, and when, moreover, it is seen distinctly by an eye in a normal condition. Objective validity then reduces itself to this: that both the process of forming the intuition and the act of judgment take place in a way which is universally valid. When there is agreement as to

the meaning of the predicate, any dispute which may now arise can only refer to the question as to whether the person giving utterance to the judgment, "This is snow," sees correctly; *i.e.*, sees in the same way as every one else, and under the conditions of correct knowledge. In particular cases this is purely a *questio facti* which differs with the individual, and cannot be determined according to any general rule. But the general question as to what authority we have for referring our ideas to real objects, and for attributing an existence independent of ourselves to what we perceive, does not belong to logic. The presupposition, upon which all ordinary thought is grounded, that we know an existent, may be affirmed in the realistic sense; or it may have its meaning so changed in the idealistic sense that Being merely denotes something which is thought of necessarily and by every one in the same way. But in either case the subjective functions, which are active in judgment, remain the same.

Concerning the metaphysical validity which we attribute to ideas, our Logic need come to no conclusion at present; it investigates thought as a subjective function, and can therefore decide nothing as to the significance of intuitions. But when an intuition and a predicate-idea are there, we cannot allow that it is possible for the inward act of unification to differ, that one man should posit like ideas as unlike, another different ideas as like; for we find within us immediate certainty of the necessity of our unification and the impossibility of the contrary, and should thus be forced to exclude from the community of thought any one who should arrive at a different result. In other words, the judgment has objective validity for us because it is necessary to unify ideas which coincide.¹

¹ It might be argued by one conversant with objective logic that after all the judgment, "This is snow," is intended to state something concerning the nature and constitution of a thing, and that the question as to its objective validity depends upon whether this is really snow or not. This would remind us of the question of the clever critic, "How do the astronomers know that the star which they call Uranus is *really* Uranus?" It is a necessary condition of all use of words that at any given stage of our knowledge "snow" shall, by general agreement, denote some definite idea, and that when we give names we shall be guarded from confusion by the fact that the distinctions in that which is presented are not more numerous than those of the ideas named. This being presupposed, then, however we may turn and twist the statement, that this is really snow, the question as to its objective validity still demands the answer given above. Instead of an idea like the above, which was sufficiently characterised by sense-intuition, I might take as the ground of my statement an exact concept with accurately determined attributes. Then the statement "This is snow," would mean, "This has all the characteristics of snow; it is white, it consists of crystals whose angles of contact are 60°; it becomes water at a temperature of 0 degrees." But this brings me no further with the question of objective validity beyond the statements: (1) That at the moment

5. We might be tempted to recognise the fundamental principle just found as that known in the traditional Logic as the PRINCIPLE OF IDENTITY; for this principle of identity is there regarded as the ground of validity in all judgments attributing a predicate to a subject, and hence as a fundamental law of our thought.¹

Unfortunately, the word identity has in the course of time received many meanings, and the so-called law of identity has been made use of in very different senses.

my perception is correct, my senses do not deceive me and convey impressions to me different from those which the same object at other times gives to me and to other people; (2) That the elements which I distinguish in this image exactly correspond in detail with the ideas—white, crystals, melt, etc.—with which I am thoroughly conversant, and which are denoted by me as by every one else by these words—the total idea thus corresponding perfectly to what I am accustomed to think of as “snow.” Further, I am certain, in the first place, that I have not forgotten what “white,” etc., means; and, secondly, that I do not identify blue or red when seen with my idea of white, that, on the contrary, it is necessary that I should unify my perception with the idea. There is no other objective truth and subjective certainty for this proposition, nor can there be so long as the universal—as such—has its existence only in my mind, and only the particular in reality.

Again, it might be said that the proposition, “This is snow,” means that the thing present is like or similar to other particular things which I have previously perceived, and that it is this material likeness of existing things which forms the contents of my judgment. But though this is indirectly contained in the judgment, it is so only in so far as these particular things are also stated to be snow; the judgment would only be multiplied by this explanation.

But then, it may be asked, does all error in this department consist in verbal mistakes in denotation and false perception? May there not also be false subsumption of the particular under the universal in such a way that in the synthesis of two ideas unlike things may be held to be like? This certainly happens, inasmuch as in no stage of our judgment have we a sufficiency of ideas which are fixed in meaning, clearly distinguished and named, to correspond to the manifoldness of the particular. *Τὰ μὲν γὰρ ὀνόματα πεπεράνται καὶ τὸ τῶν λόγων πλῆθος, τὰ δὲ πράγματα τὸν ἀριθμὸν ἀπειρά ἐστιν* (Arist., *de Soph.*, el. 1). It is the difficult task of science to establish a complete system of predicate-ideas which shall be clearly distinguished and unambiguously named, and which shall make any error of subsumption impossible. So long as this ideal is not attained both in general and by each individual, there will always be particular ideas which cannot find their corresponding general idea amongst those which are known and familiar to us; and these, as an immediate unification is not possible, will seek their names by inferences. If these inferences are hasty, and are guided merely by analogy in extending the use of names, there will be errors; but it will in the first instance be a nominal error, anticipating the formation of concepts in a direction in which it will not follow, and it does not refute the principle given above, which is valid only where the general idea which corresponds to the particular is already formed. Only so far, again, is perfect certainty possible. When we reach the predicate by means of mere inferences of the ordinary sort, we may indeed state it in words, but we cannot attain to the certainty of the necessity of the act of judgment.

¹ Cf. with what follows my article in the *Vierteljahrsschr. für wiss. Philos.*, iv., p. 482 sq.

In the first place, under the formula A is A , it was interpreted as affirming that every object of thought is identical with itself, that it must be thought as just this and no other.

Then, as the principle of all affirmative judgments, it was said to state that subject and predicate must stand in the relation of identity if the judgment were to be possible or valid; possible, when the principle was presented as a natural law according to which we always do think; valid, when it was presented as a normal law according to which we ought to think. In the latter case it was the criterion of valid judgments.

Finally it received a metaphysical significance, and was interpreted as stating that everything which exists is absolutely identical with itself, and that Being can therefore be attributed only to that which is absolutely identical with itself, hence only to the unchangeable which contains no sort of plurality.

We may begin by trying to determine that meaning of the term identity, which its etymology points to as the most original, and which, except in connection with this logical question, is generally accepted. Here, then, identity means that what is thought of at different times, or under different names, or in different connections, is nevertheless not two, but one and the same object; it is merely thought of twice. The term cannot be applied to an idea which is quite simple and occurs only once; like every relational concept, it requires two points of reference. I cannot even recognise the identity of something which remains absolutely the same unless I am aware that I have thought of it at different times and compare the recurring ideas.

Now when we say that something which is thought of twice is the same, we may mean one of two things; we may mean either a *real* or a *logical* identity. A *real* identity is stated when two ideas, two perceptions, two accounts, two names or other denominations are referred to the same person, thing or event. It is in this sense that I state that the tragedian Seneca is identical with the philosopher Seneca; that the Hermes found in Olympia is identical with the statue of Praxiteles, mentioned by Pausanias; that the solar eclipse, predicted by Thales, is the same which took place, according to astronomical reckoning, on May 25, 585; that the person who met me to-day is the same whom I saw years ago in such and such a place. This real identity does not exclude difference in the object at different times; the tree, which I have formerly seen in foliage, is now leafless; the man is now grey whom I knew in his youth. But where we

are not concerned with the reference of our ideas to particular things or events in space or time, the identity must be *logical*, *i.e.* it must refer to the contents of the idea as such. We say that ideas which we have at different times and on different occasions do not differ, but are absolutely the same as to their contents; thus, different words or expressions denote the same concept; different formulæ the same number. In so far, then, as we abstract the attributes of different things and compare them only according to their contents, we can even say the colour of one material is the same as that of another, the length of one rod the same as that of a second; but the materials and rods are not therefore identical; they are only *alike* in the given respect. In the same way, we speak in diplomatic language of identical notes; for, when the contents of the notes are the same, we disregard the plurality of the documents.

This is the extent to which we can apply the word identity, if its original meaning—and, indeed, any definite meaning—is to be preserved. From this, it follows that identity is either complete or not at all; that identity has no degrees, and that the expressions “partial identity,” “relative identity,” if meant to denote kinds or degrees of identity, contain a *contradictio in adjecto*. We can speak of an *identitas partium* (*e.g.*, of parts of Europe and parts of the Russian Empire), but not of an *identitas partialis*.

To return to our Principle of Identity. In its first meaning, the formula A is A certainly expresses a necessary presupposition of all thought and judgment. No thought nor judgment is possible, except when particular ideal-objects can be retained, and reproduced and recognised as the same; for no definite relation could be established between ideas which were always wavering and shifting. Thus, all thought depends upon the *stability* of our particular ideal-contents as its condition. In so far as the stability is always realized to a certain extent, we may speak of a principle of stability in the sense that it expresses a fundamental fact; so far as it is recognised as the condition of all true judgment, the formula $A=A$ contains also a precept, which must always be satisfied if our thought is to be perfect.

But this principle, which refers only to the stability of each idea in itself, cannot serve also as a ground for the union of the subject and predicate in the judgment. For judgments which are meant only to state the identity of an object of thought with itself are quite barren. No one thinks of stating that a circle is a circle, and that this hand is this hand; and in propositions which apparently correspond to the formula A is A , the subject and predicate-terms really refer to different things. In the

proposition "children are children," the subject-term means only the age characteristic of childhood ; the predicate-term the other qualities which are connected with it. By the proposition "war is war," we mean to say that when once a state of warfare has arisen, we need not be surprised that all the consequences usually connected with it appear also ; thus, the predicate adds new determinations to the meaning in which the subject was first taken.

But in the case of simple denominative judgments, we cannot speak of the strictly logical identity of that which is represented by the subject and predicate terms. In a judgment concerning a particular thing the predicate-idea is generally the more indefinite, and does not exhaust all the speciality of the subject-idea ; we can only speak of the *agreement* of the two. I find again in my subject-idea the thought denoted by the predicate ; the particular thing is like the general image of my idea. Thus, it would be more correct to call the ground of these judgments the Principle of Agreement ; it states the necessity that there should be agreement of ideal content between what we connect as subject and predicate ; that the consciousness of this agreement should be expressed in the judgment. It implies, moreover, that no thinker can be mistaken as to whether there is agreement between the ideas present to him as subject and predicate, in so far as they are present. Thus, the principle states the immediate and infallible certainty of the consciousness of agreement, both as a fundamental psychological fact and as a necessary presupposition of judgment.

When the predicate of a denominative judgment is a proper name, or any verbal expression which by its sound gives rise to the idea of a single existing thing as such, and is used as a sign of the thing (this is Socrates, this watch is mine), and if the judgment is based upon immediate knowledge, then, in this case also, the agreement of the two ideas—the intuition and the memory-image—is presupposed. But here again it is not necessary that there should be absolute identity of the ideal contents. I recognise an acquaintance even in a new dress, or when he looks paler than usual. But this agreement is accompanied by the consciousness of the real identity of the subject with the particular thing denoted by the predicate. This real identity of the thing corresponding to the two ideas which have arisen at different times, and which are both ideas of the same thing, is again fundamentally different from the agreement and stability of ideas ; it refers to a determination of *being* as opposed to *being thought*. We can, however, formulate a principle from this point of view also, viz. that in the concept

of the particular thing itself there is included both its singularity and that identity with itself, which alone gives a meaning to the idea of the duration and permanency of the thing ; so that the very concept of a thing includes the assumption of things identical with themselves. But in saying this, we do not go so far as to state the Eleatic or Herbartian principle of absolute indistinguishability, or of the identity and immutability of the *what*—in (for instance) the formula “every thing is what it is.” On the contrary, our conviction of the real identity of particular things with themselves refers to their permanence during change of action, their duration under different appearances ; we constantly refer ideas having a partially different content to one and the same thing.

In the judgment “this is Socrates” we say that the person present is really identical with the definite individual known as Socrates. Here again the statement refers to the objective validity of this identity, for in making it we are conscious of the necessity of referring the two ideas to one and the same thing. To question its objective validity would be to say that the thing referred to as subject and the thing referred to as predicate may be, or are, two different things, and that it is not necessary to unify them. The law of agreement among our ideas is not sufficient to prove the necessity of referring two ideas to a single real thing, for this law only guarantees the agreement of their contents ; it is here that assumptions concerning the nature of the existent and the distinguishing characteristics of real identity become important, and these the function of judgment does not itself give us. Such is the assumption that all individuals, of certain kinds, can be confidently distinguished, and that there are no two objects so alike that they can be mistaken for one another if we examine them closely. It is on this, for example, that our conviction rests of the identity of persons known to us. When our recollection of the outward form fails to give us certainty, we have recourse to the identity of consciousness with its individual peculiarities, as Penelope did when she examined Odysseus as to his knowledge of the arrangement of their marriage bed ; but with outward things it is by their spatial determinations and the principle of impenetrability that we finally establish their identity. The necessity of believing in real identity is the result of presuppositions such as these, which arise from our knowledge of the nature of things. To these statements concerning real identity, which are supported by considerations obtained from another source, we may add the judgments which express the coincidence of a certain subject with a given member of a series, or with any other

particular thing which is determined by a relational predicate—Augustus is the first of the Cæsars ; Aristotle is the teacher of Alexander.

6. With regard to the objective validity of the judgments which predicate attributes and activities, everything which has been said with reference to naming is, from one point of view, true of these also, because of the twofold synthesis which takes place in them. The attribute or activity which is thought of in the subject must agree with the general predicate-idea. On the other hand, their objective validity can be maintained only upon the assumption that the unity of thing and attribute, of thing and activity, is a real relation, hence that we are able to know a thing by its attributes, and to look upon a change in our idea as an alteration in it. This relation of the thing to its attributes and activities has also been brought under the concept of *identity*, though here again an elasticity has been ascribed to the term which does not really belong to it. The thing is only identical with itself as the permanent support of its attributes, as the subject which in activity remains one with itself ; it is not identical with its attributes, nor with its activities, it is not the same as these. Cinnabar is not identical with its redness, nor the sun with its shining ; and the principle by which the judgments, "Cinnabar is red," "the sun shines," are to be justified, cannot be the principle of identity. The only general law of thought which we can lay down,—and it expresses also a fundamental fact—is, that we are able to distinguish, retain, and recognise anything which exists by means of these categories of inherence and action alone ; and that the Being of a thing is also the Being of its attributes and activities.

But even when we start from this law and hold that what we judge about is the existent, all we can mean is that the existent concerning which we judge *necessitates* this particular movement of thought by which we distinguish between it and its attribute, and then unify them again.

7. The general ideas of things which we use as predicates of denominative judgments, are, to some extent, accompanied at every further development of thought by those judgments of attributes and activities, of which they have been the subjects. Snow, for example, may signify not an unanalysed image, but a white, loose, cold thing ; the general name has thus become a summary of attributes. To this extent the relations of inherence and action are implicitly contained even in denominative judgments, as belonging to that meaning of the word which is present to consciousness. When there is added the real identity of things which fall

under different ideas (water, ice, steam—boy, man, old man), a substantive may serve merely to denote a complex of attributes which express the different temporal states of a particular kind of subject.

§ 15.

Everything which exists as a particular thing is given to us in time, it occupies a definite position in time, and is intuited as lasting during a period of time, as developing varying activities in time, and possibly as changing its attributes. Thus a REFERENCE TO TIME is necessarily connected with all our judgments concerning the existence, attributes, activities, and relations of particular things, and all such judgments can claim to be valid for a definite time only.

1. While this proposition is self-evident for activities the reference to time seems wanting in some of the attributive-predicates, inasmuch as they are regarded as invariable and given with the existence of the subject itself. But while there is always the general possibility that attributes may change in spite of the identity of the subject, this relation of permanence is to be found only in exceptional cases; it is not contained in the form of the judgment by itself; at the most it is found in the secondary relations attaching to the meaning of the predicate, or in the predicate itself (unchangeable, etc.). Only when we call a thing by a proper name do we exclude all reference to time, and by the nature of our predicate refer to the subject, irrespective of temporal relations. All other denominative judgments admit of this limitation of their validity to a certain time, in so far as in naming the predication of attributes and activities becomes prominent (*v.* end of § 14), and the same thing may thus be called first by one name, then by another.

2. Thus the narrative-judgment is never completely expressed unless it contains a statement of the time for which the unity of predicate and subject is objectively valid; it must be expressed in the present, past or future tense. The extent to which languages are capable of expressing temporal relations in predications is one measure of their logical perfection. It is only to the disconnected thought of the child, which is quite absorbed in the interest of the moment, that everything which occurs to it is present; the power of distinguishing times grows as self-consciousness becomes clearer and more methodical.

II. EXPLICATIVE JUDGMENTS.

§ 16.

The judgments we have so far considered make statements about particular things. Essentially different from these are judgments of which the subject consists in the meaning of the subject-word, which do not refer to the definite existence of particular things named by the subject-word, though this existence is frequently presupposed by the nature of the idea, or, as a consequence, of the source from which it is derived. Their objective validity is independent of time. As explaining the contents of a general idea, they may be regarded as the indirect expression of a rule which refers to the existent.

1. "Blood is red and snow is white,"—such judgments do not speak of this or that particular thing, nor give expression to any perception present at the moment. Since the subject-word is absolute, all that it can express is that which constitutes its meaning; and this meaning is an ideal content of indefinite generality which has been disengaged from the idea of any particular existing thing, and which in its indefiniteness cannot be said to exist. Hence all that the statement "blood is red" can tell us must refer to this ideal content, and it signifies nothing more than that the predicate is thought of in connection with the subject. What sort of unity there is between subject and predicate depends upon the nature of the ideas connected. If both belong to the same category, then the simple coincidence of the ideas is stated; that which is thought as a concrete thing has attributes and activities predicated of it which are contained in the idea of the thing itself.

It is in a similar sense that we make use of the article, particularly where the subject-idea is the idea of a thing having an individual form (the dog is a quadruped).

We must also include in the class of explicative judgments those which seem, by means of the so-called indefinite article, to make a statement concerning a particular individual, a particular state, etc.; such propositions as "a fir-tree is a conifer, an ague is accompanied by high fever," do not refer to any definite particular thing; what they mean is, that that which is a fir-tree is a conifer, and this statement can only be based upon the relation between the general ideas fir-tree and conifer, not upon knowledge of the particular.

2. The objective validity of these judgments has immediate reference

only to the sphere of ideas ; all that they tell us is, that when the subject (presupposing nominal correctness) is thought, it will be thought together with the predicate ; that what I and every one else think of as " blood " is thought of as red. It is only secondarily, when we pass from the generality of the word to the actual things contained under it, that the judgment refers to the Being of these things also ; and states concerning them the rule that when there is a thing which is denoted by the subject the predicate belongs to it.

According to one view, such judgments are from the first general judgments gained from experience by induction, having for subjects particular things thought of as indefinite in number. But here it is forgotten that the first essential for such an induction is that we should have some standard according to which we may name particular things by the same word and so comprehend them in a judgment common to all. Such a standard we can find only in the meaning of the words we bring to our process of naming ; this meaning must be already to some extent fixed before there can be any question of judgments of induction. It is perfectly true that under the influence of progressive experience, which is always adding something to our ideas, these ideas remodel themselves, and that it is generally a matter of chance where common thought comes to a halt and fixes the boundaries of its words.

The meaning of the word blood, for example, is formed in the first place from the intuition of human blood and that of mammals and birds ; hence it may, and in ordinary language, does, include the colour red amongst its contents ; but by an extension of its original meaning it might also be applied to the whiteish fluid in other kinds of animal organisms. At some stage of formation, however, the meanings of words are a necessary preliminary to the judgment of the individual ; whatever the stage at which they stop this fixed meaning is preliminary to naming and to the possibility of judgments of experience gained by induction. Thus if blood is the fluid in the veins of mammals and birds, then " red " forms a part of its meaning, and when we take it as fixed in this way we cannot use it to name fluids of another colour.

Thus before we can give utterance to a judgment having the meaning of a judgment of experience including many cases (of which we shall speak later), a simple judgment must have preceded to unfold the contents of the single idea denoted by a given word ; the general rule to be found herein is, in the first instance, a rule of naming which forbids us to call anything

blood which is not red. The inductive judgment first appears when a new attribute is discovered common to all the things which are denoted by the same name,—when we say that *B* is invariably connected with the attributes which make up the contents of the subject idea *A*, *B* not having been previously connected in thought with *A*.

Such an explicative judgment can contain the statement of a rule concerning things themselves only in so far as the idea of a permanent and lasting thing, together with the possibility of variable attributes, is contained in the substantival denomination; and the rule is that the predicate always and constantly belongs to the things which fall under the denomination, and is invariably connected with their other attributes. When the judgment extends to reality it is this invariability of the red colour belonging to everything which can be called “blood” to which it really refers.

3. Verbs occupy a peculiar position here. Strictly speaking, the only case in which a verb can be the predicate of a general subject is when we are speaking of an activity which is continuous, and which lasts as long as the thing comprehended under the subject-idea (fire burns, the wind blows); when, on the contrary, the verb expresses a variable activity, beginning and ending in time, it is only as a figure of speech that it can be used as predicate (the sheep bleats, the horse neighs); what the expression should really denote is merely a faculty or a habit, *i.e.* an attribute from which the activity may proceed, and not the activity itself.

4. When we compare this class of judgments with those first considered, we are chiefly struck by the fact that their validity does not depend upon the existence here or there, at this time or the other, of a thing corresponding to the subject-idea. Hence they are not valid for some given time, but can claim unconditional validity just because they refer merely to ideas. On the other hand, all merely narrative judgments are temporarily valid.

5. Here there appears a characteristic difference in the meaning of the Present tense, according as it is used for the statement of unconditionally valid judgments or of those amongst temporarily valid judgments which refer to the present. When we think of a thing as presented and as having a particular existence we have thereby assigned to it its place in that time which embraces everything, and is the same for all; by its existence it stands amongst other things which are contemporaneous, or precede or follow it; and by its quality, concerning which our judgment is made, it occupies a definite point of time and thus stands in a definite temporal relation to the moment of our judgment.

But when the subject of our judgment is the idea which constitutes the meaning of the word, this idea is disengaged from the temporal complex. Removed from the changes of time, it stands before us in a mental present, in which there is no distinction of yesterday and to-day, but in which the consciousness that our idea will be the same at every repetition annuls all temporal distinctions between the particular occasions upon which we actively think of it. When thought in this way the subject has predicates which belong to it independently of time, which belong to it as often as it is thought of. The proposition "the sky is blue" denotes the state of the sky at the present moment, and is thus,—as a narrative judgment,—really in the present tense; but it may also have the completely different meaning that the sky as I generally think of it, and as the fixed object of my thought, is always blue. In this case no past nor future is contrasted with the present. The validity of the judgment is not estimated by my perception of the object in a given momentary state, but by the invariability of the ideal content which I am connecting once for all with the word—an invariability which is the condition of all my speech or thought.

III. THE ACT OF JUDGMENT AS EXPRESSED IN LANGUAGE.

§ 17.

The verbal expression of the unification of subject and predicate which takes place in the judgment is to be found in developed languages in the form of inflection of the verb, which has itself arisen out of what was originally a mere juxtaposition. Even where the verb "to be" appears as the link between a substantival or adjectival predicate and the subject, the act of judgment is expressed by the verbal ending alone; the verb "to be" forms a part of the predicate.

1. Less developed languages, and, in simple cases, even advanced ones, are content to express such a unification as takes place in the judgment by the mere juxtaposition of the two words which stand for subject and predicate. This juxtaposition must not only indicate that the corresponding ideas are at the moment unified by the speaker, it must also express the objective validity of the judgments. Accentuation alone can distinguish the statement from the question, or from any other mode of connection, such as the attributive which expresses a unity of two ideas which is already present. When, on the contrary, the development of the linguistic forms has

followed all the logical distinctions, then in the case of verbal predicates the meaning of the personal ending (which blends the pronominal equivalent of the subject immediately with the stem of the verb, thus making it agree in person, number and gender) is to denote the connection of subject and predicate in judgment; and the indicative mood, combined with the emphasis and arrangement which distinguishes the statement from the question, states this connection as objectively valid. The tense, again, indicates the time for which the judgment is to be valid.

That which logicians would denote by the expression copula, that element of language which can combine words into a proposition, and so into the expression of a statement, lies in the personal ending of the indicative, and in this alone. The meaning of the unity of subject and predicate which is expressed by this terminal inflection differs according to the nature of the ideas connected.

2. Judgments in which the predicate is expressed by an adjective or substantive are not generally formed by mere juxtaposition (as in *ὁ μὲν βίος βραχύς, ἡ δὲ τέχνη μακρὴ*); the verb "to be" is employed. But this is not by virtue of its meaning the element which expresses the formation of the judgment; the function of judgment is contained in the form of inflection alone. The verb "to be" is the means by which we give the form of a verb to the predicate, and thus enable it to take the ending which is the outward sign of its predicative relation to the subject. In the judgment "cinnabar is red," nothing is added by the meaning of the verb "to be" which is not already contained in "red," for as an adjective it involves reference to a subject of which it is the attribute. As abstract nouns, "redness" and "red-being" would have just the same meaning; the verb "to be" is merely used to express the fact that "red" is not thought of as independent and abstract, but is predicated of a definite subject. In this way the verb "to be" certainly serves to facilitate the expression of this use of the word "red," and to show that it is a predicate, as contrasted with the merely attributive relation which might be expressed by juxtaposition; but in so doing it is a mere vehicle for the copula, not the copula itself; it does not constitute the judgment, but merely prepares the way for it. This function of "to be," to denote the meaning in which a word is to be used, appears still more clearly in the case of substantives. These do not, like adjectives, carry in their form a reference to something else, yet their meaning, by its nature, enables them to fulfil the function of a predicate, whenever that meaning is a general one and only appropriated to a definite particular

by means of a denominative judgment. "Man" is not the name of a definite individual, although the idea of the figure of an individual is included in its meaning; it is not a name at all, but the sign of a definite ideal-content. Only the demonstrative adjective or the article converts the word into the name of a definite man; the use of the verb "to be," on the contrary, converts it into a predicate, and it cannot be a name before it has been a predicate. Thus "Mensch," as a general idea awaiting its reference to some definite individual, and "Menschsein" have the same meaning; the verb "sein" serves only as the outward sign of the function of the idea as predicate, which might also be shown by its position and accentuation. Thus its function is that of a formal element of language; but it is not that formal element which expresses the act of judgment and merits the name of copula.

3. But how does it happen that it is just the verb "to be" which we use in this way? and what connection is there between the meaning of "to be" as an independent verb, when it stands alone as predicate, and this function, which it has in connection with adjectives and substantives?

In the fourth chapter of the first book of his *Logic*, J. S. Mill draws attention to the ambiguity contained in the word "is," inasmuch as when it is used as the so-called copula, it certainly does not state that the subject exists, but only denotes the relation of predication. Such a sentence as "A centaur is a fiction of the poets" directly denies the statement that a centaur is; and he is surprised that this ambiguity should have been overlooked by almost all authors, although existing in modern as well as in ancient languages.

Mill has paid as little attention to Herbart as to other German philosophers. Herbart, following in the footsteps of Fichte,¹ has shown, with his usual acuteness (*Einl. in die Phil.*, § 53), that the judgment A is B , and the question is A B ? certainly do not contain the statement (which, though commonly attributed to them, is really quite alien) that A is, for nothing whatever is said about A by itself, nor of its existence or validity.

There is no doubt that this remark is correct, and should never have been disputed.² A judgment of the form " A is B " can never, by the

¹ *Grundlage der gesamten Wissenschaftslehre*, part i. § 1, a passage of which I have been reminded by Bergmann (*Reine Logik*, i. p. 235).

² It is urged against this (cf. Ueberweg, p. 162) that propositions such as "God is just," "the soul is immortal," "true friends are to be valued," must involve the statement that there are such things as God, a soul, true friends; that this presupposition is contained in the indicative mood; that any one unwilling to accept the presupposition

mere fact that the subject and predicate are connected by "is," include the statement of the judgment "*A* exists." The function of this "is" is

would have to add clauses to those propositions which should make them hypothetical: "in case there is a God," etc. Such a clause, it is said, can be dispensed with only when the context (as in a romance), or the recognised meaning of a word (such as Zeus, Sphinx, Chimæra), shows that the reality is merely feigned, or that the statement is merely the explanation of a name. This objection is so far true that the reality of the subject is generally presupposed by those who speak or hear such judgments; otherwise there would be no motive present for uttering them. But this is quite another thing from saying that the judgment itself, as it stands, involves the statement of the reality of the subject, *i.e.* that this is necessarily stated by the words of the judgment, and more especially by the indicative mood. If it were so, it would be incomprehensible how there should be any exception to the rule, for if the indicative of the categorical judgment containing "is" has the force of stating the reality of the subject, then it must have this force always and under all circumstances. The exceptions which Ueberweg admits themselves prove that it does not depend upon the form of the judgment, but upon the ideas we unconsciously attach to the meaning of the subject-words, without their finding utterance in the judgment, whether or not the presupposition of their existence is "as a rule" accepted. And, indeed, what meaning can the statement of existence have when the subject does not denote individual beings as such—as in the proposition "God is just," or "true friends are to be valued"—but is of a universal form? When I say, "snow is white," in what sense does this judgment involve the statement that snow exists? Certainly not in the sense indicated by the present tense of the indicative when used of definite things having a particular existence, for then the statement would be that snow exists now at this moment. But the judgment, "snow is white," is true in summer and winter alike; and yet it is not held that the judgment tells us that snow always exists. Again, if the statement is said to be, that such bodies as those of which I think when I say snow, have actually existed somewhere and at some time, it can be only the existence of definite "snows" which is meant. This alone can be stated, nor can it be said of snow in general that it exists. But the judgment, "snow is white," does apply to snow in general and not only to this or that snow.

Still it is true, that the recollection of actually perceived snow is always connected with the idea which we attach to the word "snow," and for *this* reason, because of the way in which I have attained to the meaning of the word, it is presupposed that we are dealing with something which exists. But suppose I take the analogous judgment, "Pegasus is winged," the idea of wings is as firmly connected with the idea which I attach to the word Pegasus as that of white colour with snow. So far, however, I have never seen an existing Pegasus; indeed, I know that it is a creation of fancy, and for this reason the existence of Pegasus is not presupposed. But the judgment itself tells me neither that Pegasus exists, nor that he does not exist; only how the idea is constituted which I connect with the word. Take the judgment, "The branches of the hyperbola are infinite," it is undoubtedly a valid judgment, although it can have nothing to do with the existence of the branches of this or that particular hyperbola. The infinite branches of the hyperbola exist in exactly the same way as all other subjects of my judgment, that is, they exist as objects of my thought, of which I assume that every one else thinks alike.

W. Jordan has treated this question more carefully in his treatise, *über die Zweideutigkeit der Copula bei Stuart Mill* (Stuttgarter Gymnasialprogramm, 1870). It is true that on page 13 he says, "The 'is' certainly includes the concept of existence." But he gives a much wider scope than Ueberweg to this concept of existence, when on page 14 he says,

exactly the same whether we are talking of existing or of non-existing things, of subjects which are thought of as particular, or of subjects which are thought of as general (to these, as general, particular existence cannot

"Whenever the thinking subject finds something present which is independent of his act of thought, whether it is in the material or spiritual world, Logic will permit the use of the 'is.'" If we take this explanation literally, it is certainly true that every act of judgment—so far as it presupposes, and does not create, the subject of the judgment—recognises something which is independent of this act of thought, and this something is the idea denoted by the subject-word. Such a reality of ideas is presupposed whenever a judgment finds expression in language, and it is presupposed moreover that it is common to many individuals. Thus the question would disappear if such were the reality intended, and the "is" would be justified wherever the subject-word, and therefore the judgment, had any meaning. But just for this reason the judgment would have nothing whatever to do with the statement of the actual existence of the thought denoted by the subject-word, in the ordinary sense of existing.

However, this is not what is meant. Jordan attempts—in opposition to Herbart and Mill—to preserve for "is" the meaning of real existence. This he does in the first place by saying that the reality in question belongs to the predicate, but not to the subject. In such propositions as "self-defence is forbidden," "moderation is difficult," there is no doubt that the existence of the subject-idea is left undecided, while in the predicate, on the contrary, allusion is made to something which actually exists. The whole is an existential proposition in disguise, *i.e.* there are laws, or grounds, which forbid self-defence, circumstances which make moderation difficult. But if we once begin to paraphrase like this, we shall find at last that even the proposition, "a square circle is unthinkable," is an existential proposition; "there are logical laws which make a square circle impossible." This, however, is to abandon the ground of dispute, for the question was, whether the reality of the subject is stated? We are far from denying that every statement, just because it claims objectivity, contains the recognition of objective "laws" and "grounds"; but we do deny that for this reason the existence is stated of a thing (or attribute or event) which corresponds to the subject-idea. Jordan's other distinction, which he applies to Mill's example of the centaur, is better. Such a proposition as "the centaur is a fiction of the poets" approximates to a definition. Now amongst definitions Jordan distinguishes a particular class which he calls "rectifying," and these are the definitions which deny the idea which the subject stands for, and replace it by another. The proposition says, "the centaur does not exist in the sense indicated by the word of actually being, but the idea of the centaur is a fiction." No doubt there are a number of such predicates which degrade the subject-word into the sign of a merely mental thing, when, by its customary meaning, it might be taken to denote an existing thing. But we must not forget that at the head of these predicates there stands the verb to be—to exist. Whenever I expressly state of a subject that it exists, the subject-word is, for me, the sign of an idea; and my predicate states that an actual thing corresponds to this idea.

The view that the meaning of the word "to be" is always the same has lately received determined support from Fr. Kern (*Die deutsche Satzlehre*, p. 64 sq.), who pronounces against distinguishing two meanings in it. "In the propositions 'wooden iron is nonsense,' 'a square circle is a contradiction,' the existence of the wooden iron and of the square circle is stated with just as much clearness and emphasis as the existence of the boy in the proposition 'the boy is in the garden.'" But while . . . the boy exists outside my thought as well as within . . . the iron and the circle exist only in my idea. They exist, moreover, in connection with the property of being nonsense or a

belong); of predicates which can belong to an existing thing, or of predicates which deny existence by their meaning. In each case its one function is to put the predicate into such a form that it can be used in the judgment, and to enable it to take the personal ending. The nature of the subject and predicate-ideas alone determine the sense in which subject and predicate are unified, and whether the existence of the subject is presupposed, left uncertain, or denied. "The square is a regular, four-angled figure" means logical identity, "this is my watch," real identity; "gold is a metal," subsumption under a more general idea; "gold is yellow," the unity of thing and attribute; "*A* is a mile away from *B*," a relation; "the movement is slow," the unity between a general idea and its more exact determination, and so on. "Socrates is ill," presupposes the existence of the subject, because Socrates is the name of an individual who is thought of as existing, and "ill" is a condition thought of as actual at a given time. "Pegasus is winged" leaves the existence of Pegasus uncertain for any one who does not know whether the name is that of an actual or of a merely fictitious being. "Pegasus is a mythological fiction" denies the existence of the subject; in every instance our information is obtained only from the meaning of the terms, either of subject or predicate.

4. With reference to the predicates we may here distinguish between two classes.

contradiction, which property I have recognised and stated; and thus they cannot possibly be found in any reality which is independent of me."

But the ambiguity of the word is immediately granted by this distinction between a subject which exists in a reality external to me, and one which exists only in my idea. For when the word "is" stands alone in the sense of "exist" what it states is that the subject does *not* exist only in my idea, but independently of it. In such a proposition as "God exists—but only in my idea" the clause directly denies the meaning in which we are, at first, impelled to understand the words "God exists." But it is not even true that a square circle exists "*in my idea*," for who could imagine one? The contradictory is impossible in my thoughts, as well as in the reality which is independent of me. What the predicate "is a contradiction" really tells us is, that I cannot think of anything corresponding to the words "square circle"; it denies existence in thought as well as in reality.

In p. 74, it is brought forward as an instance, that to any one who doubts it we say with emphasis "*A is* the person who did it," this emphasized *is* being meant to lay stress upon the existence of *A* as the person who did it. But it is obvious that the existence of *A* was never called into question, hence that there is no reason to lay special stress upon it. It was the action of *A*, and not his existence, which was disputed, and the question whether we had any right to predicate of *A* (whose existence is undisputed) that he is the person who did it. If this is not so, then the proposition "*A is not* the person who did it" must be meant to deny not merely the quality of agency, but the existence of *A*. Concerning Bergmann's objections (*Reine Logik*, i. p. 235 sq.), cf. *Vierteljahrsschrift für wiss. Philos.*, v. 113 sq.

All modal relational-predicates, which express a relation to my knowledge (with the exception of those appertaining to the senses, such as visible, or tangible), by their meaning make the subject-term the sign of a mere idea which does not involve real existence; and this is the case whether the predicates themselves affirm or deny its existence, or leave it undecided. When I use the predicates true, false, credible, incredible, etc., of anything, I denote that it is only an idea, and that the predicate is to give information concerning its relation to me and to my subjective thought. Such propositions as "Tell's apple-shooting is a fact," "the Trojan war is an historical event," "atoms are bodies which actually exist," would be impossible if "is" and "are" could by themselves state the existence of the subject.

But the verb "to be," when used absolutely and as meaning "to exist," is itself a modal relational-predicate. By expressly stating the existence of the subject it determines the question whether that which the subject-word first denotes merely as an idea is also an actual thing. Cf. above, § 12, 7, p. 72.

But with other predicates everything depends upon what the subject is and the sense in which it is used; and this cannot be seen from the mere external form of the judgment and the use of the word "is." If the subject-term is used as general and is not introduced as the name of one or more definite things, then the predicate which is formed by means of the verb "to be" can state nothing but the contents of this subject-idea, and nothing whatever is said as to the existence of the subject. When I say "gold is yellow" or "atoms are indivisible," the being yellow or indivisible appertains to the idea which I denote by the subject-word; but the propositions do not declare the existence of particular things. I must learn from some other source whether or not the subject-term is applicable to such things. But if the subject-term appears from the first as the name of things which have a particular existence—*e.g.* "this piece of gold is yellow," "this horse is black"—then existence is presupposed; not in the word "is" however, but in "this."

5. This "ambiguity of the copula" applies not only to the verb to be, but to all predicates which can by themselves denote real states and attributes, inasmuch as they sometimes state that which actually takes place in a particular case and sometimes that which belongs as attribute or activity to the subject-idea. Strictly speaking, it is only the present tense which is ambiguous; for this sometimes expresses the empirical,

temporal present, sometimes the general necessity of thought. The proposition "great souls pardon offences" states neither that great souls exist—though this is presupposed in the act of pardoning—nor that some great souls are at this moment pardoning. All it tells us is, that if a man is a great soul he must pardon offences. But the proposition, "Socrates speaks," states the existence of Socrates in the same way as the proposition "Socrates is ill"; we can speak of him only just in so far as he exists, because the word Socrates denotes an individual having a particular existence, and any attributes or actions which are ascribed to him must involve his existence.¹

6. But how does it happen that the verb "to be," which is the expression of actual existence, assumes a formal function whereby it loses its meaning—nay, even seems to contradict it? The remarkable thing is not that the ambiguity in this respect has been so little noticed, but rather that all known languages agree in containing it. The explanation is not difficult. As Ueberweg (p. 162) rightly notices, and, as we have emphasized above (p. 95 *sq.*), it is as a rule taken for granted that the things of which we speak exist; there is no need of any express assertion; we are not interested to know *that* things are, but *what* and *how* they are. When now, it is desired, not merely to express predication by mere juxtaposition, but to give the form of the verb to the predicate, the verb "to be" offers itself just because it is so general and has by itself no contents. It is always presupposed; but in order that we may know what we wish to know, we need to have it determined more exactly as "being this" and "being thus"; just as the statement of existence is determined more exactly as "being here" and "being now." The predicate "red," which by its form already denotes something appertaining to something else which exists, now appears as a modification of Being, being *red*.

Now as the present tense denotes on the one hand the empirical, sensual present, and on the other the timeless present of our thoughts; so the meaning of being in this connection is also extended. The attribute has the same relation to the thing which is thought, and to the thing which in its existence is perceptible to sense; as the presupposition of Being was only implied before, it can now be abstracted from. As objects of my thought, things do not alter; their being may cease, their being-this and being-thus remain so far as I retain them in thought.

¹ The theory according to which the judgment "*A* speaks" is altered to "*A* is speaking," in order to get the inevitable copula "is," may be held to be obsolete.

Nevertheless the verb has kept a part of its original meaning, and the most important part. The verb "to be" originally implies real existence. That which exists holds good independently of my thought, and holds good for every one. This objectivity of the connection to which my judgment gives utterance is an essential factor of the judgment itself. This it is, and not the existence of the subject, which forms a part of the statement; and for this "Being" is a very suitable expression.

By this extension of its fundamental meaning it strengthens the assertion of objectivity, of universal validity, already contained in the inflexional form.

CHAPTER III.

HOW JUDGMENTS ARISE, AND THE DISTINCTION BETWEEN ANALYTICAL AND SYNTHETICAL JUDGMENTS.

§ 18.

IMMEDIATE JUDGMENTS are those which presuppose nothing but the ideas connected in them, in order that these ideas may be united as subject and predicate with a consciousness of validity ; MEDIATE, or MEDIATED JUDGMENTS are those in which some further element is necessary in addition to these ideas.

Kant's distinction of analytical and synthetical judgments refers only to the relation between the predicate and the concept which is denoted by the subject-term and assumed as given. It is not applied by Kant to those judgments in which the subject is a particular, intuitable idea. All relational judgments, moreover, must be regarded as synthetical from the Kantian point of view, even when based upon the analysis of a complex presentation.

1. When, after analysing the functions through which the simple judgment takes place, we ask about the origin of the judgment, we are not concerned with the origin of the ideas—whether subject or predicate—united by the judgment ; in speaking merely of the analysis of the act of judging we assume these to be given. The question concerns only the genesis of the act of judging itself, and this has two aspects : that of the connection or unification of subject and predicate, and that of the consciousness of the objective validity of this connection.

This genesis may be either immediate or mediate. It is immediate when nothing is presupposed in the judgment but the subject and predicate ideas connected in it, in order that it may take place with a consciousness of objective validity. It is mediate when this can take place only by the addition of some other elements. It may be that without some mediation the subject and predicate cannot even be referred to each other in such a way that their union in a judgment might be thought of ; or it may be

that at least the consciousness of the objective validity of such *unity* must be obtained elsewhere. Let us for the present call that which brings about the unification of subject and predicate the Ground of the judgment. Then the immediate judgment is one, the ground of which lies in the connected ideas themselves, apart from anything else ; the mediate judgment is one which has its ground in these ideas only when taken together with others ; the mediation may at first merely relate subject and predicate to each other by raising the question whether *A* is *B* ; or it may at the same time settle the question and guarantee the certainty of the validity of the judgment *A* is *B*.

If the ground lie entirely in the ideas themselves which are connected by the judgment, then, according to what we have said above, their relation must be such that the unity expressed in the judgment can be immediately recognised. In a denominative judgment I am conscious, without further mediation, of the coincidence between the idea which is present and that which is reproduced and denoted by the predicate-term ; when I say "that is a fir-tree," I find in present intuition just what agrees with the general idea of the fir-tree. In immediate judgments of attributes and activities the idea corresponding to the predicate is a part of the subject-idea. I analyse the latter and emphasize a certain element, *e.g.* colour, and this I recognise as agreeing with a colour already known. Here, again, nothing beyond the complex idea given as subject is needed in order that I may discover in it the element which corresponds to the predicate.

In the case of relational judgments, it is true that an analysis of the subject-idea by itself cannot yield the element which agrees with the predicate. I may turn and twist as I will the idea of the lamp which stands before me, I can discover in it nothing which tells me that it stands to the left of the writing desk. But in this instance that which is given is a complex intuition containing two objects and their relation ; and by analysing this into its elements I obtain the judgment, for which no more is needed than the ideas connected in it. The complex presentation is the ground for the statement that the lamp stands to the left of the writing desk.

Thus all immediate judgments are of necessity analytical, if those judgments are analytical which only re-unite the elements obtained by analysis of a presentation, judgments, that is, in which either the contents of the predicate are already presented in the subject, as in those of naming, attributes or of activities ; or in which subject and predicate, together with the relation between them, form parts of a complex presentation, as in

relational judgments. If this is so, then synthetical judgments must be those which are inferred, and those which need some other ground lying outside the ideas given, in order that the synthesis of the judgment may take place.

2. That all immediate judgments should be analytical in this sense in no way contradicts the essence of the judgment, which is to be a *σύνθεσις νοημάτων*. For the analysis or *decomposition* is only the preparation for the act of judgment, not that act itself; the work of judgment is to restore union between the elements which have been thus distinguished (cf. § 81).

3. Still, to use the terms analytical and synthetical simply in this sense is forbidden by the use of them introduced by Kant. The above distinction between immediate and mediate judgments is based upon a ground essentially different from that of the Kantian distinction between analytical and synthetical. In the former case it depends entirely upon how the judgment arises at any time in the judging subject, whether it is immediate or mediate, and whether it has its origin in the separation or combination of elements; and we cannot generally discover what this genesis is from the verbal expression of the judgment. Kant, on the other hand, depends in the first instance upon the presupposition of definite concepts which are expressed by the terms appearing as subjects.

"In all judgments," he says in the well-known passage of the *Kr. d. r. V.* (ed. i., p. 6; ed. ii., Int. IV.), "in which we think of a subject as related to the predicate, it may be related in two ways. Either the predicate *B* belongs to the subject *A* as something which is contained (implicitly) in this concept *A*, or else *B*, though connected with the concept *A*, lies quite outside of it. In the first case I call the judgment analytical, in the second synthetical. Thus analytical judgments (affirmative) are those in which the connection of the predicate with the subject is conceived through identity, but those in which this connection is thought without identity are to be called synthetical. We might also call the former explicative, the others ampliative, judgments, for the former add nothing by the predicates to the concept of the subject, they merely break it up by analysis into its component concepts, which were already thought (although confusedly) in it; the latter, on the contrary, add a predicate to the concept of the subject, which was never included in the thought of it, and could not have been obtained from it by any process of analysis." The two propositions, "all bodies are extended" and "all bodies are heavy," follow as examples. In order to obtain the former judgment I need only analyse that concept

(of body), *i.e.* only become conscious of the manifold which I always think in it, in order that I may find this predicate therein. On the other hand, when I say "all bodies are heavy," the predicate is something quite different from that which I think in the mere concept of a body in general.

Just for this reason, he adds in the *Prolegomena*, § 2, 6, all analytical propositions are judgments *a priori*, even when their concepts are empirical; *e.g.*, gold is a yellow metal; for in order to know this, I require no experience beyond my concept of gold, as yellow and a metal, which is just what constitutes the concept.

"Judgments of experience, as such," continues Kant in the 2nd edition, "are all synthetical. It would be absurd to base an analytical judgment upon experience, for in order to form the judgment I have no need to pass beyond my concept or to appeal to the testimony of experience. That a body is extended is a proposition which is established *a priori*, and no judgment of experience; for before appealing to experience, I have all the conditions for my judgment in the concept, from which I can extricate the predicate according to the law of contradiction, and at the same time become conscious of the necessity of the judgment—a necessity which experience could never have taught. On the other hand, though I do not include the predicate of heaviness in the concept of body in general, still this latter denotes an object of experience by one of its parts, and to this I may add other parts of the same experience as belonging to the first. I can in the first place *know* the concept of body analytically by the characteristics of extension, impenetrability, figure, etc., which are all contained in this concept. Then I extend my knowledge, and on referring to the experience from which I had derived this concept of body, I find that heaviness is always connected with the above characteristics, and hence I add it synthetically as predicate to that concept. Thus it is experience upon which the possibility of the synthesis of the predicate heaviness with the concept body is based; for though the one concept is not contained in the other, they still belong to each other, as parts of a whole—*i.e.* of experience, which is itself a synthetical connection of intuitions, although merely accidentally."

We have given these passages in full, because of the importance of realizing the presuppositions upon which this distinction rests. In the first place Kant—according to the received way of regarding the judgment—has in view solely the *concept*, which is denoted by the subject-word, and

which constitutes its significance. The question is whether the predicate is one of the characteristics which I think in the concept of the subject, "although confusedly," or whether it is not yet contained in this concept as I now think it. In the particular judgment also, "some bodies are heavy," which is used as an example in the *Prolegomena* instead of the universal judgment of the *Kritik*, the point dealt with is only that the predicate heavy "is not really thought in the general concept of body." Here Kant presupposes, in the examples chosen by him, that the concept is derived from experience, but contains only a part of the experience of this object; or, as he expresses it in the first edition, denotes the complete experience by means of a part of it. This implies two things: first, that the concept is formed through a process of abstraction, whereby its characteristics (as common characteristics of the different things from which it is abstracted) have already been fixed; and secondly, that we are not dealing with an exhaustive concept which expresses the whole nature of an object of experience, but with a purely subjective image, in which—from causes which have no necessary connection with the nature of the thing—one part of the characteristics actually appertaining to the given class of things is made use of to denote the class itself. Thus the only ground for calling the judgment, "all bodies are extended," analytical and the other synthetical, is a generally accepted meaning (actual or assumed) of the word body.

From the *Methodenlehre* (p. 721 sq. ed. i.) it is no doubt evident that Kant regarded it as a matter of chance, so far as empirical concepts were concerned, which characteristics are employed for their formation. It is there shown that in the empirical domain there are no definitions in the strict sense, since the characteristics which belong to the object—e.g. gold or water—can never be exhaustively enumerated, and hence the rule that the definition should be complete can never be complied with. In our concepts we include such of the characteristics alone as are sufficient for distinguishing the objects; it is never certain that a word, while denoting the same object, does not mean for us more of its characteristics at one time than at another. The so-called definitions are only verbal determinations, *nominal* definitions. The same view is taken in §§ 99–106 of the *Logic*.

In calling the judgment, "all bodies are extended," analytic, and "all bodies are heavy" synthetic, Kant has therefore no other ground than a nominal definition, which happens to be universally received. This is the

main point against which Schleiermacher's criticism is directed when he declares that the distinction between analytical and synthetical judgments is only relative, because the concept is always in the process of formation. The same judgment (ice melts) may be analytical when we have already included in the concept of ice its dependence upon definite relations of temperature, and synthetical when we have not yet done so; thus the difference represents merely a different stage in the formation of the concept. Apply this to the Kantian concept: *before* the experience which justifies me in the proposition, "all bodies are heavy," I have formed my concept only from the characteristics of extension, etc.; but *after* it I can and must include the characteristic heaviness in my concept in order that it may express the complete experience, and my concept, "all bodies are heavy," thus becomes analytical. With this concept I might now proceed to further experience, *e.g.* I might go on to say that all bodies are electrical, all bodies are warm. If my concept were the expression of a complete knowledge, which indeed were impossible before all knowledge had been attained, then all judgments of this kind would be analytical.

This criticism is fully justified by Kant's own statements. I can never determine whether a judgment concerning empirical objects is analytical or not unless I know the meaning which the person making the judgment connects with his subject-word, the sum-total of the characteristics which, at this particular stage of the formation of the concept, he has comprehended in it. His progress from one meaning of the word to another takes place by means of a synthetical judgment. This judgment, we must be careful to note, is the result of an inductive inference, the only possible ground for a universal judgment drawn from experience; but for this very reason (as the *Methodenlehre* expressly emphasizes, p. 721) it cannot be necessary and apodeictic. In the case of mathematical concepts this uncertainty disappears, but only because these are formed for a purpose and contain an arbitrary synthesis (*ibid.*, p. 729).

Before a judgment, considered in itself and independently of all other considerations, can be regarded as analytical, it must evidently be assumed that there are no subjective differences between the concepts which different individuals connect with one and the same word. Assuming, then, that words have a completely fixed and circumscribed meaning, there may be judgments which are certainly analytical; in this case they are to be found in the accepted meaning of the word. The Kantian example is strictly correct, if it is presupposed that with the word body every one

always connects the characteristic "extended," and no one ever connects the characteristic heavy.

But it is just as evident that if this is the case I can no longer have any rational motive in giving utterance to such judgments, which are mere truisms and convey no information. Who would trouble himself with such judgments as "all triangles are triangular," "all squares are square"? A judgment which is analytical in this sense can never be uttered except to some one who is in danger of forgetting the meaning of a word, of thinking the characteristics of the concept only "confusedly," of extending it beyond its sphere, etc.—*i.e.* to some one for whom it is, strictly speaking, no longer analytical; for so long as he himself thinks the characteristics only confusedly, he cannot even make the judgment. Thus judgments which are analytical in this sense lead naturally to those which impart the uncomprehended meaning of a word to the ignorant—to those which no longer make statements about thought, but only about words. They are strictly analytical for any one who is master of the language; but any one who is only learning it makes synthetical judgments; his judgments, however, being grounded not upon his own knowledge, but upon a belief in the statements of other people.

4. But so far both Kant and Schleiermacher have failed to tell us how the matter stands with those judgments which are not covered by our presupposition, judgments, that is, of which the subjects are not concepts at all, and in which the verbal expression does not show what idea is present to the speaker, inasmuch as he is speaking, not of the contents of the idea denoted by the subject-term in its generality, but of a concrete thing which, though it falls under the general concept, is particular and concrete, and hence cannot be completely denoted by the subject-term.¹ But all actual and primitive judgments of experience are of this kind. Our experience is gained from the particular, and the synthesis in the synthetical judgment "all bodies are heavy" is dependent on judgments of which the subjects are definite bodies, and in the last instance on particular perception and observation. Let us realize the process which underlies any judgment of perception, *e.g.* this rose is yellow, this liquid is sour. It is quite evident, if we consider the words and their meaning, that there is a synthesis here, for yellowness is no part of the concept "rose," nor sourness of the concept "liquid"; nor from the meaning of "this," which expresses a mere relation, can anything further be derived. But then we are not

¹ Cf. Trendelenburg, *Log. Unters.*, 2nd ed., ii. 241; 3rd ed., 265.

concerned with the meaning of words ; these are always general. "This rose" denotes a concrete thing which, in its concrete particularity, can be only very imperfectly denoted by the word ; the demonstrative "this" serves merely to draw the attention of some one present to an intuition incommunicable by words, and this intuitable thing is the subject of my judgment ; it is of it that I say : it is yellow.

I might content myself with saying : this is yellow. The subject of my judgment would be the same, but would be still more indefinitely expressed in language. When I say, "this rose is yellow," there is, properly speaking, a double synthesis. First, there is a denominative judgment, "this is a rose," by which I have subsumed my concrete idea under a general image, finding that the concrete intuition agrees in the whole of its form, structure, etc., with the general image. But this denominative judgment is only incidentally made ; it does not itself appear directly, but only in its result—the subject-word by which I denote this thing.

The actual judgment before us, however, states that this, which I call a rose, is yellow. Upon what ground ? Not on the ground of a synthesis between "rose" and "yellow," but on the ground of an *analysis* of my intuition, in which, together with form and structure, is included the yellow colour in an undifferentiated unity. One element of my intuition is identical with what I call yellow, and this I then predicate of the whole in an attributive judgment.

Or, more exactly, if we describe the process from the beginning : in my intuition I have first noticed those elements in which it coincides with the general image of the rose, hence the naming of the subject ; I have then noticed one further element, which as yet is not expressed by the name, and hence the judgment.

The relation between the concepts "rose" and "yellow" of course comes into consideration as well. If "yellow" were analytically contained in "rose," as white is in snow, or cold in ice, then, generally speaking, I should have had no motive for expressly stating it ; it would have been already expressed by the name "rose." But since this is not so, I must, in order to describe my intuition completely, add the predicate yellow to the denomination "rose." Whoever hears my judgment—in the course of a description, say—performs a synthesis as he adds the particular determination of colour to the image which the word rose has awakened in him. But I, the person judging, have merely analysed my idea of the subject.

But the other example : this liquid is sour. Is there no synthesis here ? Certainly, but before the judgment, not by means of it. This judgment is distinguished from the preceding one by the fact that different senses are concerned in it. I am accustomed to determine by sight whether or not anything is a liquid, and thus the denominative judgment that is presupposed is confined to visual ideas alone. Now I put the liquid to my tongue and discover its sour taste, and I express my perception in the judgment "this liquid is sour." Before I can make this judgment I must have already referred my sensation of taste to the object which was known to me by sight ; I must be certain that what has touched my tongue is what I saw before in the glass. Otherwise I have no subject for the predicate "sour," and cannot make a judgment, cannot refer the predicate sour to the subject liquid, and express this reference in an attributive judgment. Thus my judgment analyses an act of combination which constitutes the process of perception ; but the function of the reference of the sensation of taste to its object is not the same as the function of the judgment. If we expressed the former in a judgment, it would run "that which tastes sour is the same thing that I saw before as a liquid" ; the latter, "this liquid has the attribute of being sour." I must have recognised sourness as belonging to, and being in the liquid, before I can predicate it.

5. To return to the Kantian example : if we look more closely, we shall find that there is still a sufficient ground—though nowhere indicated by Kant himself—to justify him in saying that the judgment, "all bodies are heavy," is synthetic. This ground lies, however, not in the concept "body," but in the nature of the predicate. Looked at carefully, "heavy" is really a relational-predicate ; it refers not to what a body is in itself as the isolated object of my intuition and my thought, but to what it is in relation to other bodies. The judgment, "all bodies are extended," is valid in just the same way of each particular one, even though I should think of it as being alone in the world ; the judgment, "all bodies are heavy," expresses a reference of each particular one to all others, and hence it cannot be already contained in the "concept of body in general."

If, as I believe, this, together with the historical influence of the old Cartesian definition of body, is the hidden ground of Kant's apparently unmotivated distinction, it throws light also upon his "synthetical judgments *a priori*," for the examples which he gives of these are all relational-judgments. That $7 + 5 = 12$ is a relational-judgment concerning the numbers

represented by $7 + 5$ and by 12 ; the judgment asserts their equality. The predicate "equal to B ," can of course never be contained and thought of in the subject A taken by itself, nor discovered by any analysis of it; for the idea of B is necessary as well as that of A in order that it may be thought at all. It is quite true, again, that equality with 12 is not analytically contained in the expression $7 + 5$, but must be discovered by an actual addition, by proceeding to a number which is 5 more than 7. The judgment is impossible until the addition has been completed, and two numerical expressions given thereby; but then it is analytical in so far as the intuition of the equal number of units which may be reached in either the one way or the other furnishes the ground of the judgment. It is not in the judgment itself that we pass beyond the presentation $7 + 5$, but in what precedes the judgment and makes the comparison possible; so soon as this is possible the judgment becomes a mere analysis of the given relation. It is the same with Kant's geometrical example: that the straight line is the shortest way between two points. "The shortest way" is also a relational-predicate, which cannot be contained in the idea of the straight line by itself; it presupposes comparison with other lines. But the straight line can never be presented in intuition apart from the space in which it is drawn, and which contains the possibility of other lines beside it. That which forms the ground of the judgment and is analysed by it, is the total intuition in which the straight line appears between others connecting the same points. So that even these synthetical judgments *a priori*, so far as they are immediate, are really analytical; they are not concerned with the explication of the concept expressed by the subject-word when taken alone, but with a complex object which, though partly denoted by the subject-word, contains something beyond the subject of the judgment. The ground of the judgment lies in that part of the object which is not denoted by the subject-word.

Of the principle of causality we must speak later on.

6. In the sphere of empirical notions the Kantian division of judgments into analytical and synthetical refers to judgments having different kinds of subjects, and therefore having also a different ground for their validity. His analytical judgments are those which merely unfold the contents of a concept which has received some fixed meaning in a word, without any regard to the existing object of intuition. His synthetical judgments presuppose intuition, and the synthetical connection of intuitions in experience; their subjects are "things," which fall under the word but are

only incompletely denoted by it. The former are explicative, the latter narrative.

We have shown that an analysis takes place even in judgments of perception—an analysis, however, not of the concept, but of the intuition which was formed by a synthesis preceding the judgment and not in it. We have still to test the Kantian statement that in analytical judgments we think the connection of the subject and predicate through identity, but not so in synthetical judgments. Accepting for the present the term identity, which we showed above to be inappropriate, we still fail to see how any (affirmative) judgment can be expressed without identity; *i.e.* without the consciousness of the unity of subject and predicate. The relation between subject and predicate is the same in the judgment of perception as in the judgment which deals with concepts; and the statement that identity is absent from the judgment of experience is true only if we look, not to the proper subject of the judgment of experience, but to the meaning of the word by which this subject is denoted; or when we arbitrarily limit the notion of identity to the sphere of mere concepts.

But Kant is so far right that there is a different ground for the validity of his analytical, and of his synthetical judgments *a posteriori*. The former presuppose only the habit of connecting certain ideas with a word; hence, in order that such judgments may be repeated, nothing is needed but constancy of ideas and agreement in the use of language. The latter have their final ground of validity in a fact of individual intuition, which cannot, as such, be made common property. The necessity of the former judgments has its ground in the stability which we find in our general ideas; the necessity of the latter in the laws according to which we form ideas of the particular, and are conscious of their objective reality. And here again we come upon the difference in meaning to be found in judgments, owing to the ambiguity of the copula. In the judgments which Kant calls analytical, nothing whatever is said as to the existence of their subjects, in those which he calls synthetical the subject-word denotes "objects of a possible experience."

§ 19.

Before a judgment can be passed in which the idea of the predicate is not *immediately* recognised as one with the idea of the subject, we require some mediation, both to bring about a relation between a subject

and a predicate which lies outside it, and to cause us to recognise this relation as a unity in the sense of the judgment, and to be certain of it.

1. The first and most familiar example of a mediated judgment by which a predicate is, for the first time, added to a subject and comprehended in it, is the mental act of a person who hears from some one else a judgment which he himself has neither occasion nor ground for making. All real learning is mediated judgment. The Socratic Maieutic, indeed, starting from the principle that there is no such thing as learning, nothing but recollection, is content to call into consciousness the ideas of subject and predicate by means of questions. Thus the materials alone are provided, the person questioned being left to form his judgments himself, and his conviction of their validity being based upon his own discernment. And if this process were perfectly carried out, then, indeed, all judgment elicited by it would be immediate and analytical, finding the predicates in the subjects without assistance. The questioner would merely take upon himself the work of the psychological laws of reproduction, bringing to the subject just that idea which is appropriate as predicate, for the mind in its continual eagerness to form judgments to make use of.

But teachers and taught have seldom time for this process ; it is nearer the truth to say that all learning begins with tradition, where the learner accepts the judgments he hears, and repeats them himself. In so far as he learns, what happens is that he hears a proposition, the idea of a subject is aroused in him by the subject-term, and he includes in it a predicate in respect of which the subject had as yet been undetermined. When any one learns that ice is frozen water, ice is given to him in intuition, but its mode of formation was unknown, and no reference to water contained in his intuition. For the person who learns that the earth moves, a completely new determination is added to the idea of the earth—that of movement, and he is called upon to unify subject and predicate in a way contrary to his habit of thought. Not until he has understood what he has heard, *i.e.* really made the synthesis demanded, does he reach as the result of his mental action that from which the teacher set out,—the unity of subject and predicate in the sense determined by their category. Indeed, there will still be a difference between teacher and taught as individuals ; for on the one hand the words are not absolutely fixed and equivalent in meaning for both ; on the other hand, even if they were, they leave more or less room for choice between particular degrees of meaning in their application to particular things.

In proportion as the individual is ignorant, and connects with his words only meagre ideas based upon incomplete knowledge, he must have recourse to such synthetical judgment; and by means of this his words become gradually more significant as he learns to connect with them more particular determinations. At the word "lion" the child at first thinks only of the outward, visible form which his picture-book shows to him; from tales and descriptions his idea becomes enriched with all the attributes and habits of the animal; while the zoologist possesses the idea complete.

The more perfect the knowledge, and the richer therefore the meanings of words, the less room there is for syntheses such as these, in which something new is learned. In the end synthetical judgment would be limited to that sphere which can never be denoted by words—to the particular fact as it is for any one who has not observed it himself, and to the particular changes and relations which can only be expressed by temporarily valid judgments. All judgments about the meaning of the word, the general idea of the object, are then analytical. (This is what Schleiermacher means when he limits the proper synthetical judgment to the sphere of particular facts. *Dialectik*, § 155, pp. 88, 105.)

2. In learning by tradition the ground which the learner has for the certainty of the judgment is merely the authority of the teacher; objective validity is accepted from confidence in his knowledge and veracity; he is believed. Since all narrative judgments are necessarily synthetical for the hearer, they are those which, by their nature, appeal to and demand the belief of the hearer. Besides one's own perception (and whatever may be inferred from it) there is no knowledge concerning the particular except by way of belief, which, in this case, is historical belief.

3. There is another process, similar to those of teaching and narrating, which adds predicates to a subject capable of further determination, and calls upon us to unify them. It is introduced by the inward play of our ideas, as determined by the laws of associative reproduction, and the activity of imagination as led by similarity. When we are conscious of any object through perception or recollection it does not merely call up those predicates which agree with its contents as now presented and lead to immediate judgments. Recollection, association, and analogy bring forward other ideas, which tend to combine as predicates with the subject, although they are not as yet contained in the idea of the subject now present. From one point of view the common case already dealt with in § 9, p. 55, is an instance of this—the case in which

visual images of particular objects call up the recollection of their other qualities, and these are forthwith attributed to them as predicates. (This is a bunch of grapes—this is sweet; this is a stone—hard, etc.) But though, in such instances, association takes place with such absolute certainty that when the judgment is made the idea to which it refers is already complete, we do find cases differing from these by imperceptible degrees, in which the fusion does not take place at once, but the idea called up remains—to use Herbart's phrase—in suspense, and only brings the expectation of a judgment. This is most obvious when different and mutually-exclusive ideas are excited at the same time, and a struggle takes place between them, as, for instance, when I see a human figure at a distance, and it excites in me the images of both *A* and *B*, seeming to resemble first one and then the other.

Upon such associations are founded more particularly all judgments which extend to the future. They can never proceed from analysis of the present, but are mediated by some process of inference. "The snow will melt." I cannot tell this by looking at it, but, led by former experience, I add in thought to the present intuition a predicate not yet contained in it.

4. The universal tendency to form judgments and to connect what is new with what is familiar, is so strong that, when there is no check, the same processes which suggest the predicate may also give rise to the judgment; *i.e.*, they may produce belief in the objective validity of the proposed synthesis. In proportion as thought is undisciplined it is incautious, and slow to recognise the difference between purely subjective and psychological combinations and those which are objectively valid; there is a greater tendency to believe everything which occurs to one, especially when strongly supported by a wish or an inclination. The recollection of one or a few cases in which a predicate *B* belonged to a subject *A*, is generally sufficient to make us attribute the predicate *B* to every subject which at first sight resembles *A*; and we are often scarcely conscious of the process of inference by which the synthesis of the judgment has been brought about. This credulity of our natural thought, while the source of many deceptions, hasty assumptions and superstitious opinions, is at the same time the indispensable condition on which alone we can profit by experience, and learn to pass beyond what is immediately given.

As we found with the generalization of ideas that the process has not to be learned but rather to be checked, and the power of distinguishing cultivated, so it is also with judgment when it passes beyond what is given. Our

natural tendency is always to think of many predicates and attribute them to the subject; what we have to learn is caution and doubt, to distinguish between the valid and the invalid, and to enquire as to which of these syntheses are objectively necessary, and which are merely forced upon us by nature and habit.

5. When some check prevents the judgment which passes beyond what is immediately given from taking place, there arises the question. This may be in two ways. In the one case we seek to complete a given idea in accordance with some previous experience, the desired completion not being offered by any absolutely certain association. Such is the case when I cannot recall any idea agreeing with a new and unknown object—what is that? Or when I seek the subject to a given attribute or activity—who is speaking? What is that shining? Or again, when I am uncertain what attributes or activities belong to a thing besides those perceived—what does that taste like? In the second set of cases a completion is offered by association, but the certainty of its validity is wanting; the judgment is sketched out in thought, but is not passed as a judgment. In this way there arises the question which seeks a decision as to the validity of a given predication—is *A* really *B*?

6. Both the question which aims at completion, and that which aims at confirmation, presuppose psychologically the simple and immediate judgment which is inseparably connected with the consciousness of its validity. I can only seek for that of which I have at least a general and indefinite idea; nothing but experience of perfect syntheses can create in me the desire of completing an imperfect idea by the addition of some further element. I must have acquired the habit of referring sensations to definite things before I can arrive at the point of seeking the thing belonging to a sensation which is presented without any certain reference.

In the same way the question which demands yes or no for an answer seeks a certainty which must have been experienced in immediate judgments before it can be sought. Its search for an answer implies the thought of the certainty connected with predication in other cases.

In simple and immediate judgments—that is a tree, that is red, snow is white, coal is black,—the certainty of validity is given inseparably with the synthesis of subject and predicate. I cannot ask whether coal is black, or snow white, whether the object before me is red or a tree. As soon as I am conscious of the two ideas I am also conscious of the necessity of the synthesis between them.

It is not until we attempt to pass beyond what is given, not until we would unite with the idea of the subject an element not yet contained in it, that the two elements which are united in the immediate judgment, the simple or complex synthesis between subject and predicate and the consciousness of its necessity and objective validity, can be separated. The question "is *A* really *B*?" can only arise in that sphere of thought in which judgments are formed by a process of mediation.

Hence it follows that the first stage in the formation of judgments—our psychological starting-point—is not invariably (as *e.g.* Bergmann holds ¹) a "predication without quality," a mere "idea" in which subject and predicate are thought together, the judgment taking place only when the "critical reflection" upon its validity is added. In the simplest cases the two elements are not found apart, and we cannot show the meaning of predication in general unless we start from objectively valid predication, as it occurs in the immediate positive judgment "*A* is *B*."

All such theories overlook the fundamental importance of the distinction between immediate and mediated judgments, a distinction which in logic is not less important than that between analytical and synthetical judgments in the transcendental philosophy.

7. In the question "is *A* *B*?" the elements and the connection between them have the same meaning as in the judgment. The question expresses the expectation of a synthesis between *A* and *B*, and moreover of a valid synthesis, not merely of an arbitrary combination. The judgment is already conceived, but it still needs the seal of its confirmation, for the certainty of validity is wanting. This invention and essaying of judgments which pass beyond what is given and the immediate judgments based thereon, represents the active movement and progress of thought in inventive action in the sphere of the judgment. We may say at once, questioning is thinking. Doubt, conjecture and expectation are only particular varieties of the same state. They differ in the degree to which we are conscious of the absence of a ground for the completion of the judgment; they are alike so far as concerns the meaning of the synthesis between subject and predicate.

8. The decision of a question may sometimes come from the elucidation

¹ *Reine Logik*, 1879, p. 42, 169; cf. the objections raised by Schuppe, *Vierteljahrsschr. für wiss. Phil.*, iii. 484; my own article, *Vierteljahrsschr. für wiss. Phil.*, v. 1, p. 97; and Bergmann's reply in the same journal, v. 3, p. 370. Brentano's view is dealt with in my *Impersonalien*, p. 58.

tion and completion of the subject-idea itself; when this idea is the intuition of a particular object, we discover by means of more exact apprehension and observation what was not formerly observed. Such is the case when, seeing a white powder, I ask whether it is sweet and put it to my tongue; I have then made an addition to my perception, and my answer is an analytical judgment proceeding from the new perception. When my subject-idea is not given in intuition, reflection may bring more complete recollection, and again enable me to make an analytical judgment. But if these attempts fail, then the only way of arriving at a decision is to seek for mediations which may confirm the attempted synthesis, and the mediation to which the spoken question appeals in the first instance is information from some other person.

9. It may be that a ground for the desired synthesis, which would enable it to take the form of a judgment, can be found neither by the elucidation or completion of the subject-idea, nor yet by mediating ideas. Then either the question remains unsettled and we fail to become conscious of its objective validity, or else the negation arises, from the fact that the subject-idea immediately or mediately repels the predicate-idea.

Reserving for a later investigation the former case—that of the falsely called problematical judgment—let us now turn to the negation.

CHAPTER IV.

THE NEGATION.

§ 20.

THE NEGATION is always directed against an attempted synthesis; hence it presupposes some prompting, either from within or without, to connect the subject and predicate. The object of a negation must be either a completed or an attempted judgment, and for this reason we cannot regard the negative judgment as a species equally primitive with the positive judgment, and co-ordinate with it.

1. Following the example of Aristotle, many logicians have begun by defining the judgment as either affirmative or negative, making this twofold aspect a part of its definition. This is true in so far that judgments when formed can be exhaustively divided into affirmative and negative, and that when we form judgments, it can only be by affirming or denying a predicate of a subject. But the definition is false if it implies that affirmation and negation as forms of judgment are equally primitive and independent of each other, for the negative judgment presupposes the attempt, or at least the thought, of an affirmation—*i.e.* it presupposes the positive attribution of a predicate, and has its meaning only in contradicting or annulling such an affirmation. Or rather, the primitive judgment should not be called affirmative at all; it would be better denoted as positive. The simple statement *A* is *B* is an affirmation only when opposed to the negative judgment and in so far as it rejects the possibility of a negation; but that we should think of the possibility of a negation, or that a question should be raised which must be decided by yes or no, is no necessary condition of the judgment *A* is *B*.¹

2. When we consider that only a finite number of predicates can be affirmed of every subject, while an incalculable number can be denied, it is at once evident that the negation has no meaning except in opposition

¹ Cf. Beneke, *System der Logik*, I, 140 sq.

to an attempt at a positive statement. No one would think of making all the negations which in themselves would be possible and true; he could have no motive for so doing. Before there can be any sense in saying: this stone does not read, does not write, does not sing, does not think; justice is not blue, is not green, is not five-cornered, does not rotate, etc., there must be a chance that some one will attribute these predicates to the stone or to justice.

All that the negation aims at is to confine within limits fixed by the nature of the given ideas the subjective and fortuitous movement of individual thought, which in fancies, questions, conjectures and erroneous statements extends beyond what is objectively valid. Thus it presupposes a subjectively arbitrary and contingent thought—the limitless sphere of the false, which consists in just this deviation of individual thought from the thought which is objectively necessary and universally valid, and it is a matter of chance how the negation may arise with any particular individual. It is impossible to lay down universally and exhaustively what must necessarily be denied of any subject.¹

3. We cannot further define and describe what we mean by “not,” nor what is the significance of the negation; we can only recall what always takes place in the process. The true meaning of the proposition *A* is not *B* may, however, be elucidated by contrasting it with incorrect and artificial interpretations.

In the first place subject and predicate, taken each by itself, are thought in the negative propositions in exactly the same way as in the positive; the words stand for the same ideas. When I say “snow is not black,” snow means the same as in the judgment “snow is white,” and black the same as in the judgment “coal is black”; no effect is at present produced upon them by the negation, they have their usual contents. The question raised by Aristotle (*De Interpret.*, 2 and 3) as to whether there is a *ὄνομα ἀόριστον* (ὄνκ ἀνθρώπος) and a *ῥῆμα ἀόριστον* (ὄν κάμνει) which could appear as subject or predicate of a judgment does not refer to the essence of the negative judgment, but only to the nature of the subjects and predicates which can be used in a judgment, and can be affirmed or denied of each

¹ Kant, *Kr. d. r. V. Methodenlehre*, ed. 1, p. 709 (a passage which Windelband points out in the *Strassb. Abh.*, p. 169), says: In respect of the content of our knowledge in general, negative propositions have the peculiar function of simply averting error; hence also negative propositions which aim at averting false knowledge when there is no possibility of error may indeed be very true, but they are devoid of meaning, and for that reason often very ludicrous.

other. A natural and primitive idea can never be denoted by the expression not-*A* or not-*B*, but it is always possible that these expressions may be serviceable as abbreviating formulæ, under which definite subjects, or at least predicates, may be thought. But then they act as symbols of these, and when a statement is possible at all some predicate is affirmed or denied of such subjects, or such predicates of some subject; the judgment not-*A* is *B* and the judgment *A* or not-*A* is not-*B* are affirmative, the judgments not-*A* is not *B* and *A* or not-*A* is not not-*B* are negative. Aristotle has given the right explanation of this. It is true he attempts all possible combinations with unlimited subjects and predicates (*De Interpr.*, 10), but he does not make into a particular kind of judgment those in which a subject or predicate of the form not-*A* appears. When, on the other hand, Kant places the infinite¹ or limiting judgment beside the affirmative and negative as a third kind (*Kr. d. r. V.*, § 9)—the soul is not-mortal, that is, belongs to the infinite sphere which remains when I take away the mortal—then he starts from a view of the judgment which we shall have to oppose later on, a view which takes its essential aim to be the placing of a subject within the sphere of a concept. By this view he is enabled to bring out a distinction between the propositions “the soul is not mortal,” and “the soul is not-mortal”; he does not, however, thereby gain a third judgment in addition to the positive and negative, but is himself obliged to allow that in general logic there is no ground for considering a judgment of the form *A* is not-*B*, in which a purely negative predicate is attributed to *A*, to be anything more than an affirmative assertion.

4. In opposition to the attempts to regard all negative judgments as affirming a predicate not-*B* of a subject is the largely accepted tradition that the negation affects the copula, by which we are led to speak of the affirmative or negative quality of the copula. This is so far true that the negation does not lie in the elements of the judgment, but only in the manner in which they are referred to each other. But it is incorrect to oppose a negative to an affirmative copula. If by copula we understand the expression of that mental act by which a predicate is referred in the judgment to a subject as agreeing with it, as activity or quality, then a unification is expressed by it. But that which holds subject and predicate apart, and prevents the attainment of unity, cannot be a kind of unification.

¹ The name comes from an awkward translation and application of the *ἀόριστος*, which Aristotle had used, not of the judgment but of its component parts. Cf. Trendelenburg, *Elem. Log. Ar.*, § 5.

A bond which divides is nonsense. We should rather say that the proper copula (in language the verb-ending) has exactly the same meaning in the negative as in the affirmative judgment ; that is, it expresses the positive relation in judgment between subject and predicate, the attributing of the predicate to the subject, and it arouses the thought that the predicate belongs to the subject. For it is just this thought (which is also contained in the question) which is declared to be false ; it is just this attempt at synthesis which the negation prevents. The copula does not convey the negation, but is the object of it. There is no such thing as a negative, but only a negated copula.

Three elements, then, can be primarily distinguished in the simple, positive judgment. These are the subject, the predicate, and the thought of their unity (in the particular sense of the synthesis determined by the categories), this latter being the object of the certainty which finds utterance in the positive judgment. The same three elements are present, and have the same force, in the negative judgment ; but the negation is added as a fourth in language as well as in thought. This prevents the completion of the attempted synthesis as valid, and it opposes its "No" to the whole proposition *A* is *B*. The object of the certainty, which gives to the negative proposition as well as to the affirmative the force of a statement, is just this "No." The judgment "*A* is not *B*" means, "it is false, it must not be believed that *A* is *B*": hence, immediately and directly, the negation is a judgment concerning a positive judgment that has been essayed or passed ; only indirectly is it a judgment concerning the subject of this judgment.¹

5. If the negation were formed by means of a negative copula—if, therefore, we had to regard the "is not" in the judgment "*A* is not *B*" as the expression of a simple act of thought,—then, to be consistent, those logicians who ascribe to the "is" of the affirmative judgment the force of stating the existence of the subject should hold that the "is not" in negative judgments denies the existence of the subject. But this is certainly not the case. As a rule, the judgment *A* is not *B* presupposes the existence of *A* in all cases when it would be presupposed in the judgment *A* is *B* ; *i.e.* whenever it is part of the meaning of the words. But the negative judgment in itself states no more concerning existence or non-existence than the affirmative judgment. "Socrates is not ill" presupposes in the first place the existence of Socrates, because only on the presupposition of his existence can there be any question of his being ill. But since the negation only declares it to

¹ See Appendix A.

be false that Socrates is ill, the presupposition contained in it is certainly not so definite as in the affirmative judgment "Socrates is ill"; for this may also be denied because Socrates is dead. (For further, see below, § 25.)

§ 21.

The negation follows the different forms of the positive judgment, and has for its object those different relations between subject and predicate which constitute the different meanings of the unity between them. Hence, when the judgment contains a manifold synthesis, the negation is ambiguous. Directly it can express nothing existent—neither attribute, activity, nor relation.

1. In prohibiting an attempted statement, the negation adapts itself to all the different kinds of propositions; and whatever these may state, it declares to be false.

To the judgment which would allow two ideas to coalesce as wholes, the negation opposes their difference. Such propositions as "apes are not men," "red is not blue," "freedom is not license" prohibit a threatened confusion, or a conscious annihilation of the distinction fixed between the objects. This negative judging brings into consciousness by an express act what was already contained unconsciously in the formation of our ideas and in their verbal denotation. That is, it makes us conscious of that distinction between different ideal contents which, being always drawn in the same way, gives us a fixed plurality of ideas to which the plurality and difference of words corresponds. This act of distinguishing, through which our ideas are formed, must have already taken place before it can be brought into consciousness and confirmed by the negative judgment.

When opposed to the qualitative judgment, the negation prohibits the relation of inherence between a subject and an attribute ascribed to it. The relation of inherence in itself even forms the ground for the negative judgment. The proposition "lead is not elastic" does not deny that the subject is a unity of thing and attribute, but that the attribute, which really belongs to the subject and which is not expressed in the statement, is the one in question. I cannot find in the subject "lead" the attribute denoted by "elastic"; the actual attributes of lead are other than elasticity. So that here, also, what the negation finally emphasizes is the fixed difference of certain qualitative ideas. The same holds good of judgments whose predicates are activities.

2. Now according as the movement of thought (see § 11) passes from the attribute or activity to the thing to which it belongs, or the reverse, the object of the negation also varies (the variation being expressed verbally by emphasis or position). It may either lay stress upon the fact that a given thing, with fixed determinations, does not possess the attribute or activity in question, or emphasize the fact that it is not this thing to which a given attribute or activity belongs. The judgment "I have shouted" is as false when no one has shouted at all as when it was someone else who shouted. In the first case, the reality of the predicate is denied, in the second its relation to the subject; in the latter, the subject is generally emphasized, or the negative placed before it (*I* have not shouted; it was not *I* that shouted). Finally, the negation may mean that neither predicate nor subject is to be found. Starting from the general view of the negation, the proposition "the fire does not burn" is a *contradictio in adjecto*. How can the predicate "burn" be denied of the subject fire? And yet we do not hesitate to say it; we expect to find the burning fire, the negation says "it is false that the fire burns," and this proposition is true when there is no fire at all. This applies especially to the negation of the Impersonals. "It does not thunder" means either that the name given is false—that what is heard is not thunder, or it denies the phenomenon itself, which is indicated by the predicate—the negation extending also to the presupposed reality of the subject.

3. Similar modifications appear in the case of relational judgments. That is, the synthesis of the positive judgment being here threefold, we cannot tell from the simple negation of the relational judgment what the statement of the negation is primarily aimed at, nor what is the point which the person making the negation has in view. If the judgment "*A* is walking home" be false, what is denied may be merely the direction in which he walks, or it may be the kind of movement (if he is riding or driving), or the fact that he is going away at all; or, finally, it may be disputed that it is *A* who is walking home. The proposition "*A* is not walking home" may have any of these meanings. This ambiguity of the negation, which at best can only be dealt with by emphasis, is a new proof that its only force is to state the falsity of the positive judgment as a whole, and that it is unable by itself to constitute any definite relation. In the case of causal relations, which are expressed by transitive verbs, the negation may be directed merely against the particular object of the activity, which itself really takes place, or against the activity itself, or

against the subject to which the activity is ascribed. "I have not written this sentence" may either deny the whole fact that the sentence in question has been written, or it may emphasize either *this sentence*, or *I*. "I have written nothing" denies all writing whatever by the negation of every possible kind of object of writing; "I drink no wine" denies only a particular kind of object.

4. In the same way, when an unconditionally valid judgment is denied, the negation merely tells us that the statement made by the unconditionally valid judgment is false, the statement being that the predicate is contained in the subject-idea, which constitutes the meaning of the subject-term (plants do not feel; light is not matter). How far such negations may be ambiguous (*e.g.*, the triangle is not equilateral) will be discussed below, § 25.

When opposed to temporally valid judgments, the negation applies only to their validity for the time stated, and cannot therefore say anything as to the condition of the subject at any other time. The judgment "this watch is not going" declares that the temporally valid judgment "this watch is going" is false; and what it means by that is that it is not going *now*; this negation leaves it undecided whether or not the watch goes at other times.

5. Attempts have not been wanting to supplement the poverty of the negation as merely annulling some other statement by giving it the force of a directly significant statement. In this way, what is stated by the negative judgment would stand opposed as independent and valid in itself to what is stated by the affirmation; and thus negation and affirmation, as forms of statement, would occupy the same rank.

Aristotle himself has to some extent given a precedent for this. He makes the affirmation and negation (particularly in the *Metaph.* Θ, 10, 1051, b 1 sq.) correspond to a unification (*συνκείσθαι*) and separation (*διηρησθαι*), and in so doing he makes it the primary meaning of the relation between subject and predicate in the affirmative judgment to express something compounded (from substance and accident). We have already (§ 14, p. 80 sq.) shown this view to be impossible, since the predicate of the judgment can never be looked upon as something existent; least of all as an existent thing which can be thought apart from the subject. There is no meaning in saying that, in the existent, "commensurable" is always separated from the diagonal of the square; separation and unification both belong to thought alone. But, for the same reason, neither can the nega-

tion correspond to any separation. In the first place, elements actually separate in the object could have no mutual reference whatever; and it would be inexplicable how these separate elements should be found together in *one* act of thought. Moreover, here again, since the predicate never stands for anything but an idea, we cannot say that it exists anywhere in such a way that it can either be united with the subject, or remain separate from it. The view that the proposition "man is white" is the expression of a unification of the substance "man" with the idea of "white," because the latter has an independent existence, would be impossible, except as a consequence of the Platonic doctrine of ideas. It is due to this alone that the relation between a thing and a predicate which is incompatible with it can be denoted as a "perpetual separation."¹

On the other hand, Spinoza's well-known proposition "*Determinatio est negatio*" has been made use of to express a view which goes so far as to transfer the negation into the nature of things themselves, thus ranking the negative judgment as the original expression of knowledge of these. Trendelenburg has rightly drawn attention to Thomas Campanella as one of the most decided supporters of the opinion that all things consist in yes and no, being and not-being; that everything is this particular thing only because it is not something else. "Man is," that is his affirmation. But he is only man because he is not stone, not lion, not donkey; hence he is at once being, and not-being. Spinoza has the same meaning when he says "*Determinatio est negatio*"; a figure is determined in so far as it is not the space surrounding it, and thus it can be thought of only by the aid of negation—as a limitation, *i.e.* negation of the infinite. But such views always involve a confusion between the negation itself as a function of our thought, and the presupposed objective ground of this negation—the exclusive individuality and uniqueness of each one among the many things which are real. That which they are not never belongs to their being and nature; it is only our thought, which, in making comparisons, brings such alien elements into contact with them. All that concerns us is to recognise why we have need of these subjective expedients in order to know the world of reality in which no counterpart of our negating thought is contained. It is only by a constant confusion between negation in thought and those real relations in being which are very imperfectly expressed by mere negation

¹ Prantl has very justly noticed the deficiency of the Aristotelian theory in this respect in his *Geschichte der Logik*, I, 118, 144 sq. Elsewhere Aristotle expressly recognises that the negation belongs only to the sphere of thought (*Metaph.*, vi. 4).

that the Hegelian logic succeeds in presenting it as a real power, and as the nature of things; but unless we may admit this confusion once for all, as may well be done after Trendelenburg's penetrative criticism, it must be pointed out at almost every step of the Logic.

§ 22.

When the attempt to attribute a predicate to a subject is prohibited by the negation, the ground for this prohibition lies either in the fact that the predicate in question is wanting to the subject (or, in the case of certain relational judgments, the subject to the predicate); or else in the fact that the subject, or at least one of its elements, is incompatible with the predicate. The mere statement of the negation does not show which of the two it is.

Further, the negation is equally unable to explain, or even to give complete expression to, those relations between ideas by virtue of which they are incompatible (the so-called contradictory and contrary opposition).

1. When a negative judgment is not inferred—when, therefore, the negation is not mediated by interposed terms—we have nothing but the given subject, and the attempted predicate, as materials for our negation. Hence the ground for prohibiting the predicate must lie in the relation which is given between the ideas of subject and predicate.

There are two ways in which this may be. Either the predicate is wanting in my subject-idea (or, in a relational judgment, one element is wanting in the whole ideal complex), or it is excluded by the subject-idea (or by the ideal complex present). The ground of the negation may be either a deficiency (*στέρησις*, *privatio*) or an opposition (*ἐναντιότης*, *oppositio*).

2. It may be that the subject-idea is concrete and particular, an object of intuition, and the attempted positive judgment a temporally valid one which is to be denied in the same sense in which it was supposed to be asserted. In such a case the negative judgment rests upon my consciousness of not finding the predicate in my intuition of the subject—upon immediate knowledge of the difference between the subject as it really is, and another conceivable thing possessed of the predicate, upon my consciousness, that is, of a deficiency of determination in the subject. "This watch is not going," "this flower has no scent," "the invalid is not moving," "the sun is not warm to-day,"—all these judgments proceed from

my consciousness of difference between what is given and what is merely thought of, between this watch and one which is going, between this flower and one with a scent ; it is the fact of my bringing a more fully determined idea to what is actually given which gives rise to my judgment. In the case of relational predicates, again (Socrates is not here), the complex of things expressed by the attempted judgment (Socrates and myself in the same portion of space) differs from the complex given to intuition (Socrates is absent from the portion of space in which I am).

The deficiency becomes more striking in proportion as the more complete idea is familiar and easily produced for comparison, and in proportion as the missing predicate seems to belong more closely to the whole complex. The absence becomes a deficiency in its narrower sense—the absence of something which should be there—when reference to a purpose, or an æsthetic law, demands the completeness of the predicates. Logically, however, such references, which give a colouring of disappointment to judgments such as “he does not see,” “he does not hear,” “he will not understand this,” “the sentence has no meaning,” have no value except to draw attention to the deficiency, and to enable us to realize the standard of comparison. They form no ground for any particular kind of negation as such.

3. The same absence of a predicate takes place also in the case of general ideas. The negative judgment may be based upon the fact that the predicate does not form a part of the idea which constitutes the meaning of the subject term : “plants do not feel,” “water has no taste,” etc. Comparison with things related in other respects, of plants with animal organisms and of water with other liquids, forms the ground for the privative judgment ; something which, to judge from the nature of the subject in other respects, might belong to it, does not belong.

4. The same ground for a negation would be present, if the attempt were made to attribute to a more general idea predicates belonging only to the particular ideas which fall under it, and are more fully determined. It is no part of the general idea of the triangle that it is plane, nor that it is spherical ; nor is it part of the idea of the plane triangle that it is right-angled or acute-angled. It is not contained in the idea of man in general that he is either black or white, straight-haired or woolly-haired ; nor in the general idea of motion that it is progressive or rotatory. Still, we cannot express this mere indeterminateness of the subjective general idea by the simple negation of those predicates. The propositions, “the triangle is not

spherical," "is not right-angled," "man is not black," "motion is not rotatory," would be understood in a quite different sense, as stating that the predicate was wanting to all the objects denoted by the name. So strong is the habit of passing at once from general ideas to the most concrete and determined ideas in which they are contained, that the proposition "the triangle is not rectangular," though quite true in itself, would be misunderstood. It must be expressed "the triangle is not necessarily rectangular," or "not all triangles are rectangular." Cf. below, § 25.

5. With the negation which is based upon the privative relation, and hence upon a simple difference, we may contrast that which arises from the fact that one element of the subject-idea repels the predicate-idea. Here we cannot even think, as we do in the case of privation, that the predicate might belong to the subject. (The same thing takes place in the case of relational-ideas; it is false that *A* is to the left of *B* either because *A* is not near *B* at all, or because it stands to the right of *B*; the one relation prohibits the other.) This brings us to those relations between ideas by virtue of which they are able to exclude each other as predicates of one and the same subject.

6. Suppose we are dealing with a denominative judgment, in which the attempt is made to unify subject and predicate as a whole with a whole; then the exclusiveness of the relation between different ideas is due to the fixed determination and differentiation (within the different categories) of our ideas—a determination and differentiation presupposed in all judging, since it is the condition of the continuity and harmony of consciousness itself. "Socrates is not Crito," "wood is not iron," "red is not blue," "seeing is not hearing," "right is not left,"—such judgments are based upon the fact that we have a multiplicity of ideas distinguished from each other with certainty, and guarded against all confusion and mistake; and all the judgments can do is to remind us that these differences are always present (§ 21, 1). The knowledge that two ideas differ is, indeed, generally earlier than the knowledge of how they differ; for in order to show how they differ I must finally have recourse to elements of which I simply know that they are different. I distinguish with certainty between my friend *A* and my friend *B* before I realize to myself what it is wherein they differ. Suppose I did realize it, and become conscious that the one is fair, the other dark; the one of a round, plump figure, the other lean and angular; the difference between fair and dark, round and angular, lean and

plump would still remain; and here all that I can finally say is *that* they differ, not *how*.

In treating of the affirmative judgment we were obliged to lay down (§ 14, p. 83 sq.), as the presupposition of its possibility, a principle of agreement, according to which ideas are recognised as like with unfailing certainty, and upon which all possibility of certainty in an affirmative judgment depends. We may find a similar ground for the negation in the proposition that different ideas are immediately and unfailingly recognised as different, and that it is impossible that there should be any mistake as to whether two ideas present in consciousness are different or not. If the formula "*A* is not *A*" had not been misapplied to denote everything and anything, we might make use of it to express that *A* is different from all other ideas; that everything which is thought is just itself and no other. It would then state both a law of our differentiating thought and a fundamental psychological fact.

Does any one appeal, in opposition to this, to the fact that we do confuse many things, and so fall into error? Then we answer, first, that confusions with reference to things take place because their differences are not reproduced by our momentary ideas; as *e.g.* when I mistake an artificial flower for a natural one, because I have observed it superficially—in this case my ideas do not differ as they would have done had my observation been more complete. Secondly, that confusions take place in consequence of the imperfect reproduction and constancy of our ideas, one substituting itself for the other in course of time. Thus I may greet a stranger as an old acquaintance because the image of my acquaintance has faded, and under the impression of what I now see is represented as different. But this does not mean that it is possible to hold that two ideas are not different when they are present as different in consciousness, and are retained undisturbed during an act of judgment. On the contrary, all unity and clearness in self-consciousness depends upon this power of the negation to hold asunder the manifold presented to us, and to guard it from blending in confusion; and in the same way the one possibility of being certain of the validity of a judgment, and hence the possibility of judging, depends upon an immediate and perfectly certain consciousness of difference. Where this could not be presupposed,—as, say, in a case of insanity,—all community of thought would cease.

7. In the case of attributive judgments, it is more difficult to investigate the conditions of the negation. The same thing may have different attri-

butes, and different things the same attribute ; this simple difference in the ideas of attributes affords no ground for denying attribute β of thing A because it possesses the different attribute α ; or for denying attribute α of B because A has it ; though there is ground for saying A is not B , α is not β . The question is : what is necessary in order that we may say of a thing A that attribute B cannot be united with it ? Clearly it can only be said when one of the attributes of A stands in such a relation to attribute B , that they cannot both together belong to the same subject. Thus one particular colour on a surface—*e.g.* white—excludes all other colours. The fact that snow is white enables me at once to deny all other colours of it ; the fact that a line is straight enables me to deny the predicate curved of it ; and so on. This is true also of verbal and relational predicates. Sitting and standing, standing still and walking, right and left, equal and greater or smaller, are all mutually exclusive ; if the one is true of any thing, the other must be denied of it.

8. The expressions “opposition” and “opposed” have, like that of identity, become almost useless owing to the different meanings given to them, and owing to the frequently vague relation between what was called opposition and negation on the one hand, and difference on the other. The conflict of particular ideas has been confused under the same name with the contradiction of judgments ; and with reference to the more special relations of conflicting ideas there is confusion of language like that in the Tower of Babel. Let us attempt to formulate the distinctions from the nature of the case.

There can be no reasonable ground for denoting as conflict or opposition the mere difference of ideas which is the condition of all thought. Just as all sorts of things co-exist without conflict in space, and present in their different attributes the varied image of the world, and its perpetual change in their various activities, so our thoughts contain an incalculable manifold of ideas between which there is no conflict, although each is looked upon as particular and indeed differentiated. Negation distinguishes them and suffices to give to each its due. In themselves there is no more conflict between the ideas of man and lion than between those of black and red, or black and white. Conflict arises only when rival claims are made upon the same thing ; and a relation of conflict between ideas is found only when they meet as attempted predicates of one and the same subject. Thus conflict can appear only in the sphere of subjective thought, which is trenching upon the false, for in the truth every subject is in undisputed

possession of one predicate. The members of definite, larger or smaller, groups of ideas are related in such a way that, when tried as predicates of the same subject, they repel and exclude each other; and this not because of something peculiar in the nature of a particular subject, but because of their own contents. We will call them by a current term "incompatible," for "incomprehensible," which would most accurately express the case, has too unfamiliar a sound. This relation exists originally between ideas of attributes, activities and relations; derivatively it exists also between ideas of things, in so far as these appear as predicates in judgments of denomination and subsumption, for two substantival ideas are contradictory in so far as they contain determinations which cannot be combined.¹

9. We cannot deduce from any general rule what ideas are incompatible; this is determined by the actual nature of the ideal-contents and their relations to each other. It is possible to imagine our sense of sight so constituted that we should see the same surface illuminated by different colours (as, indeed, it does emit light of differing refrangibility); just as we distinguish different overtones in one note, and the particular notes in one chord. It is purely matter of fact that different colours are incompatible as predicates of the same source of light, and that different sounds as predicates of the same source of sound are not so, and that the tactile sensations of temperature and pressure can be referred to the same subject in the most different combinations (cold and hard, cold and soft, etc.).

Nevertheless a general statement can be made of the ideas in reference to which we are most frequently conscious of incompatibility, and which are most liable to come into actual conflict. Evidently they are those which, because they are most homogeneous and most nearly akin, because they belong to homogeneous and similar subjects, can be most easily tried as predicates together; those which, just because they are akin, present themselves together as the more special determinations and modifications of a general idea. It is for this reason that the incompatibility with which we are most familiar is that of different determinations which fall under the same general idea; such as the determinations of colours, of qualities in the sense of touch, of forms, numbers, etc. It is the incompatibility of these which is at once obvious to us because we have had most frequent opportunity of becoming conscious of it. No one thinks about the incompatibility of man and kangaroo, of melting and flying, because no one will ever

Plato, in the *Phædo*, ch. 52, 103 D. sq., investigates in a masterly manner τὰ ἐναντία ἀλλήλα οὐ δεχόμενα.

have occasion to ask whether a creature is a man or a kangaroo, or whether something is melting or flying. The incompatibility between black and white, young and old, standing and lying, strikes us constantly, for the cases are innumerable in which the question may be raised whether something is black or white, whether a person is old or young, whether something is standing up or lying down. Hence the delusion of thinking that in the different determinations of a general idea we come upon a specific relation of incompatibility, which belongs to them quite apart from any judgments—as if black and white, straight and crooked, had some special enmity to each other as being sons of the same father.

10. Incompatibility has no degrees; and when we are dealing merely with the ground of the negation, the relation between black and invisible does not differ from that between black and blue, nor the relation between black and blue from that between black and white. But with the relations upon which incompatibility is based are connected others, which refer merely to the magnitude of the difference, and are easily confused with it—the relations which are commonly called opposites. Black and white are opposed in quite a different sense from that in which black and blue are opposed; the difference between the two relations is based upon the interval between homogeneous ideas, which interval gradually increases and finally reaches a maximum. It is in this sense that we oppose day and night, mouse and elephant, a drop and an ocean, to each other. For our feeling there is a sharp distinction between the sudden transition from one extreme to the other, and the transition to the idea nearest in similarity, particularly where transitions constantly take place, and so establish a connection between the more proximate differences. And when the impressions upon our feeling are themselves of an opposite nature, being beneficial and pleasing on the one hand, painful and displeasing on the other, the impression which is given of the magnitude of the objective difference is rendered especially intense by this effect upon feeling. Thus it is that light and darkness, good and bad, pain and pleasure, beautiful and ugly, are opposed to each other; and we need not point out that it is always presupposed that such ideas are homogeneous and fall under a more general idea which is common to both. But we should prefer to call this relation contrast, to avoid confusing it with that of incompatibility. It was noticed even by Aristotle, and has been shown with much acuteness by Trendelenburg,¹

¹ Trendelenburg, *Logische Unters.*, xii., 2nd ed., II. 151; 3rd ed. 171. Cf. *El. Log. Arist.* with § 10. *Arist. Cat.*, 6. 6a 12, and the passages in Waitz with *Cat.*, 11 b, 34.

that the increase of difference in such an ordered series of ideas, and the position of the extremes of the series, is represented to us by a spatial image. The spatial "opposite" represents geometrically a maximum of difference in direction and gains a physical significance from pressure and counter-pressure, action and reaction, and it finds itself re-echoed in our own will as contrast is in feeling. But this relation also, like that of contrast, is distinguished from the many incompatibilities only by characteristics which have no direct reference to the negation.

11. This is most obvious in the attempts to understand, or at least to express, by means of the negation, those relations which go by the name of opposition. Opposition was held to proceed originally from the negation, a not-*A* taking up its place by the side of an *A*. By dragging in a term which was originally framed to apply to two opposite judgments (*v.* below, § 23) a distinction was drawn between the contradictory and contrary opposition of ideas. Ideas which are contradictorily opposed, we are taught, are related as *A* and not-*A*, so that the one idea contains only the negation of the contents of the other. Those which are contrarily opposed, on the other hand, are so related, that though the one annuls the other, it also contains a positive determination. Equal and not equal, white and not white, are examples of contradictory opposites; white and black, good and bad, of contrary opposites.

In order to test the truth of this theory, we must first insist that negation has no meaning except in the sphere of the judgment. Every negation is the negative answer to a question, and prohibits a predication; no and not have no place except when opposed to a proposition, or in a proposition. Taken literally, the formula not-*A*, where *A* denotes any idea, has no meaning whatever. There is no such thing as an idea which is only the pure negation of the contents of another idea. If negation is to mean putting aside (*Aufhebung*), then indeed an idea—man, heaven, blue, green—may be there or not there, consciously presented or not presented at all, and so far "put aside." But the fact that the idea of man is not present is not itself an idea,¹ and the formula cannot mean that οὐκ ἄνθρωπος signifies that the idea of man is not present, for in order to understand it the idea of man must be present. Thus the formula fails of its purpose in the same way as Kant's memorandum, "Lampe must be forgotten."

If not-*A* were taken to denote everything not present in our ideas when we form the idea of *A* alone, everything therefore the idea of which is

¹ οὐδὲν γὰρ ἐνδέχεται νοεῖν μὴ νοοῦντα ἔν. Arist., *Met.* Γ, 49. 100 *b*, 610.

not immediately given with the idea A , then A and not- A would no longer denote incompatible determinations, and it would not be true that they are mutually exclusive. When the idea "white" is present I have nothing before me but the colour. If everything which is not this colour is not- A , then it includes round and square, heavy and soluble in sulphuric acid. These are all "not white," *i.e.* something other than "white." But these predicates are in no way incompatible with white, and form no opposition in the ordinary sense of the term; for that, we should have to pass from "white" to all white things, but does the word "white" by itself ever mean all white things?

But if not- A were a real negation, the idea A must be denied of something, and so pass, either expressly or tacitly, into a judgment. And this is what is really meant; not- A is taken to denote that which is not A , that of which A must be denied. Thus it presupposes a negative judgment, or a number of negative judgments, concerning unnamed subjects which can be denoted as not- A only upon the ground of these negations and very indirectly. If, then, any idea is to be formed of not- A , these subjects must come from somewhere; the mere fact that we are called upon to deny A does not cause them to be present. I must review in thought all possible things in order to deny A of them, and these would be the positive objects denoted by not- A . But even if there were any use in this, it would be an impossible task, and Aristotle was right in calling the expression an ὄνομα ἀόριστον.¹

If we consult Kant's *Logik* we are told that not- A as a predicate indicates that the subject is not contained within the sphere of a predicate, but that it lies somewhere outside in infinity. The proposition "the soul is not-mortal" places the soul among the unlimited number of undying beings which remains from the totality of possible beings when I subtract all the mortal. This seems to give a simple receipt for realizing what belongs to not- A . But it is only applicable when we are dealing with predicates which can be used to denote particular beings, for then we can look upon the world as an infinite number of such particular beings from which we subtract the number of the A 's. But how shall we deal with concepts which are of an abstract nature and whose extension can never signify a number of beings? If A = mortal, and I divide the extension of possible beings into mortal and not-mortal, where will justice, virtue, law, order,

¹ With reference to this not- A compare also Prantl, *Geschichte der Logik*, i. 144. Lotze, *Logik*, ed. 2, p. 61 sq.

distance, find a place? They are neither mortal beings, nor yet not-mortal beings, for they are not beings at all; they are attributes and relations of beings, and can belong to both mortal and immortal beings. If they are not to be counted under not-*A* because they may belong to a mortal being, still we cannot include them under *A*, and contrary to the presupposition from which we set out, we are left in possession of a territory lying midway between *A* and not-*A*. If *A* stands for man, then it seems easy to put men on one side away from the rest of the world, everything which remains—sun, moon and stars, minerals, plants and animals—all is not-man. But where do the attributive concepts black, green, soft, hard, belong? to *A* or to not-*A*? The intended division of all possible beings into *A* and not-*A* quite overlooks the fact that there are different categories; that every concept stands in relation partly to concepts of like categories, partly to concepts of differing categories, and that the lines which divide them cross in all directions.

But even supposing it possible to think of all that which is not-*A* as anything comprehensible, anything which would have a meaning as a predicate, where should I find a reason for denying *A* of everything which can be called not-*A*? Not in the fact that it is not-*A*; this can only be said indirectly and derivatively; but in that which it is, that which prevents us from predicating *A* of it. The opposition said to be expressed and made comprehensible by not-*A* is, on the contrary, the presupposition of the not-*A*, which is itself merely a derivative sign of the opposition, not its essence and ground.

The same indefiniteness into which contradictory opposition resolves itself, attaches also to the contrary opposition of the ordinary doctrine. If we say that everything which can be expressed by the formula not-*A* + *B* is contrarily opposed to an idea *A*, then red and virtuous, black and immortal enter into contrary opposition, not to speak of the strange confusions which arise when *A* and *B* are taken from different categories. And this is not excluded by the formula which denotes everything included in not-*A* and denoted not merely negatively but directly; so that grass-green and algebra, emotional and ellipse, fall into contrary opposition. The examples must be excused; in no other way can we make obvious the want of meaning in formulæ which are dragged over from one treatise on Logic into another.

12. The view that negation can only take place where there is a reasonable possibility of question and of affirmative answer has led others to

abandon not only this vague not- A , but also the common explanation of contrary opposition as not- $A + B$. According to them, contradictory as well as contrary opposition is to be found only when a general idea is more fully determined by mutually exclusive differences; where, for instance, a line is determined as straight or crooked by differences of direction, or the state of a body in space is determined as rest or movement. According to this view contradictory opposition is found where only two determinations are opposed to each other; where therefore the negation of the one definitely implies the other—a line which is not straight must be crooked; while contrary opposition is found where several determinations appear on an equal footing, as in the case of colours. Thus, under the names of contrary and contradictory, the distinction is again introduced which Aristotle made (*Categ.*, 10. 11b, 33) between opposites which admit of no intermediates, such as even and uneven in the case of whole numbers, illness and health in a living being; and opposites which admit of an intermediate, such as black and white.¹

This way of putting the doctrine is more rational because it at any rate gives us a subject for the negation in the general idea. But in return it conceals another danger, that of believing it possible to produce opposite concepts—and something positive therefore—by determining a general idea by A and not- A . But the general idea does not exist before its determinations, but only together with them. There is not first a line in general, which we could determine to be straight or not straight; it is part of the nature of the space containing the line that both straight and crooked lines are possible in it. Thus, it always depends upon the nature of the objects comprehended in a general idea what are the determinations they permit, and whether another predicate is permissible together with one which we recognise as possible. In the same way it depends upon the nature of the objects how great the range is of these compatible determinations. Here again the negation and the formula framed from it

¹ Some have even gone so far as to confine the name of contrary opposition to those terms in such a series of differences which are furthest apart; thus, in the case of colours, only black and white are said to be in contrary opposition, while red and yellow are merely disjunct, not contrary. This was the view taken by Trendelenburg in his *Log. Unters.*, chap. xii., in accordance with Aristotle's dictum (*Categ.*, 6, 6a, 17, and elsewhere; *vide* the passages in Waitz, *Org.*, i. p. 309)—*i.e.* that the *ἐναντία* are τὰ πλείστον ἀλλήλων διεστηκότα τῶν ἐν τῷ αὐτῷ γένει. But this, according to p. 180, introduces a new point of view, that of a comparison of the intervals between our ideas, and with this we have nothing to do when considering the grounds of the negation.

can only interpret for us that which lies in the nature of ideas ; it cannot determine this nature beforehand. On the contrary, the incompatibility of certain ideas continues, for our present observation, to be an empirical relation, which Logic, strictly speaking, has never done more than describe. The import of the procedure by which we give expression to differences in a general idea, as *A* and not-*A*, can only be treated of under the doctrine of the Concept.

13. In one case, however, it seems undeniable that opposition has its origin in negation : when, that is, one term of the opposition has really a purely negative significance. Straight and crooked are two different intuitions, each in itself definite and positive ; and it may at least be questioned whether in the case of rest and motion the one is the mere negation of the other or not,¹ either the one or the other being taken as positive. But how about blind, deaf, unhappy, unreasonable, unwise, speechless, unfeeling, and all the numerous epithets formed with “un-” and “-less” ? Can the relation between seeing and blind be expressed except by saying that blind means not-seeing, the simple privation of sight ? and is not this an opposition which has arisen entirely from negation and one term of which means nothing but a not-being ? And has not language, by thus blending the negation with the predicate, justified the not-*A* of logical theory beforehand ?

If this were so, then it must be fully equivalent whether I deny the one term of the opposition or affirm the other ; whether I say “this does not see,” or “this is blind,” “*A* is not happy,” or “*A* is unhappy.” No proof is needed that this is not the case. If all we know is that the judgment “*A* sees” is false—and the mere words “*A* does not see” never tell us more—then it is not said *why* he does not see. But blind denotes a definite state of the subject, an organic change of the apparatus of sight, in consequence of which seeing does not take place. By denying sight, then, we do not affirm blindness, as must be the case did these so-called privative predicates really contain nothing but the expression of a negation. Here again, then, negation is not sufficient to explain opposition. It is only because our negations are almost always based upon such oppositions that negation, working according to psychological laws, excites mainly the idea of opposition. Language, which makes use of psychological forces, and arbitrarily curtails the original meanings of words, is able to turn this

¹ Spinoza tells us (*Tract. de Deo*, ii. 19) that rest is not a mere negation (nothing), and upon this proposition the whole of his physics is based.

habit to account, and to denote oppositions by negations. But language always means more than it says, and logical analysis intervenes to distinguish between the necessary meaning of the negation in itself, and its ordinary meaning which attaches to it only from association and on the ground of the known relations of the predicates.

§ 23.

The PRINCIPLE OF CONTRADICTION refers to the relation between a positive judgment and its negation ; it expresses the nature and meaning of the negation by saying that the judgments “ *A* is *B* ” and “ *A* is not *B* ” cannot both be true together. This statement differs essentially from the proposition usually known as the *principium contradictionis* (*A* is not not-*A*), which refers to the relation between a predicate and its subject, and forbids that the predicate should be opposed to the subject.

We give the name of *ἀντίφασις*, *contradictio*, to the relation between a positive judgment and its negation (and hence also to the two judgments which stand in this relation). They are contradictorily opposed to each other (*ἀντιφατικῶς ἀντικείμεναι*, *contradictorie oppositum esse*).

1. A confusion similar to that concerning identity and opposition exists with respect to the so-called *principium contradictionis*. Aristotle, in a familiar passage,¹ formulates it as follows: “ It is impossible that the same

¹ *Metaph.* Γ, 3. 1005 b, 19 : Τὸ γὰρ αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτὸ (καὶ ὅσα ἄλλα προσδιορισαίμεθ' ἂν, ἔστω προσδιορισμένα πρὸς τὰς λογικὰς δυσχερείας) αὐτῇ δὴ πασῶν ἐστὶ βεβαιωτάτη τῶν ἀρχῶν . . . ἀδύνατον γὰρ ὄντινον ταῦτ' ὑπολαμβάνειν εἶναι καὶ μὴ εἶναι, καθάπερ τινὲς οἴονται λέγειν 'Ἡράκλειτον' οὐκ ἔστι γὰρ ἀναγκαῖον, ἅτις λέγει, ταῦτα καὶ ὑπολαμβάνειν' εἰ δὲ μὴ ἐνδέχεται ἅμα ὑπάρχειν τῷ αὐτῷ τὰναντία (προσδιορισθῶ δ' ἡμῖν καὶ ταύτῃ τῇ προτάσει τὰ εἰωθότα) ἐναντία δ' ἐστὶ δόξα ἢ τῆς ἀντιφάσεως, φανερόν ὅτι ἀδύνατον ἅμα ὑπολαμβάνειν τὸν αὐτὸν εἶναι καὶ μὴ εἶναι τὸ αὐτὸ ἅμα γὰρ ἂν ἔχοι τὰς ἐναντίας δόξας ὁ διεψευσμένος περὶ τούτου. Διὸ πάντες οἱ ἀποδεικνύοντες εἰς ταύτην ἀνάγουσιν ἐσχάτην δόξαν . φύσει γὰρ ἀρχὴ καὶ τῶν ἄλλων ἀξιωματῶν αὕτη πάντων. 4 1006 b, 33 : οὐκ ἄρα ἐνδέχεται ἅμα ἀληθὲς εἶναι εἰπεῖν τὸ αὐτὸ ἀνθρωπῶν εἶναι καὶ μὴ εἶναι ἀνθρωπῶν (cf. also *Metaph.* β, 2, 996 b, 31 : λέγω δὲ ἀποδεικτικὰς τὰς κοινὰς δόξας, ἐξ ὧν ἅπαντες δεικνύουσιν, οἷον ὅτι πᾶν ἀναγκαῖον ἢ φάναι ἢ ἀποφάναι, καὶ ἀδύνατον ἅμα εἶναι καὶ μὴ εἶναι). Here Aristotle makes use of the phrase that contrary attributes (*ἐναντία*) cannot belong to one subject both together, and he seems to employ it as a proof that one and the same person cannot believe that anything both is and is not at the same time. But he must not, of course, be understood as intending to lay down a principle higher than, or independent of, the principle of contradiction ; he not only denies this in the same passage, but refers again to it afterwards (*Metaph.*, iv. 6. 1011 b, 15) :—ἐπεὶ δ' ἀδύνατον τὴν ἀντίφασιν ἀληθεύσθαι ἅμα κατὰ τοῦ αὐτοῦ, φανερόν ὅτι οὐδὲ τὰναντία ἅμα ὑπάρχειν ἐνδέχεται τῷ αὐτῷ ; here stating that the principle of contradiction is the original principle upon which the other depends. The above proof should rather be called a mere *συλλογισμὸς ἐξ ὑποθέσεως* in the Aristotelian sense ; i.e., an *argumentatio ex concessis*, which is intended to show that

(predicate) should both belong and not belong at the same time in the same way to the same thing. . . . This is the most certain of fundamental principles . . . for it is impossible for any one to believe that the same thing both is and is not (as some think Heraclitus says, for a man does not necessarily really believe what he says) . . . Hence every one who brings forward an argument, falls back in the last instance upon this proposition, which by its nature serves also as the principle upon which all other axioms depend."

That is to say then: the proposition *A* is *B* and the proposition *A* is not *B* cannot both be true together. If we state the proposition *A* is not *B*, we must declare the proposition *A* is *B* to be false; and if we state the proposition *A* is *B*, we declare the proposition *A* is not *B* to be false.

2. This is nothing but a statement concerning the significance of the negation. Its nature and meaning are laid down in a proposition which cannot itself be expressed without negation, and which is, therefore, only of value as making us conscious of our own act when using the negation. If we connect the same meaning as every one else does with the word "not," we may indeed say both that *A* is *B* and that *A* is not *B*, but we cannot believe it and maintain it seriously. Or, again, we may by our words make it appear that both are true together; but only by using the words in different meanings, or by speaking of different times. It is for this reason that Aristotle guards his proposition so carefully by the determinations "at the same time" and "in the same sense."

That Aristotle intended his principle to have immediate reference only to the nature of our thought is as certain as it is that the negation has its sole origin in a movement of thought which extends beyond that which is, and endeavours to combine incompatible things. This is shown by his argument "it is impossible that any one should believe that the same thing, at the same time, both is and is not." It is shown again (*Metaph.*, iv. 4) when he says that those who say that it is possible for the same thing both to be and not to be, and that it is possible to believe this, do away with all possibility of thought and of mutual understanding; for these require that every word should have definite meaning, and that the speaker should keep to this meaning, and not contradict it.¹

the recognised proposition "contradictory attributes cannot belong to the same thing" involves the proposition "No one can believe that the same attribute can both belong and not belong to one and the same thing."

¹ Cf. Prantl, *Gesch. der Logik*, I., 131 sq., 134: The corresponding principle with reference to objectivity always arises out of the relation which exists in subjective speech

3. If this is what Aristotle meant by his principle of contradiction, then the positive interpretation of it is also obvious. It can be none other than the proposition that every one who consciously makes a statement states just that which he does state; that what he says must have a fixed meaning, because, if another meaning is substituted whilst he speaks and thinks, he does not really say anything. It must be true that "what I have written, I have written; what I say, that I do say." But it is evident that this is only a corollary to what we have called above the constancy of ideas; it is the unambiguity of the act of judgment. This unambiguity of the act of judgment would form the content of a principle of identity which would be the positive rendering of the Aristotelian principle. But it is through the rejection of simultaneous affirmation and negation that we first become conscious of this unambiguity, and it tells us nothing which is not already stated in the principle of contradiction. It is then quite natural that only the principle of contradiction should be ranked by Aristotle as a principle, while its positive rendering finds merely an incidental expression.¹ For a long time indeed it was the Aristotelian principle of contradiction which was meant by the *principium identitatis*.

and belief, and this is in accord with the subjective origin of human judgment. No doubt when the principle takes the form "it is impossible that the same thing should both be and not be" it does seem as if we had to do with a principle which is primarily metaphysical, and logical only in a secondary manner; Ueberweg, for instance, takes this view when he divides the sayings of Aristotle into those which state the metaphysical, and those which state the logical principle. But there can never have been a division such as this between metaphysics and logic as Aristotle understood them, if only because he regards the true judgment as always the expression of Being, and, indeed, frequently denotes the *ἔστιν* of predication simply as a being. Nevertheless, if we remember his express statement (*Metaph.*, vi. 4) that truth and falsehood appertain to thought, and not to things, a proposition which declares that of two propositions one must be false can only refer originally to the activity of thought in its *σύνθεσις* and *διαίρεσις*. It follows from this as a matter of course that the same thing cannot both *be* and *not be* at the same time, nor the same attribute both belong and not belong to the same (objective) thing at the same time; for if this were not so, then, owing to the Aristotelian conception of truth, the logical principle would also be without validity. For Aristotle, both modes of expression are ultimately exactly equivalent in meaning. Cf. Zeller, *Phil. d. Gr.*, II. 2, 174.

¹ Trendelenburg, *Elem. Log. Arist.*, § 9, quotes from *Anal.*, I, 32, 47a, 8 : Δεῖ πᾶν τὸ ἀληθὲς αὐτὸ ἐαντῷ ὁμολογούμενον εἶναι πάντη; but this is introduced for the sake of the later doctrine, and where it stands has not the significance of a principle. Such significance can be attributed only to the propositions in the *Metaph.*, p. vi. 4 sq., and the contents of these are correctly formulated by Prantl, *Gesch. der Logik*, I, 131, as the statement that every assumption concerning a *ὑπάρχον* (I should only have said *ὑπάρχειν*) remains constant with itself; and this again is only possible on the assumption that our verbal denotations are conceptually determined. Baumann has lately endeavoured

4. The *principium contradictionis* of modern logicians (particularly Leibnitz and Kant¹) in the formula A is not not- A , differs entirely in meaning and application from the Aristotelian proposition. This latter refers to the relation between an affirmative and a negative judgment. According to Aristotle, one judgment contradicts another. The later proposition refers to the relation between subject and predicate in a single judgment; the predicate contradicts the subject. Aristotle states that one judgment is false when another is true; the later writers state that a judgment is in itself and absolutely false, because the predicate contradicts the subject. What the later writers desire is a principle from which it can be known

(*Philosophie als Orientierung über die Welt*) to re-establish the true Aristotelian meaning of the logical principles, but he nevertheless diminishes the importance of the law by making it refer to the merely empirical fact that something has been thought of ("Es drückt nichts aus, als dass die Thatsache des Vorstellens stattgefunden hat in der Weise, wie wir sie vollzogen haben"), and by stating it as merely a special case of *factum infectum fieri nequit*. The law is not intended to establish in a subsequent judgment the fact that something has been thought; for this subsequent judgment itself is governed by the rule that it must have a definite meaning, its meaning being that just this mental act and no other has taken place. It is really intended to show *how* any act of judgment has taken place, that it contains one definite single meaning, that if we affirm anything we can affirm it in one sense only, and cannot in the same act of judgment hold the opposite view also.

¹ I do not know when it was that the name of *principium identitatis* was first given to the formula A est A , or *Ens est Ens*, instead of the Aristotelian principle, which was so denoted throughout the Middle Ages (see Prantl's *References*); nor when the *principium contradictionis* (and the *princ. exclusi tertiū*) received its altered meaning; the two changes were no doubt connected. We can see very clearly how Leibnitz passes from the one rendering to the other. In the *Nouveaux Essais*, iv. 2 (Erdm., p. 338, 9), we find as the principle of identity A est A , "chaque chose est ce qu'elle est"; while the principle of contradiction, on the other hand, is: "une proposition est ou vraie ou fausse." This, he tells us, contains two propositions: 1. "que le vrai et le faux ne sont point compatibles dans une même proposition, ou qu'une proposition ne saurait être vraie et fausse à la fois"; 2. "que l'opposé ou la négation du vrai et du faux ne sont pas compatibles, ou qu'il n'y a point de milieu entre le vrai et le faux, ou bien il ne se peut pas qu'une proposition soit ni vraie ni fausse." Here, as in the *Theod.*, i, 44, Leibnitz is in essential agreement with Aristotle. But his first example is "ce qui est A ne saurait être non- A "; and though we may still recognise in this the two judgments "the same thing is A and is not- A ," the use of the term non- A has very nearly converted it into the formula A is not non- A . This formula actually appears together with the other in the *Nouveaux Essais*, i, § 18 (Erdm., p. 211): "il est impossible qu'une chose soit et ne soit pas en même temps." In the *Princ. phil.*, on the other hand, he states that the import of the *principium contradictionis* (§ 31) is to enable us to say that everything which contains a contradiction is false, and that everything which is opposed to the contradictory or false is true. Here, then, the *contradictio* is in the predicate. Finally, in § 45, it is said that the opposite of an identical proposition contains an express contradiction, so that " A is A " and " A is non- A " are opposed to each other as necessarily true, and necessarily false.

whether certain propositions are in themselves true. From the Aristotelian proposition we cannot immediately infer the truth or falsehood of any particular proposition, but only the impossibility of believing both affirmation and negation at the same time.

Kant's polemic against Aristotle is then quite misdirected. According to him (*Kr. der r. Vern.*, Hart, p. 166 sq.) the fundamental principle is: "To no thing can there belong a predicate which contradicts it." It is a universal—though indeed, merely negative—criterion of all truth. It is valid of all knowledge whatever, irrespective of its contents, and states that knowledge is entirely annihilated by contradiction. It does not, indeed, as a rule, go so far as to guarantee the truth of a proposition, for a judgment may still be false or groundless even when it is free from internal contradiction; nevertheless it may be of positive use towards knowledge of the truth, for when the judgment is analytical, let it be affirmative or negative, the principle of contradiction always affords a sufficient test of its truth. We must always be right in denying the reverse of whatever is contained and thought of in the concept of the object as given to knowledge, and we must necessarily affirm the concept itself, since its opposite would be in contradiction to the object.

It is from this point of view, then, that Kant rejects the formula, "it is impossible for anything at the same time both to be and not to be," as containing a synthesis which has slipped in from carelessness and quite unnecessarily. The proposition, he says, is affected by the condition of time, and as much as to say: "a thing A , which is something $= B$, cannot at the same time be not- B ; but it can very well be both (B as well as not- B) one after another." "But the principle of contradiction, being a purely logical principle, must in no way limit its statements to time-relations; such a formula, then, is entirely contrary to the intention of this principle." The misunderstanding, he goes on, is due entirely to the use of synthetical propositions. A predicate (*e.g.* unlearned) is synthetically connected with the subject (man); if, at the same time, an opposite predicate (learned) is attributed to the subject, then a contradiction arises; but the contradiction is between the one predicate and the other, not between predicate and subject. If, on the other hand, we say: "no unlearned man is learned," the truth of this negative proposition is obvious from the principle of contradiction, without the addition of "at the same time" as condition. In his *Logic*, also, Kant deals in the same way with the principle of contradiction.

Little is needed to show that Kant is speaking of something quite different from the original principle of contradiction. Leibnitz divided truths into necessary and matter of fact, and offered a special principle of truth for each of the two classes: the principle of contradiction for necessary truths, which are all of them in the last instance what are called identical propositions; the principle of sufficient reason for truths of matter of fact. Kant proceeds in the same way with his two classes of analytical and synthetical judgments; what he is looking for is a principle for the truth of analytical judgments. Now analytical judgments are always concerned with subjects which are concepts, and they state what is thought in these subjects as concepts—hence quite independently of time; the predicate of an analytical judgment is always already contained in the concept which forms its subject. But what the principle of contradiction in the Kantian sense states is, “to no concept must a predicate be attributed which contradicts it.” In so far, then, as other judgments express their subject by means of a concept (in “this man is learned,” the object is known by the concept man), the principle applies to them also; they would be self-destructive if they attempted to attribute to a subject a predicate contradictory to the concept under which that subject falls. We will investigate hereafter what is meant by contradicting a concept, and whether a universal logical principle can be grounded upon this contradiction. At present it is clear that, starting from these premises, Kant is certainly right in excluding time-determinations from his principle. But when he accuses the Aristotelian formula of a misconception in containing its “at the same time,” it is due only to his own misconception in thinking Aristotle’s meaning to be the same as his own; for it is certain that Aristotle meant to prohibit, not indeed contradiction between two predicates, but contradiction between the affirmation and negation of the same predicate.

5. It may fairly be asked here, “How is it possible that two principles, differing so greatly as do the Aristotelian and the Kantian, should be generally regarded as one and the same fundamental law of human thought? And is there really no connection between them?” Some connection there certainly is. The ordinary principle of contradiction aims at giving a rule by which the validity of negative judgments may be tested. It is seen that a negation generally rests upon the exclusion of the predicate by the subject, and it is fancied that this relation of incompatibility rests again in its turn upon the negation; hence it is thought

that universally valid negations are all reducible to contradiction. But in this way we see that the formula really "moves in a circle."

But what can be the meaning of saying that a predicate *B* contradicts a subject *A*? That a proposition which attributes a predicate *B* to a subject *A* involves a contradiction? The only way in which a contradiction can take place is when the judgment which attributes this predicate *B* to a subject *A* contradicts another judgment (presupposed, if not expressly stated) which denies the predicate *B* of the subject *A*. This latter judgment (*A* is not *B*) being accepted as needing no proof, or as known from some other source, the contradiction certainly annuls the first judgment; and it does so, moreover, according to the proposition laid down by Aristotle, that they cannot both be true together. Why is the proposition in Kant's example, "an unlearned man is learned," a contradiction? Because the predicate "learned" is attributed to a subject which is denoted by the subject-term "unlearned man," a term which implicitly contains a judgment stating that the subject is not learned. In this way the proposition may be reduced to the two judgments, *X* is learned, and the same *X* is not learned. These two judgments are both stated by the proposition, which therefore contains a contradiction, and therefore is false, *i.e.* it is false that the same man is learned and not learned; and if it is true that he is not learned, it is false that he is learned.

A contradiction, then, can only take place in so far as a judgment is already implied in the subject. This is certainly the case both with the analytical propositions which Kant has in view, and with those propositions of which alone the school logic is accustomed to treat. As we have seen above, Kant's analytical propositions are possible only on the presupposition of concepts which are the same for every one; *i.e.* on the presupposition of universally valid judgments concerning the meaning of words; judgments which state that body means the same as extended thing, that the idea denoted by the word "body" contains the idea "extension." If I say, "all bodies are extended," then that means "everything which I call body I must also call extended"; the denotation of any *X* as body contains the judgment, "*X* is extended." If now I say "a body is not extended," or even "this body is not extended"—then there is a contradiction: "this is extended, and this is not extended"; and as it is absolutely established that anything which is body is extended, the contrary is necessarily false.

So far the opposition has been between affirmation and negation: A is B and A is not B . But now, instead of contradictory propositions, there appear the contradictorily opposed predicates B and not- B ; and the contradiction between affirmation and negation is transferred to the two affirmations " A is B " and " A is not- B "; if it is true that A is B , it is false that A is not- B .

On these presuppositions alone can a contradiction take place between subject and predicate; and we cannot recognise the falsity of a proposition from the contradiction between its predicate and the concept forming the subject, unless it is presupposed that the process by which concepts are formed is infallible, and their denotation by words absolutely fixed; and, when we are treating of the particular, that the process of the subsumption of the particular under the concept is also infallible. Now, so long as we have to do merely with the subjective image upon which Kant bases his analytical judgment (*v.* p. 104) no doubt it is easy to form a concept and to combine in it certain characteristics—to say "body is extended thing." Then the judgment "bodies are extended" is equivalent to "the extended is extended." Here, as Hobbes holds, we have only equations between the arbitrarily framed meanings of words. Even the statement "all bodies are extended" involves an unauthorized assumption, for it tacitly assumes that my concept is applicable to possible things, and that I can safely make this application in any particular case—for this is the only meaning of "all." Finally there can be absolutely no question raised as to whether I should say more concerning what I call body than is already contained in the name; all propositions become identical, *i.e.* without meaning or importance.

But it is just at this point that the formula " A is not not- A " appears as the expression of the *principium contradictionis*. The contradiction between a predicate and its subject is reduced to this formula by the assumption that all true judgments must admit of being finally reduced to " A is A ," and that our absolute measure of truth lies in the finished system of concepts in which alone our thought and knowledge move. But the first weak point in the formula is its ἀόριστον non- A . An attempt might indeed be made to explain this away. Starting from the so-called principle of identity, the proposition can be made, "it is false that A is not A "; because, that is, it contradicts the true proposition, A is A . By a slight perversion of language this may be contracted into the proposition: *non* [A non est A]; the given A still remains subject, and it is denied

that A as predicate can be denied of it. In the same way the formula would have a meaning if A were taken as the sign of a proposition. But this is not what is meant. Non- A is introduced deliberately: the contradictory opposition of concepts is substituted for the contradiction of propositions, and what is now prohibited is that non- A shall be predicated of A . From some points of view we might be content to accept non- A , and to acknowledge the formula as theoretically correct; but it is useless in practice. We seldom meet with the contradiction in such a crude form as "gold is not-gold," "green is not-green," "Being is not-Being." Generally speaking it must be discovered under disguises. Would that it were so easy when A is given to decide what determinations are included in non- A and therefore contradict A !

But now we see that, like a proper oracle, the *principium contradictionis* gives us no answer to the question, "what is it which may not be stated of A ?" If we take refuge in saying that A as a concept contains the characteristics a , b , c , d , and that therefore non- a , non- b , non- c , non- d must not be attributed to it, the difficulty of the non- A is only multiplied; and if we are content to say a , b , c , d must not be denied—well, that is the Aristotelian principle applied to judgments, which we already acknowledge to be valid.

But the Kantian Logic can contain no universal formula by which to determine what is opposed to a subject; for our concepts, according to Kant's express teaching, generally denote the nature of their objects by a part only of our experience of them. Hence we can never conclude from the fact that something is not contained in the concept that it does not belong to the thing; it never follows that because something does not contradict the concept, it does not contradict the thing. It is moreover a fiction that all the relations of concepts are known to us so far as opposition and exclusion are concerned.

It is because the Aristotelian principle refers only to what we know—the function of negation—and does not confine itself to judgments having concepts for their subjects, that it is a first principle, absolutely unconditioned and applicable to all our judgments. Because, moreover, there is contained in it—so far as we have any knowledge of the relations of opposition and incompatibility among concepts—the ground of the impossibility of attributing to a concept a predicate excluded by it.

But all that the ordinary *principium contradictionis* can say is, that a proposition is false if its predicate is incompatible with some determination

of the subject; and it therefore prohibits us from attributing to it an incompatible predicate. Thus it presupposes a knowledge of what is incompatible and cannot be an unconditioned first principle sufficient in itself to establish the falsity of a proposition (by doing which it would also, according to the principle of the excluded middle, establish the truth of its contradictory opposite).

§ 24.

To interpret the nature of the negation completely we must add to the principle of contradiction the further principle that the negation of the negation is affirmative, that to deny a negation is equivalent to affirming the same predicate of the same subject.

1. It is strange that logic has found no place for the proposition "*duplex negatio affirmat*," which has been abstracted by grammar from frequently recurring forms of speech. Probably it has been looked upon as a consequence of the principle of excluded middle; but its position is rather that of the indispensable link leading from the principle of contradiction to that of excluded middle. The principle of contradiction states that it is impossible for affirmation and negation both to be true together; in this way, if the affirmation holds good, it leads to the falseness of the negation. But in so doing it does not state what is meant by stating that a negation is false. Only because the denial of the negation is the affirmation itself is there no medium between affirmation and negation.

Aristotle lost sight of this simple connection between the two by treating affirmation and negation from the first as completely parallel and co-ordinate forms of statement. Owing to this he failed to give a satisfactory account of the negation itself, and, strictly speaking, left no room at all for the negation of a negation. But as soon as we see that every negation presupposes a previous synthesis, its only object being to declare this synthesis invalid; as soon as we see that the negation is a particular act in which the "not" has the force of a judgment concerning a judgment (either attempted or complete)—then it becomes clear how far the negation of a negation is possible. "*A is not B*" contains the statement that the proposition *A is B* is false, that other determinations than *B*—determinations incompatible with *B*—belong to *A*, that it is impossible to unify *B* with *A*. This statement, or the attempt at such a statement, may in its turn be denied. "It is false that *A is not B*," says that it is impossible to state that the proposition "*A is B*" is false, to ascribe any other predicate than *B* to *A*, to prevent the unification of *A* and *B*. And if the objections

to the synthesis A is B are impossible, then this synthesis must hold good.¹

2. It is in this property of the negation, that when directed against a negation it is to that extent a positive statement, that we first become fully aware of the completely subjective character of the whole movement of thought when concerned with negation. By the process of negation no truth can be produced and nothing can be created which did not exist independently of it. The presupposition of its validity is that a combination should be attempted which is merely subjective and peculiar to the individual; and which is prohibited by the immovable necessity of thought; and when the negation has been attempted without sufficient ground it vanishes, leaving no trace behind; the repeated "not" only serves to indicate the circuitous way which the thought of the individual has taken, in order to reach a truth which might have been reached directly. For in the last instance it is always out of some positive knowledge that we declare a negation to be false, and in this positive knowledge is contained the ground for the overthrow of the negation.

3. But it is not altogether to no purpose that this circuitous route is taken. In grammar it has already been recognised that resisted attacks increase the psychological firmness of conviction; the affirmation which has fought through a negation seems to stand firmer and to be more certain. So much it may gain; but it can never gain so as to contain more than before, or to be richer in meaning. " A is B " gives just the same information whether it is known directly or is the result of the negation of A is not B —that is, so long as A and B keep the same meaning, and we do not substitute the affirmation of a positive characteristic opposed to B for the mere negation " A is not B ." If we did this, we should of course have gained something new, inasmuch as A would have come into relation with a new predicate. Is it false that light is not a kind of motion, then the proposition we have gained tells us no iota more than the proposition "light is a kind of motion." If for our first proposition we substitute another "light is a kind of matter," on the ground of a disjunction accepted for other reasons, then the denial of this might give us new information, *i.e.*, a distinction between "light" and "matter"; but this would not be due to the twofold negation.

¹ Cf. Bradley, *The Principles of Logic*, 1883, p. 149 sq.; special weight is laid upon the position that every negation must be based upon positive knowledge; the only ground upon which we can deny that A is not B is our knowledge that A is B . This knowledge is therefore included in the double negation.

§ 25.

It follows of itself from the principles of contradiction and of twofold negation that of two contradictorily opposed judgments one is necessarily true; hence that there is no third statement besides affirmation and negation which would imply the falsity of both. This is the PRINCIPLE OF THE EXCLUDED MIDDLE, which, like the two previous principles, aims only at interpreting more fully the nature and meaning of the negation.

The ordinary interpretation of the *principium exclusi tertii* is by the formula *omne A est aut B aut non-B*, according to which every subject possesses one of two contradictorily-opposed predicates; but this is as different from the original and genuine principle of excluded middle as the ordinary *principium contradictionis* is different from the principle of contradiction.

1. The principle of contradiction tells us, that of two judgments "*A* is *B*" and "*A* is not *B*" one is necessarily false, because both cannot be maintained together; and by so saying it fixes the meaning of the negation. But it follows immediately that one is necessarily true because both cannot be denied together. If I deny that *A* is *B*, then by so doing I maintain that *A* is not *B*; while if I deny that *A* is not *B*, that again is only maintaining that *A* is *B*. If then I attempted at one and the same time to deny both that *A* is *B* and that *A* is not *B*, I should by one negation say that *A* is not *B*, and by the other that *A* is *B*, thus falling into a contradiction. Thus there remains no middle statement between affirmation and negation, which could contain any reference of the predicate *B* to the subject *A*; and any judgment which tries to combine *B* and *A* as predicate and subject, must either affirm *B* of *A* or deny *B* of *A*.

2. Aristotle repeatedly states this principle, and in the most important passage (*Metaph.*, iv. 7) he attempts a proof of it (one, however, which contains a *petitio principii*); elsewhere he gives it as needing no proof.¹

¹ Arist., *Metaph.* Γ, 1011 δ, 23: 'Ἀλλὰ μὴν οὐδὲ μεταξὺ ἀντιφάσεως ἐνδέχεται εἶναι οὐδὲν, ἀλλ' ἀνάγκη ἢ φάναι ἢ ἀποφάναι ἐν καθ' ἐνὸς ὁτιοῦν· δῆλον δὲ πρῶτον μὲν ὁρισαμένοις τί τὸ ἀληθὲς καὶ ψεῦδος· τὸ μὲν γὰρ λέγειν τὸ ὄν μὴ εἶναι ἢ τὸ μὴ ὄν εἶναι ψεῦδος, τὸ δὲ τὸ ὄν εἶναι καὶ τὸ μὴ ὄν μὴ εἶναι ἀληθὲς, ὥστε ὁ λέγων τοῦτο εἶναι ἢ μὴ ἀληθεύσει ἢ ψεύσεται· ἀλλ' οὔτε τὸ ὄν λέγεται μὴ εἶναι, ἢ εἶναι, οὔτε τὸ μὴ ὄν. This passage has received various interpretations; its meaning is: there is no intermediate between the members of a contradiction; everything must be either affirmed or denied of everything. This is obvious as soon as we define what true and false mean. To say that the existent is not, or that the non-existent is, is false; while to say that the existent is and that the non-existent is not, is true. Thus when we say that this (*i.e.* some definite thing which is either existent or

Its close connection with the principle of contradiction is shown by the fact that even Aristotle gives formulæ containing both, while Leibnitz expressly includes them both in the formula: "A proposition is either true or false."¹ But the "either-or" conceals an ambiguity in an apparently simple expression, and the dependence of the derivative proposition is lost sight of. It is therefore more natural to give the principle of excluded middle a special place as a corollary to those principles which immediately unfold the meaning of the negation; but it is incorrect to rank it beside the principle of contradiction upon which it depends, as if it were equally immediate, especially as its application is less easy and evident than that of the fundamental principle.

3. It is owing to the weakness of the mere negation, and the incompleteness of the information which it gives us as to the meaning of its denial, that difficulties seem to arise in applying the principle of excluded middle.

The more familiar difficulties, arising from continuous transitions and the many-sidedness of the subjects, are indeed easily solved. When the sun is rising, either of the two propositions "it has risen" or "it has not risen" may be true, according as we apply the term "risen" to the elevation above the horizon of the upper or lower edge. It may be said that in the moment

non-existent) is or is not, we speak either truth or falsehood. But if we suppose a middle statement between affirmation and negation, it tells us neither that the existent, nor that the non-existent, either is not or is; for if any one of these statements were made, it would be an affirmation or a negation, and either true or false. The middle statement could say nothing either of the existent or of the non-existent, nor could it therefore be either true or false. But that which is neither true nor false is no statement at all, for it is a part of the nature of a statement to be either true or false (*ὥστε οὔτε ἀληθεύσει τις, οὐτ' οὐκ ἀληθεύσει*, 1012 a, b). Ueberweg gives a similar interpretation, ed. 3, § 79, p. 216. It is obviously presupposed in the definition of the true and the false judgment, and in the classification of judgment into affirmative and negative, that there is no *μεταξύ*, if no more can be stated than that the existent or non-existent is or is not. The passage cannot therefore be regarded as a proof; it serves merely, together with the rest of the chapter, to point out that it is always presupposed that there is no middle course.

Categ., 10, 13 a, 37: "Ὅσα δὲ ὡς κατὰ φασιν καὶ ἀπόφασιν ἀντίκειται . . . ἐπὶ μόνων τούτων ἀναγκαῖον εἶναι τὸ μὲν ἀληθὲς τὸ δὲ ψεῦδος αὐτῶν εἶναι.—This is repeated in 13 b, 27–33.

Metaph., 1, 7, 1057 a, 33: τῶν δ' ἀντικειμένων ἀντιφάσεως μὲν οὐκ ἔστι μεταξύ· τοῦτο γὰρ ἔστιν ἀντίφασιν, ἀντίθεσις ἧς ὁπωρὸν θάτερον μῦθον πάρεστιν, οὐκ ἔχουσης οὐθέν μεταξύ. K. 12, 1069 a, 3: ἀντιφάσεως οὐδὲν ἀνά μέσον, there is a similar passage, *Phys. ausc.* v. 3, 227 a, 9.

In the *Analyt. post.*, 1, 2, 72 a, 11, the opposition which excludes a third is even used as the basis upon which to explain what a judgment is: ἀπόφανσις ἀντιφάσεως ὁποτερονοῦν μῦθον, ἀντίφασιν δὲ ἀντίθεσις ἧς οὐκ ἔστι μεταξύ καθ' αὐτήν. Cf. *De interp.*, 9, 18 a, 28.

¹ See note, p. 142.

of death it is equally false to say "he lives," and "he does not live"; but this again is incorrect, for "life" expresses a state which continues, and the dying man *in articulo mortis* does not live. And all cases in which we have to deal with spatial and temporal limits may be similarly treated. Still more clumsy are such examples as "it is false that a chess board is black and false that it is not black"; if the predicate applies to the whole, then the negation is true; if not, then the subject is not the same in the two propositions.¹ But these are not the only difficulties which arise.

Aristotle himself raised the question as to what is the relation between the two propositions "Socrates is ill," and "Socrates is not ill," when Socrates does not exist at all;² and whether it is still the case that one of the two is necessarily true. The decision he comes to is, that in this instance the two propositions expressing the material opposition, "Socrates is ill" and "Socrates is well," would indeed both be false; but that the mere negation "Socrates is not ill" remains true even in this case, thus preserving the universality of the principle. This solution does not, indeed, seem perfectly satisfactory; for, as commonly understood, the proposition "Socrates is not ill" means that Socrates does live but is not ill. If we answer the question "is Socrates ill?" by yes or no, then—according to our usual way of speaking—we accept the presupposition upon which alone the question is possible; and if we say of a dead man that he is not ill, we are guilty of using our words ambiguously. It may still, however, be claimed that, by calling such an answer ambiguous, we admit that the words do not, in themselves, exclude the other meaning; and that formally, therefore, the truth of the proposition is incontestable.

4. We may admit this justification, and nevertheless draw from it the lesson, that in the sphere of temporally valid judgments there is not much to be done with the principle of excluded middle. The question is not as to whether Socrates existed at all; if it were, then, of the two propositions "Pegasus is winged" and "Pegasus is not winged," the latter must be true. His existence is presupposed in the proposition, but existence at a previous time, and the difficulty applies to the present. Because temporally valid judgments state their affirmations only for a given point of time, it remains uncertain whether the negation of such judgments applies only to this point of time, or to the subject throughout its whole exist-

¹ With reference to these and similar objections cf. Ueberweg §§ 78–80, more especially p. 205 sq.; Drobisch, *Logik*, § 60, p. 66.

² *Categ.*, 10, 13 a, 27–b 35.

ence. It is uncertain, therefore, whether what is false is only present, past or future, or the predicate altogether and at all times. Of the two propositions "he will die—he will not die," one is necessarily true, the other false. But the proposition does not tell us whether "he will not die" is true because he is already dead, or because he will be carried up to heaven in a whirlwind like Elijah.¹ When, therefore, we can attain by means of the principle of excluded middle to the truth of an affirmation, the principle has its value even in the case of merely temporal judgments, for affirmations are not ambiguous. But it is not worth while to use it for the sake of mere negations.

5. There is less difficulty with respect to judgments of unconditional validity. Since these apply to the contents of the subject-idea, there would seem to be no room for ambiguity in the negation of them. In such judgments as "matter is heavy" and "matter is not heavy," "space is infinite" and "space is not infinite," neither affirmation nor negation seems to be ambiguous. But here a difficulty of another kind presents itself, one which we have already touched upon above (§ 22, 3, 4, p. 128) and which rises out of the generality of the subjects. Owing to this generality we are constantly tempted to extend our judgments to all the definite particular things which fall under the subject-idea; but though the Predicates of affirmative judgments are of course true both of the general idea and of the particular objects which fall under it, we cannot in like manner deny of the particular objects everything which does not form part of the general idea. It is no part of the general idea triangle to be equilateral, nor of the general idea man to be white; but we cannot on that account deny that any triangles are equilateral or any men white.

¹ Aristotle makes a strange exception with reference to the future when he says (*De interpr.*, 9, 18 a 27) that when one person says that something will happen, and another person denies it, it does not follow that one necessarily speaks the truth, for if it did all the future would be necessary and deliberation would be superfluous. But here—as Zeller allows (*Gesch. der griech. Phil.*, II. 2, p. 157)—the Stagirite has committed an oversight. He confounds the statement that it is necessary that the one or the other should be speaking the truth, with the statement that one or the other speaks necessary truth, *i.e.* speaks the truth because what he says is necessary or necessarily not; while all that is necessary is that the actual, perhaps fortuitous result should prove one of the two to have been right. What Aristotle means is that the statement that one is necessarily speaking truth involves that the truth or untruth is already determined, while really the one statement is as uncertain as the other, and neither *ἔσται* nor *οὐκ ἔσται* can be properly used in the sense of knowing. We see here how he, with his habit of referring every proposition to Being, can find no correlate for a statement which leaves the alternative between Being and Not-being undetermined.

Hence there is something wrong about the opposition between the judgments "triangles are equilateral—triangles are not equilateral." The negation is again ambiguous, for now it only denies generality, and is not meant to state that the predicate cannot be united with the subject. It is at this point that first the divisive judgments, and then the disjunctive judgments grounded upon them, appear; the former stating the compatibility of different predicates with the general idea, the latter their incompatibility amongst themselves.

6. According to the ordinary formula of the *principium exclusi tertii* the proposition that of two contradictory judgments one must necessarily be true (either A is B or A is not B , holds good) is rendered as follows: "of two contradictorily opposed predicates one must belong to every conceivable subject A " (A is either B or not- B). In this way the negation is transferred to the predicates, and strictly speaking we obtain two affirmative judgments, which allow of no third intermediate. After this transition had been made by the Wolffian Logic, Kant made use of it for his own purposes. He showed that since, when all possible predicates are compared with their opposites, one of each pair must belong to every subject, this principle passes beyond the sphere of the merely logical, and as the principle of complete determination presupposes a totality of all predicates as the total possibility. We need not here enter upon the question as to whether this transition does not contain a *quaternio terminorum* in the "all," which in one place is used as a completely indefinite generality, in the other as a definite number. At any rate, it shows clearly how Kant understood the principle—*i.e.* as concerned with the reference of a subject to all possible positive and negative predicates, in order to see by which of these it is to be determined. Thus the proposition " A is either B or not- B " instructs us to substitute for B all conceivable predicates one after the other. But even apart from the question as to the justification of not- B this is a barren task; we never gain any determination in this way, since it always remains undecided whether B or not- B , X or not- X belongs to the subject. Even if we could decide between the alternatives, still, so far as the great majority of such predicates was concerned, no conceivable combination would enable us to try the affirmative form of the predicate, and thereby challenge a negation. As regards general concepts the difficulty would still remain that with them both B and not- B are compatible; so that for this reason also the value of the principle is considerably lowered.

7. But the principle of excluded middle really derives its reputation mainly from the fact that it is a special case of a relation which is certainly most important and rich in consequences—*i.e.* the relation of disjunction. It is due to the nature of ideas that we are often in a position to limit our choice among different statements concerning the same subject to a few, often to two only. We are able, on the ground of our knowledge and of the particular contents of our subjects and predicates, to frame two positive statements, of which we know that they are so far related like contradictorily opposed judgments that while both cannot be true together, neither can both be false; and in this case we gain, by denial of either member of the disjunction, a definite, unambiguous affirmation. Now the principle of excluded middle is apt to make us think that we can attain easily, and at little cost, to such fruitful disjunctions as these; we need but say that every proposition is either true or false, and there we have an incontestable truth and a sure basis for strict investigation. But the opposition of predicates has substituted itself unnoticed for the mere negation, and the negative statement seems to tell us more than it really does; it is understood as if it applied to the truth of the proposition with the opposite predicate, this being what it is as a rule really based upon. If we could solve all difficult questions by starting right off with “It is either so or so” (a proceeding of which we might use the phrase “*trancher la question*” even more fitly than the French themselves use it) —“he is either mentally healthy or diseased in mind,” “the number is either odd or even,”—then indeed the principle of excluded middle would be an invincible weapon. But in itself it can never really do more than oppose the affirmation by the negation in its poorest and most barren form; and though the view that there is no intermediate between affirmation and negation is important as interpreting the meaning of the negation itself, yet the proposition cannot be dignified as a special principle.

8. The indirect proof, again, does not really derive its cogency from the Principle of Excluded Middle. It is true that it ends by inferring an affirmation from the falsity of a negation; but the falsity of this negation must have been shown by substituting for the purely negative and therefore indefinite contradiction one which was definite and based upon a disjunction, and this disjunction was in itself sufficient to support the proof.¹

¹ While reserving a fuller treatment of the question for a later section, we may, in

passing, show by an example that the principle of excluded middle is unnecessary for the indirect proof. In Euclid I. 29, it is proved that when a straight line falls on parallel straight lines the alternate angles are equal. For if the angles were not equal it would follow that the inner angles were together less than two right angles, therefore that the lines—according to the accepted postulate—were not parallel. The contradiction of this with the hypothesis from which we started shows that it is false that the alternate angles are not equal; true, therefore, that they are equal. Expressed in this form, the proof seems to rest upon the principle of excluded middle. But it is not really so. Unless we substitute the statement “one angle is greater than the other” for the “angles are not equal,” we can get no further with our proof. The hypothesis which proves to be false is that one angle is greater than the other; and it is from the falsity of this that we get the truth of the proposition. The ground of the proof, therefore, is not that of the two propositions: the two angles are equal

the two angles are not equal

one must necessarily be true; but that of the propositions:

the two angles are equal

the one angle is greater than the other

one must be true.

The disjunctive proposition includes the mere negation, but the negation does not include the disjunctive proposition, and it is upon the latter that the proof rests.

CHAPTER V.

PLURAL JUDGMENTS.

By Plural Judgments we mean those which, in one proposition, state one predicate of a plurality of subjects.

I. AFFIRMATIVE PLURAL JUDGMENTS.

§ 26.

When simple judgments repeat the same predicate of a number of subjects, and the person judging expresses his consciousness of this agreement by making one verbal act of predication refer to several subjects, then the first form of judgment which arises is *A* and *B* and *C* are *P* (Copulative Judgments).

When *A* and *B* and *C* fall under the same denomination *N*, so that we are enabled—or induced—to enumerate them as several *N*'s, then there arises the Plural Judgment in the narrower sense, which, with a definite or indefinite statement of the number, includes the plurality of subjects in one verbal expression (several *N*'s are *P*).

1. Our desire to form judgments exercises itself according to the material and occasions offered psychologically. The first result is a series of acts of judgments, connected only by the fact that they follow one upon the other in the judging subject, and are comprehended by one consciousness which, in passing from one to the other, does not immediately lose sight of the previous acts. The verbal connection of propositions by “and,” of all forms the most primitive and least significant, states originally nothing but this subjective fact of co-existence in one consciousness. Thus it has no objective significance; “and” serves equally to connect the most heterogeneous and the most closely allied. But according to psychological laws those judgments tend to group themselves together which either state different predicates successively of the same subject, or the same predicate of different subjects. Judgments of the former kind, which presuppose that the attention lingers over one and

the same subject-idea, link themselves naturally into the conjunctive form, *A* is *B* and *C* and *D*, etc.; and this does not merely imply that the predicates one and all belong to the subject, but also expresses the consciousness of this co-existence of different determinations. To this extent the conjunctive judgment tells us more than its component parts taken singly. But there is no occasion at present to consider this form more at length. It becomes important only when applied with a consciousness of logical claims either to narrative judgment, when it serves as a description, or to explicative judgments, when it serves as a definition.

2. The combination of judgments which ascribe the same predicate to different subjects presupposes that our attention is fixed upon the predicate, that factor of the judgment which is present in the mind as a general idea. This further implies that our thought is actively comparing ideas and referring them to each other; seeking to connect particulars and to recognise agreement in difference. Thus the judgment of the form *A* and *B* and *C* are *P* has the same end before it as all judgment; it attempts to appropriate the manifold and new by the aid of ideas which are already present and firmly established, and it therefore represents a higher development of thought as compared with the simple judgment.

3. The simplest case in which we have a repetition of predicates is the intuition of a plurality of like or similar things which are named by the same word. These things may be perceived as discrete, and form a spatial or temporal series; or they may be recognised as differentiated parts of a whole, as members of a group. The repetition of the same intuition and the same denomination comes into consciousness in the distinction of the many *A*'s from one *A*; verbally in the formation of the plural. When we are interested merely in the question what it is which we have seen, there follows a denominative judgment in the plural (those are sheep, those are letters). But by every plurality of the homogeneous we are instigated to count and to make numerical comparisons, and an indefinite or definite numerical expression appears in answer to the question "how many." In the judgment "these are three shots" either the numeral or the noun is the real predicate, which is emphasized by the speaker, according as the name or the number precedes.

4. As a rule the act of naming takes place so rapidly and unconsciously, that we form no special judgments in the process, but merely assert the name in the word by which we denote the subject. Generally also the distinction of unity and plurality, the counting of small numbers, and the

estimation of different degrees of plurality—few, some, several, many, etc.,—pass as quickly. We make special predicates of these only when the question arises how many there are, or when we wish to establish a doubtful or disputed statement; then the synthesis which is contained in the judgment is between the plurality which is presented and now counted and the definite or indefinite numerical idea.¹ Generally this again finds expression only as part of the denotation of the subject, as a completed result; and what we are concerned with is the statement to be made about such and such a number of subjects.

5. What is the nature of the function of judgment, when there arise in this way such judgments as “hail-stones are falling—some stars are becoming visible—many trees are rooted up—fifty men are wounded”?

The first view is that which regards the plural of the verb—and therefore the copula also—as the sign of a plurality of acts of judgment, which are summed up in one common expression. Before I can say, “some stars are visible,” I must have seen one here, one there, another there. The predicate belongs to each one, but either I do not know their names, or I do not wish to name them; instead of saying *a* Lyrae and *a* Cygni and *a* Bootis are visible, I denote the particular individuals by their common name only; but it is these definite particular stars to which I refer. It is only in one set of cases, however, that the plural judgment arises in this way. In others the subject is perceived as a plurality, as it were, at one glance, and the predicate is stated of this plurality; the synthesis is then, as a matter of fact, simple. This is particularly obvious in judgments where the predicate cannot belong to the particular subject by itself. “Numberless birds fill the wood with their song”—“the trees grow thickly together”—these are not judgments which can have arisen from a summation of many judgments.

It is different when the numerical term is the real predicate. The

¹ Those logicians who see in every judgment a subsumption of the subject under a more general predicate concept which is its genus, might be puzzled to say what the particulars are in respect to which three, or seven, or a hundred are general and what extension is to be ascribed to these concepts. Does the extension of “three” comprise everything in the world which I can enumerate as one, two, three? Or must we not rather say that three is in itself a fully determined idea, in which there can be no question of extension, because, as we always count in the same way, the number itself is always absolutely the same? Again, when it stands as predicate is it really the predicate of the things concerning which the statement is made, or is it not rather the predicate of their number, this number owing its existence to the fact that I am at the moment grouping together and counting just these things and no others?

proposition "many men are short-sighted" is not intended to inform me that *A* and *B* and *C*, etc., are short-sighted, nor indeed is it meant as a statement about definite individuals. The information offered is the lamentable fact that the short-sighted are many—*i.e.* many, comparatively speaking, in comparison with the total number. When news arrives from the battle field it is taken for granted that some have been killed, or at least wounded; the question is how many, and the construction of the telegram, "Dead 10, wounded 50," is logically the most correct.

We need not point out that repeated activities give occasion in the same way as things for plural judgments.

It is true, no doubt, that several particular judgments must precede all such statements of number; before I can count, each particular one must have been observed, *A* is short-sighted, *B* is short-sighted, etc. But in counting them I disregard everything which distinguishes them; I forget who is short-sighted, I know merely that I have made my observations upon people, and retain the definite number of repetitions of the same observations upon homogeneous individuals, determining its relative magnitude. I proceed like the statistician, who fills his schedules with numbers only, and who cares nothing as to who are the subjects of the enumerated births, deaths and suicides; the predicates of his judgments also are numbers.

It is necessary to treat of these obvious points at length, in order to throw some light upon the obscurities of the traditional doctrine of universal and particular judgments.

§ 27.

The "all" by which the subject of the so-called universal judgments (all *A*'s are *B*) is bound together, signifies, according to its original meaning, a definite number, and a judgment beginning with "all" presupposes a limited number of particular objects which can be counted. Thus, according to its original meaning, "all *A*'s are *B*" can only be said in reference to definite particular objects. And here from a logical point of view the "all" is the predicate (the *A*'s which are *B* are all *A*'s).

We must carefully distinguish between this EMPIRICALLY UNIVERSAL JUDGMENT and that which is UNCONDITIONALLY UNIVERSAL. In the latter we attempt inadequately to express the necessary connection between the predicate *B* and the subject-idea *A*, by falling back upon the unlimited number of the particular. (If anything is *A*, it must also be *B*.)

1. According to its original meaning "all" presupposes a definite number, for it expresses the fact that two definite numbers are equal to each other. Before I can pass a judgment of the form "All *A*'s are *B*," I must undertake a twofold process of counting. I must first count the things which are *A*, and then the *A*'s which are *B*; then if the two numbers are equal I express it in the proposition "All *A*'s are *B*" (such phrases as—"all four," "all nine," remind us directly of this process). If I say "they are all there"—*e.g.*, the guests invited—then I know how many I invited, count those who are present, and the numbers being equal, the "all" results. If I think that a card is missing from a pack, I count them, and if the number of cards which I hold in my hand is equal to the number which belongs to the pack, they are "all there."

It is not necessary that the definite number should be expressly known and named, in order that a judgment may be stated as to "all." When a room has been emptied, and I say "they are all gone out," there is no need for me to know how many people were there. It is enough that none have remained behind, that I have reviewed in thought all who were present, and now know that each particular person who was present must also have gone out; hence that the predicate is wanting to none.

Under any circumstances "all" has always passed through this twofold negation. It arises out of the assumption of a possible difference between one number and the other,—out of the question, therefore, whether there is no exception. "All" denies the exception; and by whatever method I may assure myself that there is no exception, whether by directly counting them or only by taking one after the other with the certainty that none escape me, I am equally sure of my "all." Thus the formula *nemo non*, *nullus non*, etc. is really the primitive one, and not a circumlocution; it exactly expresses the process gone through, and it is *omnes* which is the secondary expression.

2. Strictly speaking the real statement is concerned with the "all." It is this which is the predicate from a logical point of view, even though it may not appear as such grammatically. The proposition is "those *A*'s which are *B* are all *A*'s." It is implied by the plural that there are many *A*'s, it is also implicitly stated that there are *A*'s which are *B*; but the question to be dealt with and to be answered by the judgment is, whether the *A*'s to which *B* belongs are all—whether there is no exception. (When we are dealing, not with things which can be counted side by side in space, but of states or activities which take place at different times, then what we have said applies to the "always" and "every time.")

3. From this it is clear that in a judgment concerning "all," the words originally apply to particular things ; that these particular things must be present in a definite and limited number which can be counted, and that only upon this presupposition can a judgment concerning "all" be the adequate expression of my thoughts.

In other words : The words "all *A*'s are *B*" are originally the expression of an empirical generality only, *i.e.*, a generality attainable by actual counting; and they can only be used in reference to subjects which are forthcoming in a definite number which can be counted, and of each of which the predicate can be stated. They are the expression of a definite limited comparison of the cases before us, and they presuppose that I am certain that the judgment is true of each particular one before I can state it of all.

4. How then are such judgments as "all men are mortal," "all bodies are extended," etc., related to these? Their meaning is not that before making the judgment we have gone through all men or all bodies one by one and counted them ; but that the predicate, mortal or extended, belongs to whatever is a man or whatever is a body.

But there are two senses in which such judgments may be really valid. They may, in the first place, be explicative judgments (analytical in Kant's meaning of the word), because based upon the recognised meaning of the subjects-term. I can state with perfect certainty that "all animals feel" without having enumerated the particular animals, if feeling is contained in my idea of animal—if therefore I should not call anything an animal to which feeling was wanting. In this case it is only in a secondary way that the thought is expressed by "all." The expression results simply from the analysis of the idea which I connect with the word animal. The meaning of the word determines the limits within which it is applicable, and hence I can foretell from the meaning that the predicate to feel must be present wherever the denomination animal is justified (p. 9 sq. above). Because *animal* feels, all animals feel. The analytical proposition expressing the meaning which the words have in our thoughts is translated into the more familiar language of narrative judgment concerning particular things, and by my passing from the general thought to individuals it becomes more intuitible. This is why the expression "all" has become naturalized where it is not original; where, that is, we only anticipate an experience of all the particulars, and anticipate it even where, from the nature of the case, it can never be completed.

In other cases the predicate is not analytically included in the meaning

of the word. The word "man," for instance, may include only the particular formation of the body, life, ability to speak, etc. ; a definite length of life is not necessarily included in it; every one at one stage of his life is able to distinguish men from everything else with confidence, before he thinks of asking how long they live or whether they all die. When this is the case the judgment "all men are mortal"—is not analytical. Nor is it a judgment of experience in the sense that "all men" denotes only those whom I know and in whom my experience has taught me that I shall find the predicate. But this makes it only the more certain that it is the result of an inference ; either of an inference from all observed cases to all others, of which there is an indefinite and incalculable number ; or else of an inference from the determinations which are understood as expressed by the word to others which are necessarily connected with them. If we really form the judgment, and do not merely repeat it after some one else, it can only be as the result of some such inference.

* We can never tell at all from the verbal expression of the judgment in which sense it is to be taken ; whether it is to be understood as an empirically universal judgment which presupposes a definite number of subjects, or as an unconditionally universal judgment ; and, if the latter, whether as analytical or synthetical. The ordinary doctrine does not hesitate to regard every judgment beginning with "all" as belonging to the same species.

5. If a judgment concerning "all" is unconditionally universal, then it is clear that no direct statement is made about the actual existence of the subjects, though this is most certainly presupposed by empirical judgments if they refer to actual things at all. "All A 's are B " means only "what is A is B " ; or "if anything is A it is B ." It is, indeed, indefinitely presupposed that some existing particular thing is recognised and called by the name A , but this is not stated in the judgment. Just for this reason the plural, and indeed the whole mode of expression, is, strictly speaking, inadequate, a *μετάβασις εἰς ἄλλο γένος*, a relapse from the sphere of the free and independent thought which moves among our firmly-established ideas into the habits of intuition, which deals with the particulars. The adequate expression is simply A is B , man is mortal, a square is equilateral, etc.

6. The traditional doctrine finds no difficulty in introducing the universal judgment. What is generally said is that if a predicate B is stated of the whole extension of the concept A which forms the subject,

then the judgment is universal; if a part only of the extension, then it is particular. If the subject-term is a *nomen proprium*, or an equivalent expression, then its extension is exhausted by one individual; thus the judgment "Callias is rich" has so far the character of being universal.

Nevertheless this simple doctrine contains—besides the doubtful use of the *nomen proprium* as the sign of a concept—an obscurity of which the consequences are constantly recurring. It is generally taught that the extension of a concept is made up of the concepts contained under it as species, inasmuch as we can form a plurality of more definite general ideas by means of the differences admitted by the higher concept. But in the discussion of the universal judgment it is generally assumed without hesitation that the extension of a concept consists of particular existing things, and that there is no difficulty at all in reviewing, establishing, and recognising this extension, because it is presupposed that our concepts are already all that they should be—that is, the expression of the nature of things according to their fixed specific differences. Hence Logic does not as a rule distinguish between those judgments which are based upon the Concept alone (*i.e.*, the meaning of the subject-word), and which in unfolding this meaning attribute a predicate to every thing which is named by the subject-word and thus forms a part of the "extension of the concept"; and those which—perhaps on the ground of unanimous experience—state a predicate of all known things which fall under the same denomination because they have the same attributes. By neglecting this distinction Logic obscures the most important point, the transition, namely, from an empirically universal to an unconditionally universal judgment, the process by which concepts and judgments are formed from experience. The judgment "all planets move round the sun from West to East" is originally empirically universal; any one giving utterance to it before 1781 meant by all planets, all six; between 1781 and 1801 Uranus was counted amongst them, and all seven were included; from 1807 till 1845 all eleven was what was meant; and to-day we mean in the same way all the two hundred or whatever the number may have become. But what is meant is always all that are known, the same movement in its orbit being attributed to each. The proposition says that all the bodies which I call planets have a common direction of movement from West to East; that I know of no exception. But suppose that I had recognised it as necessary—upon the ground, say, of the hypothesis of Kant or Laplace—that all compact bodies moving in constant orbits around our sun should move in the same direction, because

no reversed movements were possible within the space which can contain them ; *then* I should have to include movement from West to East in the meaning of the word planet—*e.g.*, to distinguish them from meteors—and then my judgment, “all planets move from West to East,” would be analytical in the Kantian sense, and would extend to the unenumerated planets still to be discovered. It would mean that whatever can be called a planet moves from West to East ; from which it follows that anything moving in the reverse direction would not be a planet.

7. The difficulty of finding a place for the so-called Singular Judgment in the classification which distinguishes universal and particular judgments, arises—as is clear from what has preceded—from the fact that the former is in no way comparable with the latter. For in the case of universal and particular judgments we have to do with a predicate which implies an absolute or relative numerical statement ; their genus is not the judgment in general, but judgments whose predicates are numerical ideas. But in the case of the so-called singular judgment we are dealing with the question of what belongs or does not belong to a certain particular subject, and not of how many subjects are forthcoming which possess one predicate.

In the first place, then, we cannot regard the division of judgments into singular, particular and universal as correct and exhaustive. In the second place, we have no ground for treating particular and universal judgments as special kinds ; there is no more reason in the ordinary Logic for regarding judgments having “all” for a predicate as a special kind than there is in mathematics for regarding judgments having the predicate “equal” or “infinite” as a special kind. Hence it is an arbitrary proceeding on the part of the traditional Logic to ask of every judgment whether it is particular or universal. Singular judgments concerning the particular and concrete have been treated as universal (in spite of the fact that there are three kinds generally called singular—the individual judgment “Callias is rich,” the numerical judgment “one planet has a ring,” and the particular judgment of the next section “there is a comet which has broken up”) ; while plural judgments have been regarded as particular, although in no sense comparing what is given with the “whole extension of the concept” ; and simple explicative judgments, even definitions, were without a place until they consented to pass as universals. Generality plays an important part in human thought ; nevertheless in the last instance it borrows its importance from necessity.

§ 28.

The so-called PARTICULAR JUDGMENT as given in the common formula "some *A*'s are *B*" may be an empirical judgment concerning particular things. In this case it differs from the merely plural judgments only when it aims at establishing an exception to the universal, or at preparing the way for a universal judgment.

When the subject is not to be taken in an empirical sense it is a completely inadequate expression for the thought which it is meant to denote, and confuses the important distinction between empirical and unconditionally valid judgments.

1. To the universal judgment the traditional Logic opposes a Particular Judgment having for its formula "some *A*'s are *B*," and in so doing it follows Aristotle, though his meaning was not the same. This particular judgment, as ordinarily treated, is one of the most unfortunate and inconvenient creations of Logic. So far as the words go, its meaning is quite indefinite, and as a rule it is incongruent to the thought which it should express and which it really obscures. It is true that the difference between the universal and particular judgments is generally elucidated by pointing out that in the former the concept which serves as subject is taken in its whole extension, while in the latter it is taken in a part of its extension only (*ἐν μέρει*). If we allow that the extension refers to the totality of particular individuals, then this distinction holds good on the assumption that we know the whole extension and that therefore all the parts of the extension are actually given to us. This distinction between the universal and the particular judgment was rational enough according to the view of Nature taken by Aristotle; a view which starts from the idea that a system of fixed and immutable concepts has realized itself, and is constantly realizing itself, in the fixed forms of Nature, and that our empirical knowledge surveys this realisation of the concept in all its essential differences. And the distinction was all the more rational because Aristotle never actually applied it except when it was justified. But when later Logic dealing only with conceptual relations, and totally disregarding the material realization of the concept, took up the Aristotelian distinction, and made use of its formulæ—or rather of a bad translation of them—the result was a host of absurdities, rendering the ordinary doctrine completely false if the words are understood in their ordinary meaning.

2. The Aristotelian *τινὲς ὑπάρχειν μὴ παντὶ ὑπάρχειν* is generally translated

by the formula "some *A*'s are *B*," but the plural here can have a meaning only when applied to things which are particular and definite, and can therefore be counted, in which case it presupposes a narrative judgment treating of the actually existent.¹ And, in the same way, when the particular judgment is contrasted with the universal, the plural has a meaning only when it is presupposed that the extension of the concept is divided into parts, each of which itself contains a plurality of individuals; whereas there seems no reason why a single individual should not suffice to form a part of the extension of the concept.

The first condition is present in all cases where a particular judgment is opposed to an empirically universal judgment—all planets move in ellipses, some planets have moons. But when we are dealing with abstract subjects, whose extension does not consist in a plurality of things, the formula leaves us in the lurch. Must we say "some virtue is justice" or "some virtues are justice"? "some love is calf-love," or—but there we have no plural! Even in cases where number is not contrary to sense, the use of the plural shifts the ground of the judgment. Such judgments as "some parallelograms have equal diagonals," "some conic sections are parabolas," have a very strange sound from the point of view of geometry; for in geometry we do not think of its constructions as spread abroad in the world in a plurality of instances, so that we can speak of them as of some cats which have blue eyes and some quadrupeds which can fly. The universal judgments, "all parallelograms are divided by the diagonals into congruent triangles," "all conic sections are curves of the second degree," sound less strange, because "all," when used in the unconditional sense, extends beyond the empirically known. But this privilege does not belong to the particular judgment; this necessarily imprisons the thought within the sphere of the particular. Ἡ κατὰ μέρος εἰς αἴσθησιν τελευτᾷ.

But on the second point the customary plural is false and misleading; "one man is sinless" is as much a particular judgment as "some men are sinless" would be; and Aristotle himself included the singular in his *τις ἀνθρώπος λευκός*. Herbart is quite right in his correction of the ordinary doctrine on this point (*Eínl.* § 62).

3. When a judgment of the form "one *A* is *B*," or "some *A*'s are *B*," is a narrative judgment of empirical origin, then it seems to have no other meaning beyond that of stating a definite predicate of one or more subjects which are denoted indefinitely by a general word instead of being

¹ Even Kant makes the category of plurality correspond to the particular judgment.

named one by one. The second judgment does not seem to differ in its plurality from a number of judgments concerning single subjects, since the numerical determination is not emphasized.

Still in the judgment "some men mistake red for green" there is something more implied than in the copulative judgment "John and Peter and Paul mistake red for green." The individual definiteness of the statement is indeed lost by denoting John and Peter and Paul, as "some men." But through being denoted by the general name they are placed in a relation to the whole of mankind which challenges a comparison; and the meaning of the judgment indicated by the indefinite denotation of the subject, is that those who as men are like all other men differ from others and possess some peculiarity in this respect; in other words, that differences of colour-sensation are contrasted with the assumed likeness.

In so far as it thus aims at emphasizing differences and exceptions the plural judgment becomes particular. But it is clear that this aim is attained just as well by a singular judgment whenever its subject is denoted by the general name instead of the proper name. The judgment "there is a comet which has broken up into two" is particular in this sense.

4. But the traditional logic teaches that the particular judgment is not meant to exclude the universal; "some *A*'s are *B*" does not tell us that not all *A*'s are *B*. This is a new proof of the ambiguity of the formula, for, generally speaking, it is certain that what we do mean to say is just this—that some *A*'s are distinguished from the other *A*'s. Still the former interpretation has an element of truth in it, which is, that the plural judgment may both be proceeding towards a universal judgment to which it is preliminary, and be cut off from a universal as an exception to it. As a proof against the apparent immobility of the fixed stars, it was first shown that some of them have a motion of their own, and the copulative judgment "*α* Centauri, and 61 Cygni and Sirius have a movement of their own," was expressed as "some fixed stars have a movement of their own." It was meant by this not that these three are on this account not fixed stars, but that while leaving them amongst the fixed stars, we have—contrary to our old belief—perceived movement in certain of them; the judgment was opposed as an exception to the proposition "all fixed stars are absolutely immovable." It was a particular judgment meant to express a distinction amongst the fixed stars.

But as the number grew and progress was made with observations the

same judgment "some fixed stars have a movement of their own" was able to take the new meaning that it is known certainly of some and is probable of all. The former judgment presupposes the knowledge complete that a predicate belongs to some *A*'s and is wanting to others, while the latter presupposes that our knowledge is only in a state of growth, and the particularity of the judgment is merely provisional.

5. But the school logic does not generally take any notice of this progress of knowledge through experience of the particular; its particular judgments presuppose fixed conceptual relations and are only adapted to the interpretation of these. But this logic falls into difficulties when called upon to show the truth of its propositions by means of the principles of identity and contradiction. Whence do I obtain the knowledge that "some parallelograms have equal diagonals"? Not from the concept of the parallelogram, for this contains nothing about right angles; and even if I add "some" to "parallelograms," though by so doing I take only a part of the extension of the concept, the concept itself has become no more definite than before, and I cannot upon the strength of this addition say of the part anything which was not contained in the concept. If then no particular judgment can proceed from a mere explication, it must be possible for us to define the idea of the parallelogram more accurately and conceive it in a form which implies the predicate and is one among a number of other possible forms, or else this form must be present in thought as constituting the subject of my judgment. I do not however state it in denoting the subject; I mean right-angled parallelograms, but denote them merely as some parallelograms.

The more adequate expression would then be the "parallelogram *can* have equal diagonals" and "one kind of parallelogram *has* equal diagonals."

Still we should certainly not banish from logic its formula "some *A*'s are *B*" in the sense that "some *A*" denotes a part of the possible *A*'s, were it not for the constant danger of unwittingly substituting actual things for possible things, for it is actual things which are indicated by the plural in the first place.

§ 29.

II. NEGATIVE PLURAL JUDGMENTS.

Exactly the same rules hold good when one and the same predicate is denied of a plurality of subjects ; and it is particularly to be noticed that the judgment which denies universally may be either empirically or unconditionally universal, just as the affirmative judgment may.

1. The COPULATIVE NEGATIVE JUDGMENT,¹ “neither *A* nor *B* nor *C* are *P*” leads, when *A* and *B* and *C* fall under a common genus, to the plural negation “Several *N*’s are not *P*.” This again leads to the statement which refers to the number of the *N*’s, “the *N*’s which are not *P* are many, are a hundred.” The relation between this statement and the negation concerning a single thing is exactly the same as was explained with reference to the affirmative judgment (§ 26).

2. The UNIVERSAL NEGATIVE JUDGMENT—“those *A*’s which are not *B*’s are all *A*’s”—is originally obtained by the same process of reviewing a definite number as that by which the universal affirmative judgment is obtained. When I examine a definite number of trees one after the other to see whether they bear fruit, and must answer in the negative as to each one, up to the last, then I obtain the universal negation, which is appropriately expressed in the words “none of them bear fruit.”² For by means of this “none” one after the other is brought before me ; there is not one, οὐδὲ εἷς,

¹ This must be distinguished again from the conjunctive negation of different predicates with respect to the same subject (*A* is neither *B*, nor *C*, nor *D*) ; the import of this judgment cannot be made clear until later on. I consider it a burdensome superfluity of terminology to make use of the expression remote judgment for the copulative negation.

² Thus “no person”—“no one”—“nothing, etc.,” are not negative subjects such as the Aristotelian οὐκ ἄνθρωπος ; I make no statement about “nothing”—“no one.” When I say “no one is good except God alone,” the subject of my judgment is “human beings,” and their goodness is denied ; what I mean is “there is none who is just, not even one.” When I say “nothing hurts me,” I do not mean that a thing called nothing hurts me, but that everything that might perhaps hurt me does not hurt me. But the appearance of the negation in the subject is most expressive of the way in which I seek, as it were, a subject for my predicate and find none. It is the same when “no one”—“nothing”—“none,” stands in the accusative, e.g. “I see no one going, no one coming,”—grammatically the “coming no one,” appears as object of my seeing, actually it is the sight of a coming person which is denied. Again, in the proposition “I hear nothing,” not only the object, but the hearing itself disappears—“it is false that I hear anything.” From this it follows that “nothing” (as well as “no one,”) has no meaning except in a proposition ; to use it out of a proposition as the independent symbol of a concept, as in the famous Being, Nothing and Becoming, must lead to mere playing with words.

ne unus quidem, to which the essayed predicate belongs. A single *A* which was *B* would prohibit the universal proposition. This explains also the ambiguity of the negation, and the different meanings which may belong to judgments of the form "no *A* is *B*." For on the one hand they presuppose the existence of a number of *A*'s, and what they are meant to tell us is that the predicate *B* is absent from all existing *A*'s,—no tree bears fruit. On the other hand, they may be meant to deny the very existence (within the given spatial or temporal region) of subjects to which the predicate might belong—no tree is casting a shadow, no spring breaks through the sand. When I deny that an *A* exists which is *B*, it is presupposed that I am trying to find a subject *A* of which the predicate *B* may be affirmed. It may be that I succeed in finding one, or some *A*'s, but without the predicate *B*; or that I find no *A* at all, which will be the case when the predicate *B* could not be absent if an *A* were there (cf. "the fire is not burning," p. 124).

Thus the judgment "no *A* is *B*," has for its immediate meaning the denial that an *A* which is *B* exists. It is only secondarily, and in cases where the predicate might be absent from *A*, that the proposition might be expressed as "the *A*'s which are not *B* are all *A*'s."

3. From this it follows again that this formula "no *A* is *B*," is adequate only when particular *A*'s are in view, and as the result of judgments concerning particular *A*'s; when therefore it represents a narrative judgment. But if we wish to say that the predicate is excluded by the subject-idea, hence that anything which can be named as *A* is on that account not-*B*, then the adequate expression is "*A* is not *B*," or "it is impossible for *A* to be *B*." It is only our habit of always returning to the concrete and intuitable, which makes us use the unconditionally negative judgment (as well as the universal affirmative) with reference to particular things, even in cases where no question is directly raised either as to their number, or even their existence. Instead of saying "no man can know the future," it would be more correct to say "man cannot know the future"; for my judgment denies the possibility, not the existence, of the prophet. This is obvious when modal predicates question the existence of any particular thing corresponding to the subject-word. We do not say "no ghost exists," "no murder is commanded," but "ghosts do not exist," "murder can never be commanded."

III. THE NEGATION OF PLURAL JUDGMENTS.

§ 30.

When a universal judgment is denied, the negation applies to what is actually stated in the judgment; viz., that the subjects to which the predicate belongs or does not belong are all which fall under the subject-term. The negation of "all *A*'s are *B*," means "the *A*'s which are *B* are not all *A*'s"; and the way in which we understand the negation must depend upon whether the judgment was intended as empirically or as unconditionally universal.

The negation of the empirically universal judgment tells us that there actually is an exception; that of the unconditionally universal judgment only that an exception is possible.

Aristotle taught, and logic is constantly repeating his teaching, that universal affirmative and particular negative judgments, and universal negative and particular affirmative judgments, are contradictorily opposed. This doctrine leads to false conclusions unless attention is paid to the difference between empirically valid and universally valid judgments.

1. The real character of the judgments we have so far considered is brought out most clearly by the negation. The negation of a copulative or plural judgment has more than one meaning, inasmuch as that which is false may be merely the plural, or the whole connection between subject and predicate. The difficulty is especially great in obtaining any definite statement from the negation of a negative statement. If it is false that neither Peter nor Simon Magus were ever in Rome, that does not tell me whether both were there, or which of the two; from the statement that it is false that several comets have brought misfortune I do not know whether only one has brought misfortune or none at all. The negation of a numerical predicate disputes the numerical predicate itself in the first place, but it is left uncertain whether it does not extend further. If it is false that ten houses are burnt down, then either more or less or none at all are burnt down.

2. A more definite value attaches, according to the ordinary doctrine, to the negation of a universal judgment—whether affirmative or negative.

When a negation is advanced against an affirmative judgment concerning "all" it annuls the statement that the number was complete without exception; the universality is denied. Since the affirmative universal judgment says, "there is no exception," its negation says, "there is an

exception." If I know it to be false that all ravens are black, then there is at least one which is not black; hence I can say "one raven is not black."

Suppose the negation is directed against the proposition "no A is B "; according to what was said above this means "there is not an A that is B ," so that as a consequence of the negation it must be true that there is an A which is B . If it is false that no raven is white, then there is a white raven.

Thus, the principle of contradiction and of the twofold negation being applied to propositions with the predicate "all," the teaching of Aristotle (*De interpr.*, 7, 17 b, 16) follows directly from the meaning of the universal judgment, *i.e.*: ἀντικείμεθα κατάφασιν ἀποφάσει ἀντιφατικῶς τὴν τὸ καθόλου σημαίνουσαν τῷ αὐτῷ ὅτι οὐ καθόλου, οἷον πᾶς ἄνθρωπος λευκός—οὐ πᾶς ἄνθρωπος λευκός, οὐδεὶς ἄνθρωπος λευκός—ἔστι τις ἄνθρωπος λευκός. This formula is perfectly correct, and has not yet been falsified by the thoughtless habit of substituting the plural "some" for οὐ πᾶς and τις.¹

3. But it is correct only so long as we avoid passing from uncondition-

¹ The ordinary doctrine is that there is :

Contradictory opposition between,	All A 's are B
	Some A 's are not B
and between,	No A is B
	Some A 's are B
And contrary opposition between,	All A 's are B
	No A 's are B .

These two latter propositions cannot both be true, but may both be false.

Of the judgments "some A 's are B —some A 's are not B ," Aristotle rightly says (*Anal. pr.*, II. 15, 63 b, 27) τὸ πινὶ τῷ οὐ πινὶ κατὰ τὴν λέξιν ἀντίκειται μόνον—because the subjects are quite different. To these judgments modern terminology has given the absurd name of "sub-contrary"; both, it is said, may be true, but both cannot be false. (If we assume that in the proposition "some A 's are B —some A 's are not B " the subjects are the same, the denotation alone being indefinite, then the propositions would naturally be contradictorily opposed; but the expression leaves it undecided which A 's are meant.)

A difficulty into which the ordinary doctrine seems to lead us bears witness to the justice of our view as given above; viz., that the contradiction between universal and particular judgments of opposed quality is the simple consequence of regarding "all" as the predicate. As an instance of this difficulty, the two propositions "light is matter—light is not matter" are contradictorily opposed and one is necessarily true; while the equivalent propositions "all light is matter—no light is matter" are said to be in contrary opposition only, and may therefore both be false. The difficulty is solved when we notice that in the second pair of judgments an entirely new subject is introduced, which involves the assumption that we are speaking of light, not according to its unity but according to its differences. From this it follows that the proposition "all light is matter" is after all an inadequate and not completely equivalent expression for "light is matter."

ally valid judgments to judgments which are empirically valid and *vice versa*.

The negation may be directed against an unconditionally universal judgment, which is intended by means of the more intuitable universality to state, when affirmative, the necessary connection of subject and predicate; when negative, the necessary exclusion of the predicate from the subject. In this case it can deny only what was meant, and says that in the first case it is not necessary, in the second that it is not impossible, for the predicate to belong to the subject. But this negation, which has nothing to do with the presupposition that particular subjects have been enumerated, must not be taken to mean that the predicate *B* belongs or does not belong to one or some actual *A*'s; it would be quite inadmissible to apply the relation of contradiction here. The statement that it is false that all men are sinners (in the sense of the sinfulness inherent in their nature) does not tell us that some men are actually not sinners; and the empirical judgment "all men are sinners" might still be true, "because all have sinned." If it is false that no man is entirely bad, in the sense of the negation of an impossibility, it would not on that account be true that one or some were in reality entirely bad.

On the other hand, the negation of an empirically valid particular judgment can never be the ground of an unconditionally universal, but only of an empirically universal judgment. If it is false that there is anything living which has not had its origin from something living, then the proposition *omne vivum ex vivo* is true in the sense that everything living which is known to us has had its origin from something living. But whether it follows from this that the proposition states an absolute necessity, is still disputed. It may be false that there are men who live over 200 years, but this gives no ground for the judgment "no man lives over 200 years" in the sense that it would be impossible to be a man and still to be more than 200 years old.

It is characteristic of the inferences of empirical science that they pass from empirically valid judgments to universal judgments which are unconditionally valid. But their justification for so doing can be found neither in the doctrine of the contradictory opposition of universal and particular judgments, nor yet in the ambiguity of the "all"; and it is the difficult task of a theory of induction to determine under what conditions the transition may be made from an empirical judgment to one which is universally valid.

4. The real import, then, of the universal and particular judgment with which Aristotle and the traditional logic were concerned, and to which they attributed so much importance, was not that of the ordinary theory, viz., "that a predicate belongs to the whole extension or to a part of the extension of a concept," but that the connection of a predicate with the subject is necessary or possible. The whole interest attaching to the absence of an exception lies in its indication of a binding law ; the whole interest of the exception in the fact that it points to a plurality of possibilities.

This brings us naturally to the closer investigation of the necessary and the possible with reference to judgments.

CHAPTER VI.

POSSIBILITY AND NECESSITY.

It is necessary for our immediate guidance, and before proceeding to treat of the logical questions referring to the possible and the necessary, to lay down a fundamental distinction. The statement that a judgment is possible or necessary is not the same as the statement that it is possible or necessary for a predicate to belong to a subject. The former refers to the subjective possibility or necessity of judgment ; the latter to the objective possibility or necessity of what is stated in the judgment. The Kantian distinction of the differing modality of judgments, according to which they are problematical, assertorial or apodeictic, applies to the former ; the Aristotelian proposition : *πάντα πρότασις ἐστὶν ἢ τοῦ ὑπάρχειν ἢ τοῦ ἐξ ἀνάγκης ὑπάρχειν ἢ τοῦ ἐνδέχεσθαι ὑπάρχειν* (*Anal pr.*, I, 2, 24^b, 31) to the latter.

I. THE SO-CALLED MODAL DISTINCTIONS.

§ 31.

The so-called PROBLEMATIC JUDGMENT “*A* may be *B*” (meaning that *A* is perhaps *B*) in so far as the consciousness of objective validity is absent, cannot properly be called a judgment ; *i.e.*, it is not a judgment concerning that which is denoted by the subject of the proposition. It is a judgment only in so far as it states that the speaker is uncertain as to whether *A* is *B*.

The so-called ASSERTORIAL JUDGMENT (the simple statement *A* is *B*) does not differ essentially from the apodeictic judgment (it is necessary to affirm that *A* is *B*) ; for every judgment of which we are completely conscious when we give utterance to it, includes the affirmation of the necessity of its utterance. It is true that the judgments differ with respect to the way in which the certainty is attained, whether it is immediate or mediate ; but if we were to ground the distinction between assertorial and apodeictic judgments upon this difference, then the apodeictic would take the subordinate place, as having only a derivative certainty.

1. The distinction between a statement which is merely possible and one which is necessary, does not appear in the case of immediate judgments where subject and predicate are found to agree without further mediation. Such judgments are formed according to the principle of Agreement with a confidence which is not the result of reflection. But it is different with mediated (synthetical) judgment such as takes place when we form ideas of syntheses between definite subjects and definite predicates not yet contained in the subject-idea before us. These ideas may be due either to external stimulus, the questions or statements of other people, or they may arise internally from psychological combinations. But in either case there may be no ground capable of yielding a consciousness of objective validity in the synthesis, and then what we have is merely the idea of a synthesis which remains in suspense as a question or conjecture, and awaits some decisive certainty to confirm or deny the predicate. Here the judgment is thought of as *possible*; and this means that it is not necessary at the moment for the person thinking about it either to affirm or to deny it. Such a judgment (*A* is *B*) which is merely thought of as possible and not yet affirmed, may be briefly denoted as the *hypothesis A* is *B*, and the simplest expression of this stage between synthesis and judgment is the question. This is genuine only when expecting yes or no for its decision; when asked by one who has already made up his mind for the sake of testing some one else, it is not a question, properly speaking, but an imperative. But while the question expresses the first conception of the hypothesis which seeks confirmation or rejection, it is followed—if neither is found—by the consciousness of indecision which is expressed in the formula "*A* is perhaps *B*, *A* is perhaps not *B*." (The formula so often used "*A* may be *B*" is ambiguous and misleading, for it expresses both the objective "can" (*δύνασθαι*) and subjective hesitation.) This form of statement, then, differs from the question only by expressing our consciousness of inability to decide the question. While the question contains the wish for decision, this form denotes the resignation which is obliged to remain in uncertainty. Both really express the same thought—that of a synthesis concerning the validity of which we are undecided.

2. This expression of uncertainty is generally called a Problematical Judgment, and Assertorial and Apodeictic Judgments are opposed to it as expressing different degrees of certainty.¹ Kant himself, indeed, gives a

¹ Cf. e.g. Ueberweg, *Logik*, ed. 3, § 69, p. 164 sq., ed. 5, p. 207; Drobisch, § 61, § 62.

somewhat different meaning to the problematical judgment. He tells us (*Kritik d. r. V.*, § 9. 4) that modality contributes nothing to the contents of the judgment, but is concerned only with the value which the copula has for thought in general. Problematical judgments are those in which we accept the affirmation or negation as merely possible (a matter of choice). Assertorial judgments, those in which it is regarded as actual (true). Apodeictical judgments those in which we look upon it as necessary. The arbitrary acceptance of the problematical judgment Kant extends to judgments which are obviously false ; they have a problematical significance if it is thought that some one might perhaps for a moment accept such a proposition. On this view, which compares the problematical judgment to the Aristotelian *ὑπόθεσις*,¹ every judgment is problematical when its validity is not at the moment expressly stated. But this may mean one of two things, between which we must distinguish. The reason why nothing is stated concerning the validity of a judgment in Thought may be that nothing *can* be stated, because the speaker has not yet come to a decision ; or it may be that the speaker *will not* state anything concerning its validity, some ulterior purpose leading him for the time being to treat a valid judgment as invalid, or an uncertain judgment as certain. On this point the traditional Logic has not followed Kant, and Kant himself in his *Logic* (Intr. ix.) means by the problematical judgment nothing more than an uncertain acquiescence.

3. But the custom of denoting the proposition "*A* is perhaps *B*" as a problematical judgment, threatens to destroy the concept of the judgment itself, and to fall into contradiction with all other teachings. The essence of the judgment is to present a statement which claims to be true, and calls for belief. An utterance therefore which states nothing and leaves it open for the opposite to be true, is not a kind of judgment. If every judgment is either the affirmation or the negation of a question, then an utterance which neither affirms nor denies cannot be a judgment ; for to leave the question undecided is no decision, and to be uncertain is not a degree of certainty. The law of contradiction notwithstanding,

¹ According to Aristotle, the *ὑπόθεσις* is a merely assumed judgment concerning something which is or is not ; the judgment is not certain, or at least it is not proved to be certain, and it must be conceded if it is to be used in conversation or proof. Cf. my Programm : *Beiträge zur Lehre vom hypothetischen Urtheile* (Tübingen, Laupp, 1870), p. 2. A justification may also be found here for the use of the word which I have introduced above.

the two propositions " A is perhaps B " and " A is perhaps not B ," would both be true together.

So far, then, as concerns A , the so-called problematical judgment is not a judgment at all; it is only the thought of one, the unfinished attempt at a judgment. The only real statement made in the formula " A is perhaps B " is, "the hypothesis A is B is uncertain." Immediately, and in the first instance, this is nothing but a judgment concerning the person who is speaking, concerning his relation to the hypothesis A is B . The formula says: I do not know whether the hypothesis is valid or not, I have no ground either for affirming or for denying it; it states the present condition of my thought, but nothing which could have objective validity with reference to A .

We might try to give a wider meaning to the formula by taking it to mean, not merely that I do not know whether A is B , but that it is not known whether A is B ; thus regarding the uncertainty as something more than a fact peculiar to the individual, as always attaching to the proposition. But apart from the fact that the words do not mean this, even this statement could not lead to a judgment concerning A which might be co-ordinated with positive and negative judgments. We still have only a statement concerning a subjective attitude, though an attitude not peculiar to the individual but due to the present position of the sum total of knowledge, or, which is still more general, due to the limits of human intelligence. It is quite true that with reference to many questions we can get no further than stating the impossibility of a decision, and that such knowledge has its value when we are measuring our human capacity by the ideal of knowing. Nevertheless it states nothing which could be taken as a judgment concerning A . For an ideal consciousness, an omniscient intelligence, the one proposition is true, the other false; not until we are certain of the one or the other have we realized the aim of thought, a judgment which is objectively valid. Until then, the hypothesis remains an unsettled problem, and it is only confusing to subsume the expression of subjective uncertainty and the expression of the certainty of the objective validity of a proposition, under one concept as judgments. Hence the only possible negation of the problematical judgment is "the person forming the judgment is not uncertain whether A is B , but is certain either of the affirmation or of the negation."¹

¹ Not only Wundt (*Logik*, I, p. 197, cf. my reply in the *Vierteljahrsschr. für wiss. Phil.*, iv. 473 sq.), but Windelband as well (*Strassb. Abh.*, p. 185 sq.), has defended the

We must, then, cease to teach that the so-called problematical judgment is a kind of judgment, if we hold that the concept of the judgment

problematical judgment against this view. The latter endeavours to show from the position which has been discussed above, p. 122 sq. that the problematical judgment is co-ordinate with the affirmative and negative judgment; that it is a third kind of the "practical relation" which is expressed in the criticism of a given combination of ideas. "The critical judgment, like all other functions of approval and repudiation, is capable of gradual variations." The gradation of certainty appears in the different degrees of probability; these represent different intensities of the feeling of conviction, a feeling which applies both to negative and to affirmative propositions. "We may think of these different intensities of probability as represented by a line; at each end we have complete certainty, of affirmation at the one, and of negation at the other; and these, by a gradual diminution of the certainty, approximate to a point of indifference where there is neither affirmation nor negation." This zero point has two meanings, for the indifference between positive and negative reaction may be either total or critical. Total indifference occurs, first, in all those trains of thought which are accompanied by no estimation of the truth; secondly, in the question where ideas are combined without any decision as to the truth of their combination, but still with the desire for such a decision. Now since the question contains no decision as to the validity of the thought, Windelband does not co-ordinate it (as Lotze attempts to do) with the affirmation and negation as another kind of judgment. But when we are led by consideration of the ideas combined in a question to the conclusion that there is no sufficient ground for certain, or even probable affirmation or negation, then we get the problematical judgment, which signifies that nothing is to be said concerning the validity of the ideal combination $A-B$. This is intentional suspension of criticism, critical indifference. Such conscious refusal, to come to any conclusion is, says Windelband, quite decisive as to the position assumed by the person judging towards the combination of ideas, contained in the question; therefore, when we divide judgments according to their quality, the problematical must be co-ordinated with affirmative and negative judgments.

Of course I fully recognise the truth of what is here said as to the import of the so-called problematical judgment and acknowledge my obligation for the distinction (which I have adopted) between the problematical judgment and the simple question. But I cannot help drawing an exactly opposite conclusion from it. Even from Windelband's point of view, that affirmation and negation are both alike criticisms of the value of an ideal combination, I fail to see that the suspension of criticism is a kind of criticism. The relation between the three "forms of judgment" cannot be that of co-ordination. Either I cannot come to any decision or I can; if I do come to a decision, I must decide either in the affirmative or in the negative. Thus it is only affirmation and negation which are co-ordinate because they are divergent kinds of decision, and both are opposed to indecision; this is what I wished to show. If I have no knowledge on any matter unless I can either affirm or deny, then the knowledge that I can neither affirm nor deny is only a knowledge of my subjective inability; it is therefore a judgment about myself and not about the subject of my proposition. Nor can I agree that because certainty is a feeling and all feelings present differences of intensity, therefore there are degrees of certainty. Certainty, if we take the word in its strict sense, is either present or not; anything which is not absolutely certain is uncertain. It is true that certainty manifests itself immediately in consciousness, that there is a feeling of certainty; just as uncertainty, the wavering between opposite probabilities, manifests itself in feeling. But then the opposition is not between certain affirmation as one extreme and certain negation as the other, with uncertainty as a transition between the two; it is certainty and uncertainty which are

includes the affirmation of the truth of the statement made, and if we teach that a judgment must be either true or false.

4. The traditional doctrine is not much more successful in its distinction between Assertorial and Apodeictical Judgments. Kant says (*Krit. d. r. V.*, § 9. 4; *Logik*, § 30) that the assertorial judgment is accompanied by the consciousness of the actuality of judging, the apodeictical judgment by the consciousness of its necessity; and according to this all that is needed in the assertorial judgment is that a statement should find utterance in words, the consciousness of the necessity of the judgment being unessential. In the introduction to the *Logik*, ix., again, the assertorial judgment appears as the expression of a merely subjective belief, which is valid only for me; while, on the other hand, that which I know is said to be apodeictically certain, *i.e.* necessarily having a universal and objective validity for every one—even though the object of this assured conviction should be a merely empirical truth.

According to this distinction the assertorial judgment would also be excluded by our definition of the judgment; for in this we make the claim to objective validity an essential characteristic. Indeed, from this point of view the judgment has but one meaning, that every one must affirm and believe the same, because it is necessary to affirm and believe it. Our speech would lose all serious meaning, and degenerate into mere child's play or lies if, when we stated a proposition, we did not also mean to say that its negation is false, and that any one making an incompatible statement is wrong; *i.e.* if the distinction between an assertorial and an apodeictical judgment were that while the latter is necessary the former is not, that while the latter is true for every one the former is true only for me. Truth has no meaning except this necessity of our subjective action. Even the merely temporal statement concerning the most casual particular phenomenon—this iron is hot—presupposes that at the moment it is necessary to judge in this way and not otherwise. My sensation makes the connection between this subject and this predicate inevitable, and if the question were raised whether or not the iron is hot, I should maintain against all contradiction that in no other statement could I find expression for my sensation.

opposed, and certainty belongs both to affirmation and negation. If the doctrine that certainty has degrees were consistently carried out, we must regard the distinction between opinion and knowledge as merely relative. It is really only the hope of certainty which has degrees.

5. Here, then, there disappears all essential difference between the assertorial and apodeictical judgment. When I say "this is so," it is not a perfectly complete judgment unless it means "I must of necessity judge that this is so"; the certainty of my statement rests entirely upon the presupposition of this necessity.

All that remains of the distinction is that the necessity rests upon different grounds in different cases, and that we become conscious of it in different ways.

6. With reference to the first point, we may begin by distinguishing between immediate and mediate judgments. In the case of immediate judgments (in specie analytical) the necessity of affirming (or denying) the predicate of the subject is grounded upon the principle of agreement (or of difference); in the case of mediate judgments, either upon authority or upon some process of inference. Immediate judgments depend either (as in perception) upon individual experience as the ground for attributing a predicate to a subject, or upon the generally recognised meaning of a word. The same distinction between a ground peculiar to the individual and one accessible to all, divides mediate judgments into those which depend upon authority and those which depend upon a process of inference. The fact that I take some one as an authority is a ground which holds good only for me as an individual, so long as his credibility is not established and proved as universally valid; but a process of inference cannot be binding upon me unless it is binding for every one who starts from the same data.

In this way the distinction has been made between immediate certainty (which is grounded upon perception by ourselves or by others), and mediated certainty which is grounded upon proof—though really the certainty grounded upon the perception of others ought rather to be counted as mediated, and immediate certainty is not confined to perceptions alone. The assertorial judgment has been reserved for the former kind of certainty; the apodeictical (*πρότασις ἀποδεικτική*) for the latter. The distinction is borne out, moreover, by the customary formulæ, "*A* is *B*," and "*A* must be *B*" ("must" being taken as the expression of that which is merely inferred, as in the proposition "it must have rained last night.") But then it becomes necessary to give up the ordinary notion that the apodeictical judgment denotes something higher than the assertorial; and that in passing from the problematical to the apodeictical there is an increase of certainty, and therewith of the value and dignity

of the judgment. For every mediated certainty must ultimately rest upon that which is immediate; every proof upon premisses which need no proof themselves. It is curiously at variance with the emphasis laid upon apodeictical certainty that in ordinary life the "apodeictical" judgment "it must be so," "it must have happened so," denotes only a very moderate degree of confidence; and this because we have good grounds for distrusting the certainty of ordinary inferences, and prefer to trust to that which is immediately perceived. But even when the most rigid proof is presupposed that which is proved can never claim a higher degree of certainty than the data from which it is proved.

Other writers seem to have in view rather the difference between propositions which hold good with absolute universality, and those which depend upon conditions peculiar to the individual; as when, e.g., the necessity of reason is characterized as apodeictical, in opposition to matter of fact. In this way Leibnitz distinguished between necessary truths and truths of matter of fact.¹ Necessary truths are those whose opposite contains a contradiction; truths of matter of fact are those whose opposite is possible. The former may be reduced to identical propositions; the latter rest upon immediate sensation. This way of formulating the distinction fails to show that the subjects referred to by necessary truths, and those referred to by truths of matter of fact, are not of the same kind. Necessary truths of reason set forth comparisons between concepts, which are assumed to be the unalterable possession of all. It is only upon this presupposition (according to § 23, p. 145 sq.) that it can be said of a proposition that it is contradictory, hence that its opposite is true; such truths correspond to Kant's analytical judgments. The subjects of truths of matter of fact are things having a particular existence, and truths of matter of fact, so far as they have reference to existence and to changeable events, certainly tell us something which is not contained in the concept of the thing; for it is no part of the concept of the thing that it exists, nor yet that it happens to be constituted in one

¹ Leibnitz, *Princ. phil.*, § 33 (Erdm., p. 707): Il y a deux sortes de vérités, celles de raisonnement et celles de fait. Les vérités de raisonnement sont nécessaires et leur opposé est impossible, et celles de fait sont contingentes et leur opposé est possible. Quand une vérité est nécessaire, on peut en trouver la raison par l'analyse, la résolvant en idées et en vérités plus simples jusqu'à ce qu'on vienne aux primitives . . . 35 : ce sont les énonciations identiques, dont l'opposé contient une contradiction expresse. *Nouv. Ess.*, iv. § 1; Erdm., 340 : Pour ce qui est des vérités primitives de fait, ce sont les expériences immédiates internes, d'une immédiation de sentiment. Cf. *De scientia universali*, Erdm., p. 83.

particular way. To deny them, therefore, involves no such logical contradiction as to say that a triangle is not triangular. But because the opposite of a truth of matter of fact is not *a priori* impossible, it does not follow that it is not necessary for me to state the fact after it has happened; or that the opposite statement would be possible for any one knowing the fact. Even truths of matter-of-fact are truths only because it is impossible to state their opposite; but with them the impossibility rests upon the ground of individual experience, not upon the ground of the unalterable concepts from which I start. Even when immediate consciousness is translated into a proposition of objective validity, it is presupposed that the sensation is referred to existence and an existing thing according to universally valid principles. Truths of reason are contained even in truths of matter of fact, inasmuch as no true judgment can arise out of individual experience except in accordance with general principles (*e.g.* the principle that every change presupposes a permanent subject in which it takes place). On the other hand, our possession of general concepts upon which identical propositions are based, is in the last instance matter of fact, something which must be there before the principle of identity can be applied and give rise to a necessary judgment. Thus the necessity of both kinds of truths is, finally, hypothetical. If I think in definite concepts, I must predicate of them what I think in them; and if I have certain perceptions I must predicate of the perceived subjects what these perceptions force me to predicate.¹ Even this dis-

¹ It may be objected here that the senses deceive us, and that while it is impossible to doubt the proposition $A=A$, it is possible to doubt the existence of the whole material universe. This is quite true, but in no way invalidates the proposition that judgments are true only in so far as they are necessary. For if we assume that our empirical sensations are purely fortuitous, differing for each individual and as inexplicable as dreams in their occurrence, we could make no statement as to an existent, or indeed any universally valid statement at all, but then neither would any empirical judgment, except such as referred to our momentary affections, be possible. If, on the other hand, we assume that though sensations are indeed subject to a necessity which is the same for every one, yet both this necessity and its law are unknown to us—then again we can form no judgment concerning external existence. This is really the case so far as concerns the question as to the ultimate nature of the existent which we feel, and for this reason we get here only conjecture and hypothesis, not judgments which can be announced as true. But wherever we are persuaded that we recognise necessity, in the processes by which we form judgments from sensations, there we find judgment which is assured and accompanied by a conviction of its truth. We know that we have a sensation of this or that colour, that we are obliged to assign to it a definite position in space and that we must regard it as the colour of some definite phenomenal object; whether this object is mere phenomenon or the phenomenon of something existent and

inction disappears so far as the nature of the necessity is concerned, and it is only the ground of the necessity which differs because the subjects of the judgments differ.

There is no doubt that our consciousness of the necessity of connecting a subject with a predicate may arise in different ways. Many immediate judgments are affirmed by us unhesitatingly and without reflection, without a suspicion that we may be wrong, or that the facts may be otherwise. Absolute certainty and unalloyed confidence in our act of thought is here inseparable from that act itself, and with such judgment our thought always begins. The announcements of immediate self-consciousness,—as well as statements of what is immediately evident, whether in intuition or universal judgments—are accompanied by no feeling of constraint, such as we might expect from the necessity of which we have spoken, nor yet by the thought of the possibility of the opposite; not until some attempt is made at contradiction are we conscious of this. With other judgments we are constrained to certainty because all other possibilities are cut off; and in the consciousness of this constraint the judgment and its certainty present themselves together. If we define necessity as the impossibility of being otherwise, then we may say that only the latter judgments are accompanied by a consciousness of necessity, not the former.

But it is that immediate security and certainty which is the original and genuine form which necessity assumes in the sphere of thought; in it we see thought at work in its most active and powerful form. And nothing can completely replace this immediate self-evidence. The attempt to maintain a contradiction may indeed serve to confirm the sense of security, and to measure the force which is exerted in a statement; but the validity of the original proposition must, generally speaking, be presupposed before we can see that its opposite is impossible. Only when it has been established that *A* is *B*, is it immediately clear to us that the proposition *A* is not *B* contains a contradiction. The twofold negation does not create the proposition, it merely circumscribes it by cutting it off from its opposite; still it is in this form that we become expressly aware of the truth by withdrawing from it and again returning to it. Just as we first become explicitly conscious of identity through negation of the other, of affirmation through negation of the negation; so also we become

what the nature of this existent may be—all this is matter of dispute, and the view which is taken determines the sense in which a judgment concerning matter of fact is true.

conscious of necessity through the impossibility that it should be otherwise. But necessity is already contained in the thought which elucidates it; the negation of the negation confirms the affirmation only because this process is itself immediately certain in every step it takes. The true and original necessity is that unreflecting necessity which predominates in all our thought, and just for that reason cannot be brought into consciousness at every point.

We might distinguish between assertorial and apodeictical judgments by saying that in the case of the latter we are expressly conscious of their necessity, which therefore finds expression in words; while in the case of the former the necessity is undifferentiated from the act of judgment itself. Then we should have hit upon a really existing difference, which attaches, not indeed to the degree, but to the kind of certainty belonging to a proposition. But it is a difference which is entirely psychological, and which shows how that which is dependent upon conditions peculiar to the individual may—though accompanying the same judgment—appear now in one way, now in another. It is a difference, moreover, which signifies the exact opposite of what we mean by the ordinary expressions, for the apodeictical form “*A* must be *B*” reminds us of the doubt, and of the conceivability of the opposite; it proceeds with circumspection from *A* to *B*, while the assertorial judgment makes straight for its mark. It is just when the judgment is an inferred one that the assertorial form expresses more complete confidence than the apodeictical, which seems to challenge us to test the proof. Thus the former is always the more natural, because the more direct expression of apodeictical certainty; even logic states the conclusions of its syllogisms in the assertorial form.

It might be objected to this that many statements are made at random, the speaker troubling himself but little as to the necessity of his statement. This is true, as true as it is that many lies are told. But it does not refute the proposition that the act of which the sober statement is the adequate expression involves the declaration of the necessity of the judgment, and that it is in this way that every one understands the statement. Else would speech be without thought, using words without meaning; or burdened with falsehood and presenting as certain that of which the speaker is not certain. It has nothing to do with logic that many falsehoods are thus spoken in the strife of interests and party; for logic presupposes the desire of speaking, as well as of thinking, in accordance with truth. We

grant also that it is only gradually that this desire of speaking and thinking the truth becomes conscious ; at first it is merely an impulse unconscious of its aim. But before this consciousness is clear the speaker knows not what he is doing ; judgment is indeed there, but it is not free and conscious and has not yet come to full maturity.

8. The necessity of thought which is manifested in the certainty of particular acts of judgment owes its distinctive character in the last instance to the unity of self-consciousness. Every particular judgment may be repeated with the consciousness of the identity of subject and predicate as well as of the act of judgment ; starting from the same data it is always the same synthesis which takes place, and our self-consciousness cannot exist apart from this invariability. Thus our judging ego, with its unvarying activity, is opposed to particular acts of judgment as a universal, as the same and the permanent which binds together the different and temporally separated acts of thought. With the confidence of the movement in each particular case is connected the consciousness of unvarying repetition, of return to the same point. In this constancy, which presents a general law in contrast with the particular act, we are conscious of judgment as something withdrawn from the sphere in which we have a subjective choice and are free to bring about alterations ; we are conscious of it in the same way as when it maintains itself in some particular act against contradiction. Because this identity and constancy of our action is the condition of our consciousness as one and undivided, it is also the final and fundamental basis upon which we can fall back. While this consciousness, in its completeness and comprehensiveness, is not yet there—as in immature childhood—so long the psychological conditions of judgment are not yet fully developed. The case is the same in dreams, where only a few trains of imperfectly coherent ideas are aroused.

From this it follows that each single act of judgment points, by its meaning, to necessary and universally valid laws ; laws, that is, which are universally valid both for the single subject in his temporally different moments, and for the various thinking subjects with whom we stand in one community of thought ; laws which, at first unconscious, serve only to bring about the certainty of the judgment, but which, when raised into consciousness, give us our fundamental intuition of something which is necessary.

9. The necessity of thought, which first becomes apparent in the certainty of the particular act of judgment and the invariability of its repetition, is something essentially positive ; it is the immediate expression of

intelligence, the form of self-consciousness itself, and when brought into consciousness it is as much an immediate intuition as the thought of the ego, or of being. For this reason, it is also the measure of the further concepts of possibility and impossibility. In the sphere of judgment the possible is that which it is not necessary either to affirm or to deny ; it is the suggestion, the attempt of a judgment, which is never decided and completed, and cannot be taken into the unity of self-consciousness, or woven into the lasting fabric which is as certain as my own existence. Mere possibility is a privation. The impossible, on the other hand, has two meanings ; that which it is impossible to think, would never be thought, at the most it might be spoken in words. There can be no thought corresponding to the words "the circle is square," and this is what Aristotle means when he says that it is impossible to think of the same thing as at the same time being and not being—"for it is not necessary to accept what we say as true." On the other hand we have the possible which must necessarily be denied, the hypothesis which is quite feasible when taken by itself, but the affirmation of which would conflict with a valid proposition, and so place thought at variance with itself. This impossible is to be found only in the sphere of the mediated judgment ; the predicate may be thought of as compatible with the subject because their incompatibility is not known analytically, and thus the proposition may be temporarily accepted so long as the truth opposed to it escapes consciousness ; only a thorough comparison between all our judgments brings about the negation of the possible. It is in this sense only that Leibnitz's distinction is correct, when he says that the negation of necessary truths is impossible, that of truths of matter of fact possible. We may attempt to state them, but our experience refuses to confirm, and forces us to deny them.

10. From what we have said, it follows that an actual affirmation or negation—*i.e.* a judgment which we utter with a consciousness of its validity—is only possible for those to whom it is necessary ; as far as the judgment itself is concerned possibility and necessity coincide. On the other hand the hypothesis is possible if—and so long as—it is not necessary, and therefore impossible, either to affirm or deny it. As the expression of a subjective state of indetermination it may certainly be said to take a third place beside affirmation and negation ; but just for that reason it is not a judgment.

§ 32.

The so-called LAW OF SUFFICIENT REASON, as originally conceived by Leibnitz, is not a logical law at all, but a metaphysical axiom, which is applicable to a part only of our judgments.

Inasmuch as every judgment presupposes the certainty of its validity, we may lay down the proposition that no judgment finds utterance which has not some psychological ground for its certainty; and inasmuch as it is justified only when it is *logically* necessary, every judgment claims to have a logical ground which makes it necessary for all who think. But it does not do more than raise a claim, the justice of which it falls to logic to investigate.

The nature of necessity in thought is expressed by the proposition that when the ground is affirmed its consequences are also necessarily affirmed, and that when the consequences are denied the ground also is denied. This principle of Ground and Consequence corresponds to the principle of contradiction as a fundamental law of the operation of thought.

1. The results of the preceding paragraph seem to find their natural expression in the proposition that judgment cannot take place without a ground, for by the ground we mean just that which makes a judgment necessary. Thus an analysis of the import of every judgment framed and spoken would give us the fourth of the so-called laws of thought; it would express the characteristic common to all judgment whatever—*i.e.*, that belief in the validity of the judgment involves belief in its necessity.

2. But the law of Sufficient Reason has had various meanings assigned to it, and in this has shared the fate of the other so-called laws of thought. Leibnitz was the first to give it a co-ordinate place as a first principle with the law of contradiction. "Our inferences," he says,¹

¹ *Princ. phil.*, 31 sq.; Erdm. 707: Nos raisonnements sont fondés sur deux grands principes, celui de la contradiction . . . et celui de la raison suffisante, en vertu du quel nous considérons qu'aucun fait ne saurait se trouver vrai ou existant, aucune énonciation véritable, sans qu'il-y-ait une raison suffisante pourquoi il en soit ainsi et non pas autrement, quoique ces raisons le plus souvent ne puissent point nous être connues. In the *de Scientia Universali* (Erdm., p. 83), Leibnitz formulated the same principle as follows: Omnis veritatis (quæ immediata sive identica non est) reddi posse rationem, hoc est, notionem prædicati semper notioni sui subjecti vel expresse vel implicite inesse. It is evident, therefore, that the principle as he understood it here was a purely logical one, according to which all propositions which are not identical are true only in so far as their necessity is syllogistically proved. On the other hand, there are places in which he emphasizes only the metaphysical aspect, *e.g.* in the *Theod.*, 44 (Erdm., p. 515): . . . l'autre principe est celui de la raison déterminante, c'est

“are based upon two great principles, that of contradiction . . . and that of the *ratio sufficiens*, by virtue of which we assume that no fact is true or actual, no proposition true, unless there is a sufficient reason why it is this and not something else, although in most cases these reasons may be unknown to us.” It is not difficult here to distinguish the two sides of the question, and to see that he is speaking partly of the actual existence of real things and events, partly of the truth of propositions. But we must remember that Leibnitz intends this principle as the ground for truths of matter of fact alone, *i.e.* for the truth of propositions which state a fact, while necessary truths rest upon the principle of contradiction; and that with him the ultimate *ratio sufficiens* is always the Divine Will. Then it becomes evident that this distinction signifies nothing, and that the principle of Leibnitz is no other than the principle of causation in the real world; the principle, that is, that the existence of every real thing, and the reality of every event, must have a cause. The truth of propositions which state facts is based upon the reality of those facts, their truth being thus dependent upon the reality of the fact stated and this reality upon the sufficient cause; so that in giving the real ground of a truth of matter of fact, I name the cause which has given rise to the actual event or thing. But this just shows how little we are justified in regarding this principle as an absolutely universal logical law, which should rank beside the law of contradiction as valid in reference to the same propositions, or in trying to find in the Leibnitzian proposition a logical ground distinct from the real cause. Such an interpretation is indeed excluded by the repeated remark that the *ratio sufficiens* may frequently be unknown to us. This is true only of real causes. A logical ground which we do not know is, strictly speaking, a contradiction; for it only becomes a ground through our knowing it. Unless we believe in the fiction that a judgment can be true apart from any intelligence by which it is thought, we cannot believe that its ground has an independent existence of its own.

que jamais rien n'arrive, sans qu'il-y-ait une cause ou du moins une raison déterminante, c'est-à-dire quelque chose qui puisse servir à rendre raison *a priori*, pourquoi cela est existant plutôt que non, et de telle plutôt que de toute autre façon. Ce grand principe a lieu dans tous les événements . . . *Pr. de la Nature et de la Grace*, § 7 (Erdm., p. 716): rien ne se fait sans raison suffisante. Cf. *Trois, écrit à Mr. Clark* (Erdm., p. 751). While yet again in the fifth letter to Clark, § 125 (Erdm., p. 778) we find the full formula: Ce principe est celui du besoin d'une raison suffisante, pour qu'une chose existe, qu'un événement arrive, qu'une vérité ait lieu.

If therefore we lay it down as a logical law that nothing should be thought without a ground, we must certainly mean something quite different from what Leibnitz meant.

3. If we distinguish between the Real Cause and the Ground of the Judgment, between what makes it necessary for an existing thing or event to be actual in a given way, and that upon which the judgment is based as an act of thought, there still remain two very different meanings in which the word "ground" may be taken.

From one point of view every judgment, as an actual psychological event in a thinking individual, may itself be regarded as something existent, and the conception of the causal relation and its fundamental principle is so far applicable that every event must have its sufficient cause. The cause of an act of judgment must be sought, in the first place, amongst psychological facts, for no judgment is possible unless certain ideas are present to consciousness; thus the psychological cause of a judgment is the sum total of the necessary conditions of this particular act of judgment. The principal cause is the judging subject itself, together with its faculty of thought, and the laws which govern the manifestations of this faculty; and it further consists in those states and preceding acts out of which this given judgment arises. Amongst these are the following conditions:—

a. The ideas both of subject and predicate must have been present in consciousness; (this presence in consciousness implies causes which are still further removed, and which we may call the *causæ remotiores* of the judgment, one of the most important being the will, as led by some interest in its desire to know and to think about an object).

b. A synthesis must have taken place between the ideas of subject and predicate, either because of an agreement which has led the activity of thought, in accordance with its laws, to connect them, or because their synthesis was suggested by the manner of their entrance into consciousness, which gave rise to the thought of their possible connection.

c. In the latter case something must have occurred to bring about a decision—either affirmative or negative, and, inasmuch as every judgment includes the consciousness of its validity, to afford a psychological explanation of the certainty as an actual state of mind.

From this point of view a distinction must be made amongst immediate judgments between those which connect mere ideas and those which contain a reference to the existent. With immediate judgments of the first

kind the principle of agreement (as expressing a law of motion for thought) is sufficient to explain both the synthesis and the certainty attaching to it; while statements which refer to the existent, as *e.g.* judgments of perception (it lightens; this iron is hot) depend upon more complicated pre-suppositions. Their occasion is a momentary feeling, or complex of feelings, which in its turn points to a number of causes which have brought me into a position in which my senses may be thus affected. But, besides an actual matter of fact, we must include amongst the causes of the judgment the sum total of all the psychological forces which are incessantly re-creating the ideas of actual things and their attributes out of sensations, and bring about in each particular case the certainty that we perceive and know the existent. The principle of agreement explains only the way in which we identify present perception with a former idea; it can never explain the general conviction of the material reality of things, nor the conviction that what we say at the moment is a true judgment concerning matter of fact. While then the merely explicative judgment is sufficiently explained by the causes which give rise to the ideas and our consciousness of them, together with the principle of agreement, the other judgments call for special explanations of our belief in the reality of things. Here we see the Kantian distinction between analytical and synthetical judgments reappear, and the meaning of the question how synthetical judgments (in the Kantian sense) are possible becomes obvious. It is clear also that recognition of the existing causes which give rise to our belief in reality, and in the actual validity of our judgments of perception, can tell us nothing as to the correctness of this belief; it is due, for instance, to existing causes that sun and moon appear larger to all of us when rising than when in the meridian.

In mediated judgments, however, the mediation which brings about a decision need not consist solely in premises which can themselves be stated in the form of judgments; it may also consist in unconscious habits of combination, and in the influence of authority which is rooted in unanalysable impressions.

Amongst the totality of psychological conditions we may now distinguish: 1. the occasion, which brings subject and predicate into consciousness, thus giving rise, in the case of mediated judgments, to the question; 2. the ground of decision which enables us to affirm the judgment, and to state the subjective synthesis as objectively valid, and which is therefore the ground also of the subjective certainty of the judg-

ment. The change in the objects concerning which we judge depends upon the occasion, which may, so far as the content of thought is concerned, be quite a matter of chance, and come entirely from without; but the ground of the decision always brings us back, in the last instance, to a psychical force working in accordance with laws, and any particular psychological event can only be called the ground in so far as it introduces the judgment by virtue of a constant connection. Thus in all immediate analytical judgment the ground upon which we attribute the predicate is the subject-idea, but only inasmuch as, in accordance with the principle of agreement, the presence of subjects and predicates which agree necessitates their synthesis.

5. Of this psychological ground of certainty the law holds good that no judgment can be made without a ground, *i.e.* unless the consciousness of its validity has in one way or another been produced; and thus without violence to veracity no proposition can be stated which is not accompanied by the consciousness of the validity of the judgment. This is involved in the nature of the judgment itself, as maintaining the validity of a synthesis, and in the fact that a purely arbitrary act, a *sic volo, sic iubeo*, could not produce that consciousness of validity which implies that the synthesis is not arbitrary. We do not, however, mean by this that we are always conscious of the ground as soon as we have uttered the judgment.

6. But the necessity asserted by every judgment stated with a full understanding of its meaning is not this psychological necessity, but objective truth; and the ground of its certainty, which is implicitly affirmed with it, is not one peculiar to the individual, but a universal ground which must make the judgment necessary for every one, and can lie only in the ideas themselves as such, since these only and not the individual frame of mind, etc., can be common to all. This alone is the logical ground, the ground of truth as distinguished from the ground of certainty. All error and strife are due in the last instance to a difference between the psychological ground of certainty and the ground of truth; to the possibility that momentary belief may err, and the temporary feeling of certainty deceive us. So far the law holds good that no proposition is true without a ground; but just for this reason the investigation of what is a logical ground, and what the conditions are which justify us in affirming a proposition, falls beyond our present task. The object of our analysis of the judgment was only to show that every statement claims by its meaning

to have a logical ground, and that this also involves the problem of how the ground may be known.

7. A further distinction is called for here, which arises from the fact that in thought we always start from data which have arisen involuntarily and without reflection. Absolute necessity would belong only to judgments necessarily developed in every being capable of judgments as such, both the ideas connected and the connection itself being inevitable; and this is what is presupposed in every theory which is based upon innate ideas (in the older and original sense) and innate truths. The ground of these judgments is reason itself, and so far as they are concerned there can be no difference between logical and psychological ground. But other judgments have only a hypothetical necessity, *i.e.* it is logically necessary to maintain them on the presupposition that something else has preceded in consciousness. So far, then, as it depends upon external conditions what ideas arise in a subject and come together in thought, the judgment *A* is *B* is indeed necessary so soon as we are conscious of *A* and *B* as agreeing; but that it should come into thought at all is not universally and absolutely necessary. Only when we assume an ideal thought, comprehending all truth, is the logical necessity a material necessity also, giving rise to actual thought. For the individual whose voluntary thought aims at this ideal, the necessity is *moral* and conditioned by his ability.

From one point of view, then, we can accept as a ground in its fullest and truest sense only that which must itself of necessity be thought. But from another we may look upon every actual premise as a ground so far as we acknowledge that a further judgment results from it with logical necessity. We may indeed go a step further and find a relation of ground and consequence between propositions which we think of only as hypotheses, and in reference to which therefore there is not even a psychological certainty. To say that a hypothesis is a ground with respect to another hypothesis means, that if the former is accepted as true, then the latter must also be accepted as true. Thus in the first case ground signifies that which compels us to complete a judgment so soon as it has actually come into conscious thought; here it means the hypothesis which, when accepted as valid, forces us to maintain the validity of another hypothesis.

It is of the ground in this latter sense that the law holds good which was formulated by Aristotle,¹ and which in later times has ranked only as

¹ Aristotle, *Anal.*, Pr. II., 4, 57 b. 1: "Ὅταν δύο ἐχῇ οὕτω πρὸς ἀλλήλα ὥστε θατέρου

the principle of hypothetical inferences ; the law, that is, that when the ground is affirmed the consequence is also affirmed, when the consequence is denied the ground is also denied. All that is expressed by this formula is the nature and meaning of logical necessity, just as the nature of the negation is expressed by the principle of contradiction. It tells us that if the proposition *A* is recognised as the ground of *B*, then the affirmation of *A* involves that of *B* also, the negation of *B* that of *A* also. This law alone deserves to rank beside the principle of contradiction, for, like it, it applies to a fundamental form in which thought moves, a progress according to necessary connections. But just as the principle of contradiction leaves it undecided which of the opposed statements holds good, so also this law leaves it undecided whether either ground or consequence is true.

8. Material causality must in no way be confused with the logical relation of ground and consequence. The proposition that every thing, or every change, has its cause, stands in just the same relation to the logical necessity of our judgments as every other general proposition which serves as a ground for extending our statements, or enables us to pass with logical necessity from one proposition to another. If we use the expression "ground" of material causality also, and say that the attraction of the earth is the ground of the falling of bodies, all that we are immediately told by this is, that the one produces the other *realiter*. But so far as the causal relation when recognised enables and forces us to infer from the fact that the cause takes place that the effect also takes place, so far that recognition is a logical ground ; and only by assuming the causal relation can we pass from the truth of one fact to that of another distinct from it. Thus the propositions which state causal relations play an important part amongst our logical grounds ; but it is far from being true that every logical ground is based upon a causal relation, and still less true that the way in which our judgments depend upon each other at all resembles that in which material causality takes effect. The distinction still remains between the ground of knowledge and the material ground, and it becomes obvious whenever an effect leads to recognition of the cause.

9. Finally, we must distinguish between logical grounds and grounds of probability. These give the preference to one of several hypotheses, none of which we have sufficient ground for affirming ; and this they do by

ὄντος ἐξ ἀνάγκης εἶναι θάτερον, τούτου μὴ ὄντος μὲν οὐδὲ θάτερον ἔσται, ὄντος δ' οὐκ ἀνάγκη εἶναι θάτερον.

intensifying the expectation that this one is, and will prove to be, valid. Thus their value is in the first instance only psychological, but they have sometimes a practical significance; *e.g.*, when it is necessary, upon practical grounds, to come to some decision even in the absence of certainty. The significance which they have for the growth of knowledge cannot be investigated until the third part of the book.

II. POSSIBLE AND NECESSARY AS PREDICATES OF ACTUAL JUDGMENTS.

§ 33.

In its objective sense, "necessary" is always in the last instance a predicate of that which is expressed in a judgment. It is necessary either that a thing is, or that it has certain attributes, or exerts certain activities, or stands in certain relations. This necessity is either internal and due to the nature of the thing, or external and due to causality. In either case it is hypothetical. It is knowable only in the form of general rules, which govern the particular; while, on the other hand, it is the aim of unconditionally universal judgments to express this necessity.

1. While the assertorial judgment is separated from the apodeictical by no essential, inherent difference, the statement that something must be or must happen differs in its contents from the statement that it is or happens when these statements extend beyond the sphere of our ideas to the existent, and refer to a material necessity. No doubt, wherever the "must" and the "necessity" appear in merely explicative judgments—such as "all bodies are necessarily extended," and "an effect must have a cause"—what is meant is the logical necessity contained in the fixed meanings of our words which compels us to connect a predicate idea with a subject idea, and hence to predicate the former of everything to which the latter can be applied. The judgment "bodies are extended" makes the same statement as the judgment "bodies must be extended"; the only difference is that the latter expressly reminds any one who may be in danger of forgetting it of the meaning of the words.

But when we are speaking of the existent as such, then our statements concerning its necessity refer to something which is binding upon the existent itself, and not merely upon our judgment. "Necessary" then becomes a significant predicate which is affirmed or denied in just the same way as any other predicates of actual judgments.

It is true that of things as such necessity in its proper sense cannot be used as a predicate ; it is not an attributive word. Such phrases as "God is a necessary Being," "the world is not necessary," are no adequate expression of our thought ; what we mean is, that God necessarily exists. Just as *δεῖ* or *oportet* require a proposition to follow, and "must" is a so-called auxiliary verb, so also it is only what is stated in a proposition—the existence of a thing, its possession of a quality, its development of an activity—which can be predicated as necessary. Only to abstract nouns which take the place of a proposition can "necessary" be attributed as a predicate ; e.g., the existence of God is necessary. (The necessity of the judgment is no exception to this ; it is necessary that I, and that every thinking being, should judge thus.)

This introduces a new class of statements—those in which the subject is that which is stated in a judgment (not the judgment itself, as in the case of the predicates true, false, credible, logically necessary). Thus we can only speak of material necessity in so far as a material unity, that of the thing with its attribute and activity, corresponds to the synthesis of the judgment.

2. What is it which connects existence with an object of thought, an attribute or activity with a definite existing subject, or different things in one relation ? If we disregard for the present necessity of existence and assume that certain things are, there remains for investigation their kind of being and their mode of action. This appears to us primarily as merely matter of fact and empirical, and what we want is to see that it is also necessary, since thus alone can it be understood and penetrated by thought. The ground of the subjective necessity of that part of our judgment which refers to the matter of fact in our knowledge must be found in material necessity. What we are investigating here is not the origin of the attempt to find such a bond of necessity in the world, and to attain, beyond the knowledge that something is and happens, to the insight that it must be so ; nor do we seek a metaphysical justification for this assumption. It is enough that the attempt is there and governs our popular as well as our scientific thought, so that the problem arises of establishing its meaning.

Here we must begin by distinguishing between the presupposition which guides all our thought, that there is necessity in the world, and the ground of the statement that this or that particular thing necessarily is. Necessity is knowable only where there is an invariability of connection in the

existent like that which governs the connection between thoughts in the sphere of logic (§ 31. 8); when therefore the particular event results invariably and with infallible certainty from what has preceded it, as the judgment always recurs in the same way when the same data are present; and when therefore a complete coincidence of material with logical necessity is possible. In such invariability alone do we find the ground of the necessity that something is. That which is to be a material ground must have the invariability and universal validity which belongs to the logical ground; it must be universal as opposed to the particular case, and invariable as opposed to mutability in time. To know something as necessary always means to know it as the consequence of something which is constant and universal. For this reason it is that we can never see the necessity of that which is purely individual, and as such incomparable with anything else, even though we may believe that it is necessary.

3. We regard the necessity which binds together subject and predicate as sometimes internal, sometimes external; and the distinction reminds us of that between analytical and synthetical judgments. When a subject is in itself sufficient to make its determinations necessary, we look upon the necessity as internal; when something else must supervene in order to give rise to a determination, we look upon it as external.

It is for us an internal necessity that the spirit is self-conscious and thinks; it is external that a body moves when it is pushed. In the first instance the attribute and the activity result of themselves from the subject alone so soon as it is there, in the latter case not until something else is there also.

4. When we speak of internal necessity, we contrast the unity of the thing with the plurality of its attributes and activities, and regard the former as the permanent ground, unaffected by temporal distinctions, which makes the attribute or activity necessary, either as constant or in a fixed order of change. In so far as the unity of the thing involves the necessity of certain attributes it is called the essence (the nature) of the thing, and everything which proceeds from its essence alone is essential to it. In no philosophical conception has this thought played a more important part than in the doctrine of Leibnitz that there is none but internal necessity, and that each particular monad develops its series of activities from its own inner nature alone. Here the nature of the particular individual is the sole ground of necessity, and the whole course of

its existence is only the unfolding of this nature. It is recognisable partly in immutable attributes and permanent activities, partly in the law of development which prescribes that one activity shall give rise to another.

It is this idea of a permanent ground governing the manifestations of a thing in which the thought of a thing as being identical with itself and yet having changeable attributes and varying activities culminates. If we want to establish the full identity of the thing, we must look deeper than its actuality at any given time. Here we find change; and since attributes are not external to the thing, the thing being what it is through its attributes, the identity of the thing itself seems in danger of disappearing if there is no basis permanent in change and giving rise to that very change. And the unity and identity implied in a thing can be represented in thought only if one and the same ideal content, which recurs always in the same way, may be taken as the counterpart of a real existence identical with itself.

This same thought of the nature of the thing as the permanent, timeless ground of its varying, temporal actuality, affords an objective justification for our primarily subjective general ideas. The comprehension of spatially and temporally different things under one general idea, and their denotation by one and the same word, is an arbitrary act guided only by subjective caprice or grounds of expediency, unless there belongs to the various things something really common to all and identical in all beyond the similarity which appears to us. But this common element lies behind the distinguishable and particular phenomena, with their individual peculiarities, and must be found in the fact that a common nature necessitates uniformity in attributes and activities, and that variations are looked upon as due to external causes, as accidental, and not essential.

5. External necessity is opposed to internal, and determination by circumstances to the development of nature. Every particular phenomenon is this because another is that; every change in a particular thing takes place because a certain change has taken place in another thing. Things have the power of mutually prescribing each other's action. The order of the world consists in this necessity which passes from one to the other, and which is causal necessity in the narrower meaning of cause as *causa transiens*. The knowledge that something is what it is, and happens as it happens, from external necessity, is made up of two elements—the general law and the particular datum to which this law is applicable. It is necessary for the planets to move in ellipses round the sun: our knowledge of

this rests on the one hand upon our knowledge of the general principles of mechanics, on the other hand upon our knowledge of the actual mass of the sun and the planets and of the relation between tangential velocity and attraction ; a different relation would give rise to different orbits. We cannot get rid of this purely empirical element, and for this reason our knowledge of the necessity as such is expressed in hypothetical forms only, which state that if this or that occurs something else necessarily occurs. The occurrence of the former is again necessary from other causes ; but in explanation of these we always come upon some further empirical fact, and so on *ad infinitum*.

Thus the necessity of each particular phenomenon is never more than a conditioned necessity, an *ἀνάγκη ἐξ ὑποθέσεως*. When something is said to be necessary, it is not the cause, but the fact that it results from the causes present, which is called necessary.¹

6. The hypothetical necessity of the existent due to its nature and its cause seems distinct from necessity due to the end. Man must breathe in order that he may live ; to maintain peace we must be prepared for war. But if we look more closely, this teleological necessity may be analysed into logical and causal necessity. Only as the thought of an actual, thinking and willing being is the end something actual which can be a ground of necessity. It is something which is thought of as in the future and willed, and the realization of which is intended, though, according to the causal order of nature, which connects every given result with given causes, it can be realized only by means of given causes. Hence whoever wills the end must also will the means ; assuming that we will a definite end, it is necessary that we will definite means. The connection between the thought of the end and the thought of the means as objects of our will is logical ; but the necessity of thought rests upon our knowledge of the causal necessity of being. The facts presupposed in the final cause are on the one hand that the end is willed, on the other that efficient causes are present which cannot be arbitrarily changed or multiplied. From our knowledge of these we may infer with logical necessity the means for a given end, which must be willed if the end is willed. But in our interpretation of nature we look upon results as ends even where there is no reference to the will of any thinking subject and its realization, because the end here serves as the point of connection for a plurality of causes ; and hence arises the appearance of a special kind of necessity, distinct from that which is logical or

¹ These concepts will be more fully explained in Part III.

causal. "Man must breathe in order that he may live"—this expresses nothing more than our recognition of the fact that the order of nature has connected death inevitably with the cessation of breathing, and that breathing cannot be replaced by any existing contrivance. Should life be willed as an end, breathing must also be willed as a means.

Suppose again that the thought were in itself creative ; then it would not be an end, needing the means for its realization ; it would be simply a cause having the power to produce a material reality, so that here again there would be no teleological necessity.

The same considerations apply to what has been called moral necessity. There are certain normal laws which a rational being with a will is forced to recognise as principles valid for his will, and by which he feels himself bound ; and the recognition of such obligation is a necessity of his nature which is regarded as an essential characteristic of the rational subject. If these normal laws are actually willed and accepted as a highest end, then it becomes logically necessary to apply them to the particular, to transfer the obligation to particular cases. But to regard the obligation itself in the light of a necessity because it brings with it a feeling of constraint, is to confuse our conceptions and to conceal the cleft which divides the recognition of obligation and actual willing.

7. When the necessity refers to existence itself, then, from the point of view of external causality, it is obvious that in affirming the existence of any particular thing to be necessary we assume a creative power which produces it either from blind necessity or for the sake of some actual end. We may, for instance, try to understand the existence of the world by explaining it as the self-manifestation of God ; in so doing, we derive its existence from a higher cause, and thus attribute to it a conditional necessity.

But suppose it is maintained that something has of itself necessary existence, as in the ontological proof where existence is said to belong to the essence of God, and to proceed necessarily therefrom, an attempt being thus made to convert hypothetical into absolute necessity ; then the light leaves us which the experience of our own self-consciousness had thrown upon the thought of necessity, and which had shown it to be merely a bond of connection between distinguishable data, either of thought or of being. It is a bond which breaks away as soon as we attempt to connect the existent with a mere concept, a concept, moreover, which belongs to no one's thought ; and if the idea is that something already

existent undertakes the superfluous task of making itself necessarily existent (although existence is already presupposed in such necessity), then there is nothing left for the bond to connect. Just as we cannot speak comprehensibly of logical necessity without presupposing an existing, thinking subject whose nature it is to think, so also an ultimate and simple being is always presupposed in necessity of existence. In the restlessness of our questioning, we imagine that we must have necessity even in the first member of our series; and when we accept the answer that God is *causa sui* and has the grounds of His being in Himself, we allow ourselves to be misled by words into stating empty formulæ,¹ the fictitious value of which is nowhere more obvious than when they are taken seriously and lead to the metaphysical myth of a ground distinct from God Himself. Somewhere or other we must end with simple being. That view of the world which confines itself to the cycle of finite causes must regard the whole complex of mutually conditioned beings as something simply existent; while, if we would understand the world as necessary, we make it dependent upon God, where it is even more certain that all distinction between being and necessary being absolutely fails us.

8. Mathematical necessity is often taken as the most perfect type of what we call necessity. Spinoza's standing illustration of the material necessity in the production of an effect by its cause is "in the same way as it follows from the nature of the triangle that its angles are equal to two right angles." This is not the place to investigate the nature of mathematical knowledge, or to determine the question whether its necessity is logical or material. It is, however, clear from what has been said that the very nature of mathematical objects involves that constancy and invariability by virtue of which they are always presented in the same way, and because of which every particular has the significance of a general in that it may be repeated in the same way in actual intuition; while in material objects we are forced to seek for a constant element and disengage it from casual and varying connections. Space and plurality, our intuition of space and counting, are all no doubt given in the first instance; but they are given in such a way that we are certain of their absolute immutability.

9. Now it is none other than this objective necessity which our universal judgments are meant to express; the necessity, that is, of the union between the subject and definite attributes, or of the connection

¹ Cf. Arnauld's criticism of Descartes in the *Objectiones Quarta*.

between definite attributes, activities, and relations, and other attributes, activities, and relations. Only when we are convinced of this necessity is the unconditionally universal judgment justified. The judgments "all matter is heavy," "that which is matter is necessarily heavy," "it belongs to the nature of matter to be heavy," are all equivalent. The connection of the predicate with the subject is necessary from the nature of the subject; whenever the subject exists, its predicate is actually combined with it. The judgments "every thrown body describes a parabola," "a body when thrown necessarily describes a parabola," again state the same thing; the universal judgment tells us nothing but the causal necessity of forces working according to fixed laws.

When such a judgment is denied, then the necessity is denied; and it is said that the subject might be without the predicate. This is expressed in the traditional doctrine by the particular judgment "some matter is not heavy."

10. When universal judgments express the essential predicates of things, they coincide with explicative judgments, and the logical necessity of the judgment with the objective necessity which finds utterance in it. The explicative judgment, while it states the contents of an idea, has also regard to the things corresponding to the idea, and thus it gains an objective significance whenever the idea comprehends the permanent and invariable elements necessarily involved in the existence of a given subject or subjects, when, therefore, the idea corresponds to the essence of the things. The explicative judgment "water is fluid" expresses only the contents of the idea of a thing which has been noticed in certain casual states. It does not represent the essential nature or the kind of matter which we call water, for this is also found in a solid or vaporous state; fluidity is not a part of its essential nature. The judgment "water is a compound of oxygen and hydrogen" is both explicative and an expression of the essential nature of water. It is the attempt to bring the two into perfect harmony which guides the problem of definition.

§ 34.

That alone is possible in the completely objective sense which is removed from the sphere of necessity as the manifestation of free subjects. Within the sphere of necessity we can speak of possibility only under two suppositions. Either we must think of things as removed from the temporal course of their actual existence and thus isolated from the con-

ditions of their actual being, in order that we may represent as grounded in their permanent nature predicates which really belong to them only when co-existing with others; or else we must isolate in thought a part of the conditions upon which the actuality of what is stated in a proposition depends. When in the latter case we are ignorant of the conditions, the judgment passes from objective possibility into the subjective possibility of conjecture, and thus into the expression of uncertainty.

The judgment "it is possible that A is B " stands in contradictory opposition to the judgment "it is necessary that A is not B ."

1. In investigating the many meanings of the expression "possible," we will first distinguish between the possibility of being something which is stated of a subject and the possibility of its mere existence. The former finds expression in the propositions " A is possibly B , A may be B ," the latter in the propositions " A is possible, A may be." The former judgments, again, may be sometimes stated of the particular as such, their subjects being definite things (or attributes, activities, and relations of definite things); sometimes of subjects which are thought as universal.

2. If we state of a particular thing that it *can* or *may*, we make a statement originally connected with a presupposition from which alone it derives its full meaning; the presupposition, that is, of free subjects which, as such, have the power of acting in different ways, but which exert this power only at the command of the will and upon the ground of a choice between doing and not doing, doing this and doing something else.

We have first the thought of different activities which the will alone is able to realize; which of them it will realize depends upon a decision which is neither externally necessary, nor yet a necessary consequence of former activity. Opposed to this freedom, we have on the one hand inability, when the material causality which could realize the thought is wanting to the will; on the other hand there is the "must," when choice is cut off, and necessity prescribes the path of action. On closer inspection, however, inability proves to be only another form of "must"; it is the necessity of not-doing.

3. The position of a free subject towards the activities between which it has a choice, strikingly resembles the position of the judging subject towards different hypotheses, neither of which it finds itself forced to affirm or deny. In both cases an act is planned in thought, but not yet actually realized; in both cases the question arises, "what should I do?"

But affirmation or negation can enter only when necessity appears, and the matter is removed from the sphere of free action; while in the other case it is the undetermined and arbitrary act which realizes one of the thoughts and thereby refuses reality to the other. We are not dealing here with the metaphysical truth of this view, but with the presuppositions which determine the thought of the possible in this direction. While in the one case the different hypotheses cannot become actual judgments, and so long as the choice remains the judgment is impossible, in the other the power of realization lies in the will, and stands in the same position with regard to either alternative, thus causing them to appear as really possible. Thus we speak of the real possibility of a scheme or plan, when we have convinced ourselves that all the conditions of its realization lie in our power, and that this realization depends upon our will alone.

The opposite, therefore, of the materially necessary, is that which results from freedom; here alone we find the absence of necessity. It is not without reason that language has combined the ideas of "will" and "can" in the meaning of the word "may."

4. But the idea of the possible extends even to that which is not free. This also may be regarded in such a way as to make it comparable with the free. Even the thing which is not free acts in different ways in so far as it is changeable, and is not constrained to be and to do the same thing at all times. When *we* look to its future a number of different predicates lie before us, and the thought is aroused of a choice between them. The sun will alternately shine and be concealed by clouds; the brook will at one time freeze, at another be dried up; our thought in picturing the future wavers between different predicates. But it does not depend upon the decision of the thing itself which of these predicates shall actually appear at a given time; this is determined by necessity. The necessity may be merely that of its own nature, which prescribes a certain development through different stages, and in this case it must become all that it can become; and it is only a distinction of time which makes us speak of the future, not as something which is, but as something merely possible. Or the necessity may appertain to both the nature of the thing and its circumstances; and when we are ignorant of the manifold combinations of circumstances and their changing course, or, disregarding them, comprehend in one thought that which in time is successive, then the thing is to *us* like a free being of whose future

decisions we are ignorant, and its actual states appear to issue from its arbitrary will and caprice.

It is in the former way that we regard the totality of the universe so far as we think of it apart from freedom. It contains the whole ground of everything which in the future will be and happen, but which is not yet actual. This is full possibility, *potentia* in its most significant meaning.

We regard in the second way the particular things or events which take their place in the order of the world, and which are both determined and hindered by circumstances in their natural development. In so far as the particular thing contains the partial ground of that which will be, there belongs to it the mere possibility of future states. Thus the seed contains the possibility of becoming a plant.

But this "can" has a perfectly objective and material significance only when we are certain that the predicate will actually appear under certain circumstances, because it depends upon the nature of the subject what series of predicates it will assume under different circumstances. As a rule we gather our knowledge of that which a thing may be from our experience of the past; but we mean it as a statement of certain knowledge when we say that the moon can be eclipsed.

5. This meaning of "can" is particularly obvious when we make general statements concerning our subjects: "water may freeze or evaporate"; "iron can be melted"; "cooking-salt is soluble in water," etc. Here we have perfectly definite and positive statements, referring to an attribute of the subject. Indeed, there is no way of making general statements concerning changeable attributes except by passing beyond the subject to the conditions and causes which determine its varying states. When I isolate the idea of a thing and disengage it in thought from the conditions of existence under which the actual always stands, retaining it by itself, then there remains to it at first nothing but those attributes which cannot be separated from it because they are essential. But as thought surveys the cycle of changes which must and will be introduced by varying relations and refers them merely to the general thought of the thing, it makes use of expressions such as "can," "faculty," "capability," etc., to turn the subject into a permanent ground for the changeable. This ground, however, is not in itself sufficient to bring about actuality; it must be supplemented by circumstances. But in proportion as all knowable attributes of things resolve themselves into relations with other things

we are able to express their unchangeable nature only as that which they may be under varying circumstances.

Judgments of possibility, which state the further determinations that a general idea admits of, are analogous with these. That which in the previous case falls asunder into the temporal series of successive states here breaks up into the plurality of ideas which contain a common element needing further determination before it can agree with any definite thing or can even be thought as particular. A triangle may be acute, right-angled, or obtuse. No intuitable figure contains *only* the properties which I think under the word triangle. Before I can form the image of one, some definite relation of sides and angles is necessary; and when in constructing it I try different determinations or recall them from memory, the general idea presents me with a choice of various closer determinations. No definite colour is necessarily connected with those attributes of an animal which constitute the contents of the idea horse. The horse can be black, white, brown, etc. So far as concerns the content of my idea, these judgments are perfectly definite statements concerning the plurality of differences. So far as they are intended to apply to the nature of the existent, they express in the same way a material possibility which connects variation of colour with the organization of a certain animal. Only when applied to a definite particular thing does the judgment take upon itself the problematical significance of ignorance. When all I know of a thing is that it is a horse, I cannot state of it that it is black or white; when I know only that something is a triangle, I do not know whether it is right-angled or not.

When we are dealing with subjects thought of as general, then the judgment "*A* can be *B*," etc., is the adequate expression of the so-called particular judgment.

6. It follows from the meaning of the judgments so far considered that even when they apply to the particular they are intended as universally valid, and their validity is not limited to any given time. Possibility and "can" take on another meaning when we speak of a particular instance, and state what may be and happen here and to-day. When we say, "it may freeze to-night," "the invalid can be saved," "the answer may come to-day," then our thought does not weigh the future by isolating its subject; on the contrary, it reviews the present circumstances and undertakes to calculate from them the result. But such certain prediction is prohibited by our want of knowledge, whether of all the circumstances, or of

the exact laws according to which they take effect. All that the judgments mean is, the sick man will be saved if the right cure is applied, if no unexpected disturbance appears, etc. Thus some of the conditions upon which the actual result depends are known, and form the ground of the judgment; and in estimating the known and certain conditions in comparison with the unknown and uncertain we enter upon the calculation of the probability of that which we call possible. Still it is our ignorance alone that makes it possible for us; and thus these judgments lead imperceptibly to those which merely state the subjective impossibility of a decision. While they appear to be concerned with things, they are really only concerned with the measure of our knowledge of things; and they are the expression of the resignation of our limited knowledge. This is proved by the fact that exactly the same expressions are used of that which already exists or is past. When the historian attempts to clear up a fact from fragmentary or contradictory information, or the judge in taking evidence endeavours to discover from clues exactly what has happened, then different combinations offer themselves; it may have happened this way, but then it may also have happened that way. This "may" or "can" is the expression of subjective indecision; and its significance lies in its refusal to allow the opposite alternative to be decisively established. When it is said that the accused may be innocent in spite of weighty evidence, this means only that the evidence is not sufficient to prove the guilt; that the judgment "he is guilty" is not necessary, nor, therefore, possible. But there is no question as to "can" in the objective sense, since objectively it is already absolutely determined whether affirmation or negation is true.

But the statement that such and such a thing is possible becomes the more empty and insignificant according as the extent of our ignorance is greater, and we have fewer positive grounds to give as a reason why our conjecture should come to pass. When we say that spontaneous generation is possible, this is true in so far as we cannot prove that it is impossible. But all the grounds contained in the order of nature as known to us speak against it, and it is a possibility which lies only in the dark regions into which our knowledge has not yet penetrated.

7. Only in this subjective sphere does it hold good that everything is possible which contains no contradiction, nor leads to any. Every hypothesis accepted as possible breaks down at once if it falls into contradiction with a proposition recognised as valid; hence it can be assumed only so long as we do not find it to be contradictory to a valid truth, *i.e.* so long as

its opposite remains unproved. But this absence of contradiction has nothing whatever to do with the question as to what may happen in reality.

8. Nevertheless the attempt has been made to find, in the absence of contradiction, a criterion of possibility in the other sense also, particularly where the question is not as to the possibility of being this or being that, but as to the possibility of being in general. This is what Leibnitz understood by the possible. It is that which is thinkable (*conceivable*) because it contains no contradiction; and it was in this sense that he demanded that the proof of the possibility of God should precede that of His existence. But this absence of contradiction is meaningless unless it be first established what is contradictory when thought of as the determination of one and the same thing, and what is not (§ 22, p. 127 sq.). Moreover, Leibnitz has to secure a relation to reality for this abstract determination by postulating that everything possible demands existence, and will therefore exist if nothing prevents it.¹ It is against this forced transition from the merely conceivable to something of which we can state that it is possible for it to be that Kant's criticism (*Postulate des empir. Denkens überhaupt*) is directed, which limits the conception of the possible by the formal conditions of experience. But Kant himself allows too much scope to the concept, inasmuch as he would still use it as a predicate of things, in the same sense as Leibnitz. Against his view we must here again insist that possibility can only be stated of that which finds utterance in the judgment, hence that all possibility, as well as all necessity, is hypothetical, and presupposes an existent. The statement that it is possible for something to be has no meaning, if it lays claim to material validity, unless it points out some power which can produce the thing and shows that the existing order of the universe raises no conclusive objection against it. This alone distinguishes the possible thing from the possible idea or possible concept. A bare possibility is self-contradictory.

9. The possible stands in a somewhat special relation towards the negation.

It would seem to be a matter of course that when we state the possibility that *A* is *B* we include the possibility that *A* is not *B*; for the opposition between the merely possible and the necessary lies just in the fact that it may *not* be as well as may be. But on closer inspection it becomes necessary to impose important limitations on the proposition that every *A*

¹ *De Verit. Primit.*, Erdm., p. 99. Cf. *Princ. Phil.*, § 45, Erdm., p. 708.

potest esse B is accompanied by an equally valid *A potest non esse B*; if, that is, we would deal with significant propositions, and not with empty formulæ.

For instance, our thought, in its function of comprehension, predicates of the different phases of that which is changeable, capable of development or of being determined from without, that they are possible. Then the statement that the negation is also possible either has no meaning, or when opposed to the original proposition alters the meaning of this. The judgment "cooking-salt may be dissolved in water" is intended to state an attribute of cooking-salt. What then is the meaning of the proposition "it is possible that cooking-salt is not dissolved in water"? The proposition that a pair of mice may have millions of descendants in a few years is meant as an estimate of their power of increase, and hence as the statement of an organic law; but what about the proposition that the pair may also not have these descendants? When the positive statement expressly isolates its subject from varying conditions, there is no sense in returning to these and suddenly resuming the standpoint of the multiplicity of actual events.

When, however, we are speaking of the *particular case* in temporally valid judgments, then the possibility of not-being accompanies that of being, and has the same significance. The answer may come to-day, but it may also not come till to-morrow, or not at all; it may freeze to-night, but then again the frost may keep off. The ground of the negation is the presupposition that, besides the known relations which would bring about the result, there may be others also to counteract or prevent it; in the first instance there may be delay on the part of the writer or the messenger, in the second the introduction of a warmer current of air. This relation between causes which bring about a result and causes which counteract and prevent it, is presupposed when the possibilities of being and of not-being are opposed as equally justifiable propositions. But its ground is only our ignorance as to whether the causes actually present and taking effect are favourable or counteracting.

The case is the same when we state the relation between a generic concept and its mutually exclusive specific determinations. A triangle may be equilateral or not equilateral. If all I know is that it is a triangle, I have no ground for either affirming or denying that it is equilateral; the general idea leaves both possibilities open.

10. The negation of possibility leads to necessity, the negation of necessity to possibility.

- a.* It is possible that *A* is *B* contradicts
It is not possible that *A* is *B*, and this is equivalent to
It is necessary that *A* is not *B*.
- b.* It is necessary that *A* is *B* contradicts
It is not necessary that *A* is *B*, and this is equivalent to
It is possible that *A* is not *B*.

Thus there arises the twofold contradictory opposition which runs parallel to the contradictory opposition of the universal affirmative and particular negative, and the universal negative and particular affirmative judgments.

But in both cases we can only avoid running into absurdities if we are careful to interpret the formulæ always in the same sense.

These formulæ hold good when possible and necessary are used in the subjective sense of a hypothesis; they hold good also when necessary and possible are uniformly used of the essential necessity of some determinations and of the material possibility of others which are opposed to each other; they hold good, finally, when possibility and necessity are stated of some particular case in temporally valid judgments.

11. On reviewing all the questions to which the concepts of the possible and the necessary have led us, we see that the function of judgment has undergone a further development in two directions. On the one hand, the different stages in the formation of the judgment which were traversed by the immediate judgment with one step have been definitely separated by the mediated judgment. The mere attempt at a judgment—the question—has appeared upon the scene, and has led us to reflect concerning the relation of the judging subject to this question; and by means of the antithesis between the question and the decision, NECESSITY, the innermost and essential meaning of all judgment, has been brought to light. On the other hand, judgment has advanced a step further in that particular simple subjects, or a number of these, have been replaced by the statement of the judgment itself, the material unity of subject and predicate, which has become the subject of new predicates, primarily of the necessary and possible. In this way new categories have become manifest which rank above those first discovered, inasmuch as they are founded upon the earlier ones and place them in relation to each other, thereby enabling us to know not only particular phenomena, but also their connection. Thus they afford a positive counterpart to the mere negation, which, like them, has reference to the synthesis of judgment.

If we thus look upon the progress of thought as pressing onward from the mere attempt, the hypothesis, and the possible to the necessary, we can give their natural significance to the special forms which are generally co-ordinated with the judgment which states or denies a definite predicate of a subject, that is, to the hypothetical and disjunctive judgments. The former is the simple expression of necessity, the latter the complete expression of mutually exclusive possibilities. The former shows a necessary connection between possibilities, and in this way limits the sphere of possibility by necessity; the latter is the preliminary step towards establishing the necessity of one possibility by the denial of certain others.

CHAPTER VII.

HYPOTHETICAL AND DISJUNCTIVE JUDGMENTS.

FROM the point of view of what is called relation, modern logic generally classifies judgments as CATEGORICAL (A is B ; A is not B), HYPOTHETICAL (if A is, B is), and DISJUNCTIVE (A is either B or C). But this is not in accordance with the Aristotelian logic, nor are there any grounds for regarding it as an exhaustive division of the forms of judgment.¹ It often happens that, so far as concerns the matter of the statement, categorical and hypothetical, or hypothetical and disjunctive propositions express the same thought with a difference which is merely grammatical. If, on the other hand, it is the verbal expression to which we look, then the hypothetical and disjunctive forms cannot be co-ordinate with the categorical form of judgment, since this is contained in both of them. If, again, the ground of distinction is sought in this circumstance, and composite judgments, which are expressed in complex propositions, are opposed to those which are simple, then we shall find that there are many other complex propositions besides hypothetical and disjunctive judgments, and there will be no apparent reason for excluding them from logical treatment.

As a matter of fact, logicians did for a long time follow the example of the Stoics in distinguishing the composite judgment from the simple judgment which is expressed in a single proposition; and this custom, which had fallen out of use more particularly since Kant's time, has been lately revived (*e.g.* by Ueberweg).

In an investigation of which the chief aim is to analyse actual judgment, and which must therefore begin by investigating the expression of that judgment in language, we must begin by reconsidering this former custom. This is the more necessary because a number of misconceptions concerning the hypothetical judgment have arisen owing to insufficient reflection on the logical significance of the forms of language.

¹ With what follows cf. my *Programm : Beiträge zur Lehre vom hypothetischen Urtheil* (Laupp, Tübingen), 1870.

I. THE DIFFERENT WAYS IN WHICH PROPOSITIONS MAY BE COMBINED AND THEIR LOGICAL SIGNIFICANCE.

§ 35.

There are certain forms of speech in which different propositions are connected by particles, conjunctions, and relative pronouns. This connection may be such as to express a relation between complete propositions, which state definite judgments and are comprehensible by themselves; or it may show that the one proposition is essential to the completion of the other.

In the first case the relation may be merely verbal, as in relative propositions; or it may express a subjective relation of the propositions peculiar to the individual speaker; or it may be equivalent to another judgment, having for its predicate either the logical relation between the syntheses expressed by the propositions, or the relation between the facts stated in the propositions (states, events, etc.).

In the second case we may either make a statement by means of modal relational predicates concerning the grammatically dependent judgment, or we may make a statement concerning the fact expressed in that judgment.

1. The simplest complex propositions, and the most easy to analyse, are those in which two propositions, each comprehensible by itself, and expressing an independently valid judgment, are placed in such a relation to each other that more is expressed by the two together than by the simple utterance of the one proposition after the other. The verbal forms expressing this relation are the particles, and it is of their significance that we must speak.

a. We have already seen (p. 157) that the particle "and," and all equivalent expressions, can do no more than tell us that the two judgments are, at the moment of speaking, both present in the consciousness of the speaker; and since this subjective fact is already established by the utterance of both propositions by the same person, such mere links have in themselves no objective significance, though they may assume the function of indicating a corresponding connection in the objects of thought (*e.g.* temporal succession when some event is being narrated). These particles therefore are not equivalent to a judgment.

b. Nor can the adversative particles be taken as the signs of a definite objective statement. It is true that in conversation they are often directed

against some spoken proposition, which they confront with an objection, limitation, or contradiction. Still they have not the force of a negation, for it happens just as often that they only repudiate what might have been conjectured or inferred from the proposition by means of some combination. When used by one person speaking alone they serve on the one hand to contend in this way against some looked-for statement, and on the other merely to introduce some contrasting or unexpected element, such as an affirmative proposition after a negative, or an unexpected predicate. Thus, while the negation denies a definitely uttered statement, the adversative particles often anticipate tacit combinations merely assumed as possible; the negation, therefore, which these express, is indefinite, and cannot take the form of a proper judgment.

c. It is different when we come to the so-called causal particles and particles of inference. Where these express the logical relation of the judgments connected, they state that the one judgment is the logical ground (or consequence) of the other; but when they are meant to express the relation between the facts stated in the judgment, they tell us that the fact stated in the one judgment is the real ground (or consequence) of the fact stated in the other. Thus the relation they express is that of a logical, or material, necessary connection; and to that extent they have all the force of a proper definite judgment. "It is getting cold, for the thermometer is falling"; "it is getting cold, therefore the thermometer is falling"; here we have in each case three complete judgments: "it is getting cold; the thermometer is falling; the former proposition is inferred from the latter"; "it is getting cold; the thermometer is falling; the former change is the cause of the latter."

d. With the causal particles expressing real necessity may be classed all determinations which express real relations between the states, events, etc., spoken of in the propositions, such, for instance, as the temporal relations in a narrative—contemporaneousness, succession, etc.—and relations of place. These also stand in place of definite relational judgments, by which they might be expressed.

e. Under the name of explicable judgments, the older logic has included judgments which, while apparently making only one statement, really contain several. Chief amongst these are the judgments containing restrictive words—"only," "none but," etc. The proposition "only the wise man is happy" tells us first that the wise man is happy, then that the man who is not wise is not happy, or that all the happy are wise.

2. Grammar distinguishes between the co-ordinate and subordinate connection of propositions; but in this general form the distinction corresponds to no essential distinction in thought. The grammatical form, it is true, seems to signify that the speaker is mainly concerned with the statement of the principal proposition, for the sake of which alone the dependent propositions are introduced; that is, the speaker has no object in stating the dependent propositions except for the sake of reminding us that they are accepted truths. But language as actually used has not retained the distinction between co-ordinate and subordinate propositions in all its strictness; it uses conjunctions and co-ordinating particles in the same sense, and the utmost distinction made is a slight difference in the subjective emphasis laid upon the propositions connected by them—a difference which has no objective significance with respect to the matter of the statement. The particles “for” and “because” are used to denote one and the same relation; and the relation expressed by “at the same time” may also be announced by “whilst.”

The significance of relative connection is especially susceptible of manifold gradation. When relative pronouns follow a term which is already fully determined as it stands, they serve merely to enable us to make a further statement concerning some portion of a given statement. The relative pronoun, whilst obviating the express repetition of the definite denotation, gives more prominence to the identity of the two elements than would be attained by mere juxtaposition; but the relations between the two propositions thus connected may vary greatly. The most decided subordination takes place when the proposition introduced by the relative enables us to recognise one element of the main proposition more easily by reminding us of something we already know; and in this case the statement introduced has no independent value whatever, but is more in the nature of an attributive adjective, or an apposition. There is complete equality of position, on the other hand, when the relative proposition introduces a new and independent statement (this happens most frequently in Latin). But here the relative does not take the place of a proper judgment. It tells us nothing which is not already stated in the two propositions connected by it, and its function is merely verbal—that of establishing the identity between the verbal denotations. The proposition “*A*, which is *B*, is *C*,” tells us no more than “*A* is *B*, and *A* is *C*”; all that it does is to leave no room for doubt that the *A* of the one proposition is the same *A* as that of the other.

Relative propositions have quite another function when an element of the proposition which is in itself indefinite receives through them its first determination. They then appear as a part of the denotation of subject or predicate, and limit a more general denotation to a definite sphere; that is, they are determining. In the proposition "men who live in cold climates need more nourishment than others," the subject is incomplete without the relative proposition, which plays the same part as a determining adjective in other cases—*e.g.* "elastic bodies are resilient." In this way simple denotation by a definite word may be paraphrased by a relative proposition; the phrase "parallelograms, which are right-angled and equilateral," is a synonym for "squares."

Here we must also add the indefinite relatives ("any one who," "anything that," *ὅστις ἄν, quisquis*), which tell us nothing but that subjects of which the one predicate is true have the other predicate also. The expression is then equivalent to a universal judgment, and, like it, may have either an empirical or an unconditional universality; while, again, any universal judgment may be expressed in this form. The propositions "man is mortal"; "all men are mortal"; "anything which is a man is mortal," all mean absolutely the same thing—the necessary connection between humanity and mortality. The only difference is that in the form "anything which is a man is mortal" the act of naming, which has already taken place in "all men," now takes place before our eyes; and it is left, moreover, an open question as to what particular thing, if any, can be so named. The formula "all men are mortal," on the contrary, although it does not actually state that subjects are forthcoming, still presupposes them if we understand the words in their ordinary sense.

Much the same may be said of "when" (*wenn*) and "where" (*wo*) as relatives of time and place. The use of "when" to denote one definite point of time in the past, while still retained in English, is lost to the German language. Here it is used principally of the future, and hence there often attaches to it a kind of indefiniteness and uncertainty as to whether the future will really come to pass; and this may be, although perhaps in a very slight degree, even where there is no intention of expressing anything more than that one event will happen at the same time as another (when it strikes twelve o'clock, the new year will begin; when the war is ended, we shall return). Where, again, it stands as a general relative (whenever = as often as), its only direct meaning is the universality of the co-existence of two states or events; this universality being either

merely empirical, as expressing a perception to which there has been no exception, or absolutely universal (when it grows dusk, the bats begin to fly). But we cannot state that two events will happen together in the future, nor that they will always and unconditionally take place together, unless there is some necessary connection between them. To this extent, therefore, the meaning of what was originally no more than a temporal particle comes to include this necessary connection, and thus to serve as the conditional conjunction in the hypothetical judgment, of which we shall have to speak later. "Where" (*wo*) in its general significance passes through the same process.

3. From connections such as these we must distinguish others in which a proposition as such forms part of another proposition, either as subject, or as part of a relative predicate (object). The proposition may then represent either the judgment as a subjective thought or utterance, or the fact expressed in the judgment; and this again may be sometimes merely thought or assumed, sometimes objectively valid as matter of fact.

a. Those statements in which modal relational predicates refer to judgments have propositions for their component parts. The statements that a judgment is true, false, credible, doubtful, possible, or necessary; the statements that I believe, repudiate, dispute, or doubt something—all these have reference to a hypothesis which is expressed in a proposition, and they state how the hypothesis is related to my thought or to thought in general. All final propositions belong to this class. When I do anything in order that something else may happen, the purpose makes its first appearance as my thought; and the statement refers to the relation between a result as expressed in the form of judgment and my thought and will, with the action to which it leads.

Since every judgment as such must stand in definite modal relations, these may always be predicated. To say that the judgment " A is B " is true or necessary means no more than the simple statement " A is B ." To say that I maintain, I know, I am certain that A is B , only gives special emphasis to the certainty already contained in the fact of the statement " A is B ." But in such a rendering the proposition " A is B " becomes the expression of a merely conceived judgment or hypothesis; the completion of the judgment is transferred to the modal predicate.

b. Statements of which the component parts are states or events expressed in the form of propositions differ only verbally from those which contain adjectival or verbal abstract substantives amongst their elements.

The thought remains the same whether I say, "the greater warmth of the summer is due to the higher position of the sun," or, "the fact that the summer is warmer is due to the fact that the sun stands higher"; all that is necessary to such a statement is that fresh predications can be made concerning what is originally expressed in a judgment.

4. Although this brief survey lays no claim to completeness, it may at least suffice to show that the manifold grammatical forms in which propositions are connected together are no ground for special kinds of function in judgment, which are not forthcoming in simple judgments. The meaning of such forms can always be expressed by the predicates of simple statements, and logical theory has therefore been fully justified in leaving such connections as those of time and place to grammar, which treats of the expression of thought in language. The expression "compound judgment" is misleading and inappropriate; that which is composed of judgments may be a combination of judgments, but we cannot therefore say that this combination is itself a judgment. The fact is that when the component parts of a judgment are propositions these propositions are not themselves judgments; that is, they are not at the moment intended as statements: they enter into new judgments, either as hypotheses, or as the results of previous judgment, and therefore as an indication of what is meant in the judgment.

5. Amongst all the different combinations of propositions conventional logic has distinguished only what are called hypothetical and disjunctive judgments, and in so doing has rightly judged that all other forms deal only with definite statements, however varied; that is, with the attribution of definite predicates to definite subjects. But in these two forms we have to deal with such statements concerning hypotheses as are of direct importance for the final aim of all thought—progress from the subjective to the objective, from the possible to the necessary—and therefore of universal importance for all kinds of statement. Reflection concerning the value and significance of hypotheses is always necessary where we cannot straightway form a definite judgment, but must endeavour to arrive at truth by way of a preliminary experiment. Thus hypothetical and disjunctive judgments rank with the negation, which, like them, is a judgment concerning a tentative judgment; they both apply to that stage of thought which lies between the question and decision.

II. THE HYPOTHETICAL JUDGMENT.

§ 36.

THE HYPOTHETICAL JUDGMENT states that two hypotheses are related to each other as GROUND and CONSEQUENCE ; its predicate is "NECESSARY CONSEQUENCE." "If *A* is true, then *B* is true," means, therefore, "*B* is the necessary consequence of *A*."

1. The ordinary expression of the hypothetical judgment, and that in which its meaning is most obvious, is a combination of propositions of the form "if *A* is *B*, then *C* is *D*." More briefly, and taking *A* and *B* as signs of propositions, "if *A* is true, then *B* is true."

2. In grammar such propositions are generally called conditional, and in giving them this name we take what seems to be the most natural view, *i.e.* that the point in question is the validity of the consequent. This validity cannot be directly stated, but presupposes the validity of the antecedent, the whole being thus a conditional statement of the consequent, hence a statement concerning the subject of the consequent.¹ But the consequent is not to be stated until the antecedent has been confirmed, and a conditional proposition is therefore an expression of uncertainty with reference to both antecedent and consequent. Both, as the expression goes, are problematically stated, or, as we should say, express mere hypotheses. So far as concerns the two propositions themselves, therefore, there really seems to be no judgment at all in the proper sense, no utterance, that is, which is stated as true and necessary ; and this view is confirmed by the fact that conditional propositions are sometimes stated with the avowed consciousness of the falsity of both antecedent and consequent (*si tacuisses, philosophus mansisses*).

3. Nevertheless such a combination of propositions does contain a statement which is a judgment in the proper sense of the word, and this the Stoics² were the first to recognise definitely. Such a combination tells us that antecedent and consequent are related to each other as ground and consequence (p. 195), that to accept the antecedent makes it necessary to accept the consequent, and that the validity of the consequent is infallibly connected with that of the antecedent. It is this relation of necessary consequence which is the true predicate of the hypothetical judgment,³ and

¹ This is how Wolff defines the hypothetical judgment in his *Logik*. See my *Programm* mentioned above, p. 23 sq.

² Cf. my *Programm*, p. 12.

³ J. S. Mill, *Logic*, bk. I, ch. 4, § 3.

the two points of reference for the relation are antecedent and consequent. So far as concerns the statement of this necessary connection, the question as to the validity of the antecedent and our conjectures as to its truth, probability, improbability, or falsity, are of no importance, just as in a simple judgment concerning some object of thought it is quite immaterial whether we regard the object as existing, as possible, or as merely fictitious. In this way we may explain how it is that judgments beginning with "if" (*wenn*) seem sometimes to express uncertainty alone, and at other times to be the expression of sequence amongst actual phenomena.¹

This necessity, which in hypothetical judgments refers to propositions which are merely assumed, is also stated with reference to valid judgments by the so-called causal connection of propositions. *B* is true because *A* is true, a judgment which may refer either to the ground of our knowledge or to the material ground. (It is warmer because the thermometer is rising ; the thermometer is rising because it is warmer.)

4. The nature of the statement remains, of course, the same whether the judgments which appear as antecedent and consequent be affirmative or negative, universal or particular, narrative or explicative. The attempts to find differences of quantity, etc., in the hypothetical judgment are based upon a confusion between hypothetical judgments and statements concerning mere time-relations or some purely empirical and casual coincidence.

No one would maintain that judgments such as "whenever it strikes twelve some one dies" are hypothetical. The confusion is especially apparent in the judgments which have been called particular hypotheticals : "when it is fine the barometer generally stands high." If the connection is not invariable, it cannot be necessary, and such judgments can never express more than empirical or casual coincidence in a relatively greater or smaller number of instances. The proposition "sometimes when a triangle is right-angled it has two equal angles" tells us only that the attribute right-angled is occasionally to be found together with, and does not exclude, equality of the two remaining angles. The phrase "sometimes when" does not connect ground and consequence ; it only states the coincidence of attributes or events in the same or different things, the statement being merely empirical, and saying nothing as to the ground of the coincidence. (Cf. my *Programm*, p. 45, and *Vierteljahrsschr. f. wiss. Phil.* v. 1, p. 109 sq.)

5. The co-ordination of the hypothetical with the categorical as a special kind of judgment, distinguished by the difference of its logical

¹ See Appendix B.

function, may be said to date from Kant. In the categorical judgment, he says, ideas are subordinated to each other as predicate to subject, in the hypothetical as consequence to ground (*Krit. der r. V.*, § 9. 3; Hart., p. 106). The necessary sequence thought of in hypothetical judgments, corresponds to the copula in categorical judgments: it is that which unifies the different ideas. Thus the category which corresponds to the logical function of the hypothetical judgment is that of causality.

But the classification is throughout superficial and inadequate, if only because the ideas which are related as subject and predicate are, in the Kantian phraseology, concepts, while the ideas which are related as ground and consequence are judgments. Kant's distinction gave rise to the further doctrine—which he, however, did not himself teach—that categorical judgments are the expression of the relation of inherence, hypothetical judgments of the relation of causality, a doctrine which is quite untenable if the expressions hypothetical and categorical are taken in their ordinary meaning. The judgment "God is the cause of the world" is certainly categorical in the ordinary sense, nevertheless it expresses causal relation; the judgment "if the soul is material, it is extended," is hypothetical, yet it treats entirely of relations of inherence. When, again, we disregard the verbal form and distinguish between judgments of quality and relational judgments (as, *e.g.*, Drobisch does), we may find some justification for the division, so far as concerns the meaning of definite statements. But in this classification the true hypothetical judgment is not included at all, and only by straining a point can it be subsumed under those relational judgments which are statements of real relations between things.

6. The exact meaning of the hypothetical judgment differs according to the nature of the propositions related by it as ground and consequence. When two propositions, which standing alone would express unconditional validity, are connected by "if—then," the statement is merely that if we accept the one, we must also accept the other. "If the soul is corporeal, then it is extended"; "if the soul is simple, then it is indestructible"; "if God is almighty and good to all, then the world is perfect"—such propositions make the truth of the consequent depend as necessary consequence upon the truth of the antecedent, and state that whoever accepts the one must also accept the other. The ground of this necessity is not apparent in the hypothetical judgment. It may consist in the simple relations of the ideas (corporeal and extended) by virtue of which predica-

tion of the one involves predication of the other; or it may consist in assumptions concerning the nature of things, such as "the simple is indestructible," or in assumptions concerning the necessary mode of action of certain causes; the hypothetical judgment does not tell us which. From this point of view we may draw the distinction between analytical and synthetical judgments. If the second proposition is contained in the first in such a way that the generally recognised meanings of the words enable us to infer the one from the other, then the judgment is analytical. But if the connection must be mediated by a ground of necessity derived from some other source, then it is synthetical. This, however, is a distinction which cannot be more exactly defined until we come to the doctrine of inferences. We find instances of a similar nature in applications of a general rule to a particular case: "if death be the penalty of murder, then this murderer must be punished by death." Such propositions express the logical necessity by which the particular instance is contained in the general rule.

7. When antecedent and consequent refer to the particular and are statements having a temporal validity only, there are two cases to be distinguished. Here, again, we may find that the second synthesis is contained in the first, and follows from it by virtue of the meaning of the predicates, these being invariably connected: "if this man is drunk, he is irresponsible." Or the necessary sequence may be due to the special circumstances of the case before us, and inferable from it according to certain laws; necessity of connection is found here also, but its conditions are not contained in the antecedent. When we say, "if the sky clears, it will freeze to-night," we presuppose the given temperature, etc.; the necessary sequence depends upon the laws of the radiation of heat, but these will not produce the result except at the given temperature.

8. There is a special application of the hypothetical judgment in which the propositions connected by it have no definite subjects. The subjects themselves are left indefinite; either absolutely indefinite, when they are denoted by some such word as "something," or partially indefinite, when they are denoted only by a general term. "If anything is corporeal, it is extended"; "if any one is just, he gives to every one his due"; "if a triangle is equilateral, it is equiangular," etc. Here not only is the validity of a given statement left *in suspenso*, its necessary consequence alone being given; we are also left uncertain whether there is any subject belonging to the predicates, or where it may be found. But as it is stated

that any subject in which the one predicate is found must contain the other as well, it is essential that in these judgments the antecedent and consequent should have the same subject—in meaning, if not in words. In the judgment “if a triangle has equal angles, its sides are equal,” there are, no doubt, two grammatical subjects ; but the second refers by means of the possessive pronoun to the first, and it is the first of which the statement is really made.

They are then precisely equivalent to universal relational propositions : “he who is just gives to every one his due” ; “every triangle which is equilateral is also equiangular.” Propositions such as these state the identity of the subjects to which the two predicates belong, but they can do so only because the second predicate is necessarily connected with the first ; this necessity shows itself in the fact that the two predicates are always and without exception found in the same subject.

The course of thought presupposed by these modes of expression is clear. We find them where particular things, whose existence is for the time assumed, are being characterized. With the definite predicate of the antecedent in our minds, we look abroad into the manifold ; and in the expectation that the predicate will be somewhere applicable, we state that if it is, then the other predicate must be necessarily connected with it.

9. But these judgments tell us absolutely nothing which is not contained in the unconditionally universal categorical judgment. “All bodies are extended” does not refer to any limited and definite number ; it merely states that “whatever is a body is extended,” or that “if anything is a body it is extended.” The word by which the subject of the statement is denoted contains by implication the antecedent of a hypothetical judgment. Such a proposition, therefore, as “the good man thinks of himself last” is as truly hypothetical as if the subject were introduced as “a good man” with a view to leaving it an open question whether and where such a subject is to be found—though not when it is merely the incomplete denotation of a definite subject. (The distinction may be seen from the examples, “an emperor must die standing” ; “an emperor was a Stoic.”)

This, then, is the answer to the much-disputed question as to the relation between hypothetical and categorical judgments. All categorical judgments which are unconditionally universal are exactly equivalent in meaning to hypothetical judgments, for (see § 27, p. 162 sq.) they state nothing whatever but that subject and predicate necessarily belong to

each other, so that to predicate the subject of a particular thing necessarily involves the predication of the predicate. Inasmuch, indeed, as the "all" is ambiguous, and may introduce either a judgment which is empirically universal, or one which is unconditionally universal, the hypothetical form is the more accurate and adequate expression. On the other hand, judgments in which definite predicates are attributed to definite particular subjects naturally resist the attempt to convert them into the hypothetical form. The hypothetical judgment, moreover, has a wider meaning and application than can be naturally expressed in the categorical form.

10. It is different when the antecedent predicates changeable attributes, activities, and relations of a subject which is only indefinitely denoted. "When the temperature of water falls below zero, it becomes solid"; "when a body projected with a given velocity is subject to the action of a force which varies as the inverse square of the distance, it describes a conic section"; "illumination is strongest when the rays of light fall perpendicularly on a surface"—such statements may refer either to repeated instances in which the subject remains the same, or to instances where the subjects are different; and for this reason they would be inadequately expressed by a universal categorical judgment. If we are to express the necessity as unconditional universality, we must make use of the general relational phrases "every time that," "as often as"; and from this it follows that here the hypothetical judgment is also subject to time-relations. Changes must take place at definite points of time, and to limit the validity of the antecedent to a definite period of time is to appoint a definite time for the validity of the consequent—either the same time, or the moment immediately before or after. It is these judgments which are, from the nature of the case, based upon causal relations whenever their subjects are to be found amongst real things; for when change in one thing involves a second change in the same (or another) thing, it must be due to causal connection.

11. Amongst hypothetical propositions having indefinite subjects we must also include those equations of analytical geometry and mechanics which contain variables. The equation of the parabola $y^2 = px$ is not an equation in the ordinary sense—*i.e.* a judgment that two lines, numbers, or figures are equal—because of the indefinite value of the variable. It states only that if the *abscissa* has any definite value, then the ordinate belonging to it has a value which is determined by its arithmetical rela-

tion to the constant. In the same way all algebraical formulæ containing general symbols may be translated into hypothetical judgments, *e.g.* $a(b+c)=ab+bc$.

12. Passing to hypothetical judgments containing negations, we find that the form "if A is, B is not," represents the negation of a proposition as the necessary consequence of an affirmation, thus affirming that the hypotheses A and B are incompatible. This incompatibility may be due either to the incompatibility of certain predicates, or to the real relations of hindering or counteracting causes. The relation is always mutual. If the negation of B necessarily follows from the affirmation of A , then (according to the law of ground and consequence) the negation of A follows necessarily from the affirmation (*i.e.* the denial of the negation) of B ; and this is true whether A and B represent universally and unconditionally valid judgments or temporally valid judgments concerning the particular, or whether their subjects are indefinite (when the sky is clouded no dew falls; when dew falls the sky is not clouded). To such a hypothetical judgment there corresponds the categorical judgment, which is universal and negative. The proposition "no right-angled triangle is equilateral" makes the same statement as the proposition "if a triangle is right-angled, it is not equilateral"; the negation of the predicate equilateral is stated as the necessary consequence of the determination right-angled.¹

When a negation appears as the necessary consequence of another negation (if A is not, B is not) the ground of the relation must be the necessary connection between the corresponding affirmatives. This is the sole condition upon which the negation of the one can have the negation of the other for its consequence. The invalidity of A is an infallible ground from which to infer the invalidity of B only when A is the necessary consequence of B . I cannot say that when the sky is not clear no dew falls unless I am certain that when dew falls the sky must be clear; only if it

¹ The difficulty raised by Twisten (*Logic*, § 64) against the view that the hypothetical judgment with a negative consequent is affirmative is easily removed. If the categorical judgment "no equilateral triangle is right-angled" is negative, how, he asks, can the corresponding hypothetical "if a triangle is equilateral, it is not right-angled," be affirmative? It certainly could not if the hypothetical judgment were a statement about the equilateral triangle, and not about the necessity of a consequence; but why should we be unable to affirm that a negative proposition is a necessary consequence? The very possibility of the unconditional negative "no A is B " depends upon our knowing that the determinations contained in the thought of A necessitate the negation of B ; and this, which is the meaning of a universal negation, the hypothetical judgment expresses by affirmation of this necessity.

is true that every equiangular triangle is equilateral can it be said that if a triangle is not equilateral, it is not equiangular. From the proposition "if A is, B is," we may always deduce the other proposition "if B is not, A is not," according to the principle that the denial of the consequent involves the denial of the ground—the principle, that is, which expresses the meaning of the necessary consequence stated by the hypothetical judgment.

When an affirmation appears as the consequence of a negation—if A is not, B is—the judgment is always immediately or mediately grounded upon the position that of different possible hypotheses which are mutually exclusive one must necessarily be valid; the position, that is, which finds utterance in the disjunctive judgment. But it is not true that the judgment "if A is not, B is" is, as it stands, equivalent to the disjunctive "either A is or B is."¹

13. The negation of a hypothetical judgment can take place only through the denial of its predicate—that is, of the necessary consequence stated by it. The contradictory opposite of the judgment "if A is, B is," is the proposition " B is not the necessary consequence of the proposition A "; i.e. B is not true because A is (even if A is, nevertheless B is not)."² In the same way the negation of the statement "it does not follow that because A is, B is" leads to the judgment "if A is, B is."

14. Hypothetical judgments of the form "if A is, and B is, and C is, then D is" should not be called copulative hypothetical judgments; they do not state of different relations that they are all necessary consequences, as in the judgment "both when A is and when B is, C is." The latter judgment is copulative, but the former gives only one ground consisting of several conditions, and therefore it cannot be resolved into several hypothetical judgments.

15. The possibility of the consequence can be connected with each part of the ground, but only when we understand this possibility as having reference to the partial ground (§ 34, p. 208). "When the moon is in

¹ Vide my *Programm*, p. 54 sq. The proposition "when the centre of the moon is not in the plane of the ecliptic it forms a triangle with the centres of the sun and the earth" does not mean the same as the proposition "either the centre of the moon is in the plane of the ecliptic or it forms a triangle with the two other central points." For it may also form a triangle when it is in the plane of the ecliptic, but the node does not fall upon the straight line passing through the centres of the sun and the earth.

² This shows also the logical position of the so-called concessive propositions, their meaning consists in their rejection of some immediate or mediate consequence which might be inferred from the antecedent.

conjunction or opposition, and at the same time the lunar node is in a line with the sun and earth, then there must be eclipses," the proposition may be resolved into the two judgments "when the moon is in conjunction or opposition there may be eclipses," and "when the lunar node is in a line with the sun and the earth there may be eclipses." We find the same "may" when the antecedent expresses the invalidity of a judgment which would annul the consequent; "if the radiation of heat from the sun does not diminish, organic life may have an unlimited duration upon the earth."

III. THE DISJUNCTIVE JUDGMENT.

§ 37.

The DISJUNCTIVE JUDGMENT states that of a definite number of mutually exclusive hypotheses one is necessarily true. When it does not refer to the two members of an antiphrasis as in the principle of excluded middle, it always presupposes a simple judgment upon which the different hypotheses are based, and which, by its contents, determines and limits the range of possibilities. Most frequently it happens that either the subject or the predicate admits of a closed series of mutually exclusive determinations, and these are enumerated by the divisive judgment.

1. The simplest way of expressing that a hypothesis *A* is *B* is uncertain is to say that neither its affirmation nor its negation can be asserted; I am confronted by an undecided alternative. But I know that if the affirmation is true the negation is false, and *vice versa*; and that if the affirmation is false the negation is true and *vice versa*.

But it is not only between affirmation and negation that such a choice amongst different hypotheses may be presented. Various hypotheses may be possible in reference to the same subject—*A* is perhaps *B*, perhaps *C*, perhaps *D*. Where these predicates *B*, *C*, *D*, are quite independent of each other there is no further relation between the hypotheses (thus I might say of Queen Semiramis that she was perhaps tall, perhaps black-eyed, etc.); but if the one predicate necessarily involves the other, then there arises the hypothetical judgment. If again they are incompatible, to accept the one predicate is to exclude the others, and I am confronted by irreconcilable propositions, each of which by itself is a possible hypothesis.

When irreconcilable propositions such as these are both equally un-

certain, they are connected by the particle "or." Such hypotheses are not necessarily predicates of one and the same subject, but may be any assumptions whatever which for some reason are mutually exclusive, and the relation between which can therefore be expressed in a hypothetical judgment connecting the affirmation of one with the negation of the other. Thus the particle "or" contains two statements—that the propositions are uncertain, and that they are mutually exclusive. "*A* is *B* or *C*" means that *A* is perhaps *B*, perhaps *C*; if it is *B*, it is not *C*; if it is *C*, it is not *B*.

2. A similar juxtaposition of predicates occurs in judgments which state a possibility. The judgments "water can be liquid, solid and vaporous—man can wake and sleep" may also be expressed, when any one point of time is referred to, in the form "water is liquid, solid or vaporous—man sleeps or wakes." We find the "or" again when a general idea admits of further determinations: "the triangle is plain or spherical, etc.—a plane rectilinear figure has three, or four, or five angles, etc." Whenever anything is denoted only by a general term there is room for further determination by means of mutually exclusive predicates. When all I know and say of a thing is that it is a triangle, there are many different and incompatible determinations, any of which may belong to it.

3. When we say of a number of such hypotheses that one is necessarily true, the statement implies that all the mutually exclusive predicates which are subjectively or objectively possible are included in the enumeration. Then we have the disjunctive proposition: either *A* is *B* or *C* is *D*; *A* is either *B* or *C* or *D*. Thus in stating a disjunctive judgment we have in view the necessary validity of one out of a given number of possible and incompatible hypotheses.

4. The simplest case of such a disjunction is to be found in contradictory opposition of judgments, so far as it is subject to the law of excluded middle. Of the two propositions *A* is *B*, and *A* is not *B*, one is necessarily true, the other false. But just because this disjunction is so self-evident it is of little value (see § 25, p. 154). The important disjunctions are those which limit our choice amongst positive judgments with definite predicates.

5. First amongst these are the disjunctions which state the limited number of mutually exclusive attributes by which a general idea may be further determined. A line is either straight or crooked; a triangle is either right-angled or oblique-angled; a human being is either male or female; water is either fluid, solid or vaporous. The kind of possibility possessed

by the single members of the disjunction is that discussed in § 34, 5, p. 206-7. The meaning of the disjunction is that a thing known to me only as falling under the general idea *A* must have some one or another of the various attributes possible in *A*. This meaning is most obvious in the hypothetical expression, which gives the full force of disjunctions such as these; if anything is a line, it is either a straight line or a crooked. Here then there is presupposed both a judgment ascribing a general predicate to the subject, and the knowledge of a closed series of mutually exclusive determinations which are possible to that predicate.

6. When we have in our minds all the particular subjects which may be included in *A*, thus thinking of the different determinations as actual, this relation may be expressed also in what may be called the *DIVISIVE JUDGMENT*. "Lines are some of them straight, the others crooked"; "human beings are some of them male, the others female." When we review a complete series of changes in the same thing the corresponding form is "water is sometimes liquid, sometimes solid, sometimes vaporous." Here we must notice that the relation between divisive and disjunctive judgments varies according to circumstances. When our knowledge is derived from experience alone, the disjunctive judgment is grounded upon the divisive; since it is a matter of fact that all human beings may be divided into males and females, we infer that a third kind is impossible; and upon this inference we ground the disjunctive judgment "every human being is either a man or a woman." In mathematics, on the other hand, the disjunctive judgment comes first; only after the judgment that a triangle is either right-angled, or acute-angled, or obtuse-angled, are we certain that our enumeration of the kinds of triangle is complete. It is the same with the judgment "a plane which intersects a right cone is either parallel to the base, or not; if not parallel, it either intersects all the generating lines, or not all; and in the latter case it is either parallel to one generating line or not." This judgment comes first, and only when we know that it exhausts all the possibilities do we obtain the division of conic sections into circles, ellipses, parabolas, and hyperbolas. The divisive judgment may also be expressed in the form of a copulative judgment "the circle, ellipse, parabola and hyperbola are the conic sections," the definite article indicating the identity of extension of subject and predicate.

7. The need of expressing the completeness of the enumeration more definitely than by the use of "some" has led to the divisive judgment

being expressed in the form of a disjunction : "all lines are either straight or crooked" ; "human beings are either male or female." But this mode of expression is ambiguous. The judgments said to be disjunct are not "all lines are straight—all lines are crooked" ; as, for instance, in the judgment "the human race is descended from one pair or from several," where the disjunction is between the propositions "the human race is descended from one pair," and "the human race is descended from several pairs." In the former instance the disjunction applies to each particular line ; so that here again, if we would avoid ambiguity, we must make use of the hypothetical form "anything which is a line, is either straight or crooked."

8. These disjunctions have for their members the more exact determinations of the subject, and may therefore be reduced to divisive judgments which divide the subject-concept into its species. From them we must distinguish judgments which unfold the different possibilities in the predicate of a given subject.¹ When we say that the planets either shine by their own light or derive their light from the sun, we do not mean that the fact of being a planet includes both possibilities, and that some shine by their own light, while others are illuminated by the sun. What we really do is to start from the judgment "the planets shine," and to investigate the exact nature of their light and the possibility of finding an explanation amongst the given circumstances. Take the proposition "the world has either existed from eternity, or has had a beginning ; and its beginning was due either to a free cause or to blind necessity." In the first disjunction we start from the existence of the world as datum, and enquire as to the duration of this existence ; in the second, our datum is the beginning of the world from a cause, and our enquiry concerns the different kinds of causes. When we say "he is either a hypocrite or a fool," it is presupposed that he behaves unreasonably, and the question is, as to the source of this behaviour. A further distinction may be made according as the closer determinations of the predicate form a part of its meaning, or are gained from a consideration of the concrete possibilities of the particular instance.

¹ Trendelenburg's doctrine that the disjunctive judgment is a statement of the extension of the subject concept applies only to those disjunctive judgments which are based upon a division of the subject concept. It is not applicable when the disjunction refers to changeable states, and it is not applicable in this sense when the concept of which the possible determinations are developed is predicate. Cf. my *Programm*, pp. 60, 61.

9. Judgments such as "either the wicked will be punished, or there is no divine justice," are grounded upon hypothetical judgments, and are governed by the principle that the denial of the consequent involves the denial of the ground, taken together with the principle of excluded middle. If there is a divine justice the wicked will be punished—either the wicked will be punished or not—if not, the supposed ground of their punishment cannot exist.

10. It is of course true that the disjunctive judgment "*A* is either *B* or *C*," may be resolved into two hypotheticals "if *A* is not *B*, it is *C*," and "if *A* is *B*, it is not *C*"; but it does not follow from this that the disjunctive judgment has no importance of its own independently of the hypothetical judgment. We cannot state a negation as the ground of an affirmation, unless the disjunction has already been established. The judgment that if light is not matter it is motion, cannot precede the knowledge that light is either matter or motion.

11. It follows from the nature of the statement contained in the disjunctive judgment that the propositions "*A* is either *B* or *C*—*A* may be either *B* or *C*—*A* must be either *B* or *C*," are all exactly equivalent.

§ 38.

RESULTS.

THE FUNCTION OF JUDGMENT is always the same in so far as it always states categorically that a predicate belongs to a subject. The differences which we find depend partly upon whether the synthesis between subject and predicate is simple, as in the denominative judgment; or complex as in judgments governed by the categories of attribute, activity and relation. Partly, again, upon whether the subject of a judgment is a simple idea, or is itself a synthesis of the nature of judgment, or a combination of such syntheses of which we may predicate that it is false, possible, necessary, etc.

The distinctions generally made are based upon differences of predicates and subjects, and not upon differences in the function of judgment; any real variations in function are to be found in one class alone, that of categorical judgments.

For this reason the importance of the predicates presupposed in all judgment becomes more evident; to recognise these as remaining the

same throughout changing subjects is the essential characteristic of judgment.

1. So far our investigation has shown that the customary classification of judgments, which has been established mainly by the authority of Kant, is inadequate.

The basis and presupposition of all judgment is the judgment which is immediate, simple and positive; and this consists in the synthesis of a subject and predicate accompanied by the consciousness of objective validity. The meaning of this synthesis and of its objective validity depends upon the nature of the ideas connected in the judgment; it is simple when the act is merely one of denomination, complex when it is grounded upon the categories of attribute, activity and relation. That which takes place in judgment is always the recognition of the agreement between an idea previously known and one element of the subject; and, according to the original meaning of "to know," every judgment is the cognition and recognition of something in the subject which is already known. But it does not follow that every judgment consists only in this knowledge, and is nothing more than the statement of a subsumption. In one class of judgments it is just as indispensable that there should also be present the consciousness of unity between attribute or activity and thing, or the consciousness of relation between two things; and subsumption cannot take place in judgments concerning attributes, activities, and relations without the differentiation and synthesis of different elements in a presentation.

Even the judgments which have numerical determinations for their predicates show no essentially different function. It is not peculiar to the numerical judgment that it must have been preceded by other judgments; every judgment which refers to the particular, and names it by the subject-term presupposes a previous judgment. All that is peculiar to numerical predicates is the nature of the preliminary operations which they necessitate, just as other relational predicates necessitate preliminary operations appropriate to their peculiar nature—*e.g.* equality and inequality presuppose measurement; the number given is compared with a number previously known.

2. But the course of thought extends beyond immediate knowledge, and leads us to think of judgments which might take place as distinct from actual judgments. Then new judgments arise having reference to these ideas of judgments or to the relations between them.

Every statement contains the judgment that the synthesis expressed by it is necessary or true, and in the negation this judgment is contradicted by the statement that the synthesis is false. Besides this affirmative or negative decision, we find on the one hand the judgment that a hypothesis is possible—*i.e.* that it is not subjectively necessary either to affirm or to deny it; on the other, the judgment that it is the necessary consequence of another hypothesis, or that amongst a number of given hypotheses one must necessarily be true.

These judgments all resemble simple judgments in that they state simple modal predicates concerning a subject; their peculiarity lies, therefore, not in the synthesis, but in the nature of their subjects and predicates. Since, however, these subjects form an essential element of the function of judgment, and since the predicates refer to just that characteristic of the function which expresses its relation to the final aim of all serious thought, such judgments are in an eminent degree logical. Indeed, there could be no conscious judgment at all unless some account were taken of this relation between the originally subjective combination of subject and predicate and these modal determinations. We are therefore justified in giving special attention to negative, hypothetical and disjunctive judgments; not because they are special kinds of the judgment, but because they have reference to hypotheses and to their logical value and significance.

Thus there is really only one sort of judgment, the categorical statement that a predicate belongs to a subject. If any distinction at all is made between the FORM and the MATTER of the judgment, then by form we must mean that mode of activity in thought through which a judgment as such is formed; and this mode of activity is in all cases essentially the same. The diversity which is generally represented as attaching to the forms of judgment is really a diversity of matter, and depends upon the nature of subjects or predicates. As this nature varies, we certainly find a modification, sometimes in the movement of thought which precedes the judgment, sometimes in the meaning of the predication—that is, in the meaning of that unity between subject and predicate which is the thought present in all judgments. The conventional doctrine of the different “forms” of judgment (due in part to the importance generally attached to the verbal expression) loses sight of the one idea which is the sole reason why we give the name of judgment to all the different kinds.

All judgment, therefore, presupposes first of all the presence of a number of predicate-ideas which may be recognised in the subjects. It

requires, moreover, that we should have ideas of different kinds of synthesis between subject and predicate ; these differences being determined by the nature of the subjects and predicates, and constituting the meaning of the simple statement that the predicate belongs to the subject.

If, however, we still continue to denote the resulting kinds of synthesis as “forms” of judgment, it is after all a question as to the use of language. But in so doing we must guard against thinking that the name “judgment” denotes a number of originally different and co-ordinate acts of thought,—a mistake which is constantly made when affirmative and negative judgments are distinguished as opposite forms. Unless we do so we are in danger of losing sight of the one concept which should correspond to the term, and the term itself becomes a mere homonym.

PART II.



REGULATIVE.

INTRODUCTION.

§ 39.

IN order that the function of judgment may actually succeed in attaining to certain and universally valid judgments, it is before all necessary that the certainty of the particular judgment in question should be invariable, and accompanied by the consciousness of its universal validity. Two conditions are essential to this. In the first place the person judging must be conscious of the logical ground of his judgment; and secondly, the elements of the judgment itself must be completely determined and constant, and thought by every one in the same way.

For the latter condition to be fulfilled it is necessary that the elements of our judgments, and more especially their predicates, should be logically perfect concepts; for the former, that the judgments themselves should be grounded in accordance with universally valid and necessary laws of thought.

1. In Part I. we have taken thought as we actually find it, and have analyzed the function of judgment in which it always manifests itself when aiming at truth and universal validity. We have sought to show the meaning and significance of the judgment in all its relations, and we have found that one essential element of every statement is its claim to be true, *i.e.* necessary and therefore valid for all thinking beings.

We must now test this claim, and investigate the conditions under which judgment attains its end. The conditions, that is, under which the momentary certainty which accompanies every actual judgment involves no delusion, but is the expression of objective necessity; and under which the universal validity of the individual act of judgment is guaranteed.

2. In order that a judgment may be perfect, it is necessary in the first place that it should remain unaltered for the person who judges; that he should always repeat it in just the same way whenever the same subjects and predicates recur; and that its certainty should therefore also be invariable. If the same synthesis were for him at one time certain, at

another uncertain; if the same subjects and predicates were not always connected in the same way within the limits of one and the same consciousness; if I thought it possible that, starting from the same data I might judge otherwise in the future than I now do—then the act of judgment must have failed, for its success involves confidence in the irrefragable validity of the judgment.

But the certainty that a judgment is permanent, that the synthesis is irrevocable, that I shall always say the same,—this certainty can only be forthcoming when it is known to depend, not upon momentary psychological motives, which vary as time goes on, but upon something which is immutably the same every time I think and is unaffected by any change. This something is, on the one hand, my self-consciousness itself, the certainty that I am I, the same person who now thinks and who thought before, who thinks both one thing and another. On the other hand, it is that about which I judge, my thought itself as far as regards its unvariable content, which I recognise as identical each time, and which is quite independent of the state of mind of the individual thinker.

The certainty that I am and think is final and fundamental, the condition of all thought and all certainty whatever. Here there can be none but immediate and self-evident certainty; we cannot even say that it is necessary, for it is prior to all necessity. In the same way the certainty of my consciousness that I think this or that is immediate and self-evident; it is inextricably interwoven with my self-consciousness; the one involves the other.

If now we can say that there is a necessity which obliges me to judge of an idea in one particular way so soon as I am aware of it; if I can know that as surely as I am I, I must connect subject and predicate in this way, for no other reason than that I have thought of them, then my discernment of this necessity is the ground of certainty for every judgment. This constitutes my consciousness of its logical ground, and a connection is thus established between the judgment and my self-consciousness itself; I know that so surely as I myself continue the same I must always repeat the judgment in the same way.

This then is the first condition: *for a judgment to be perfect the person judging must be conscious of its logical ground.*

3. What conditions must be fulfilled in order that we may attain to this consciousness?

If an *A* of which I am conscious is to be the ground which makes a

judgment B logically necessary, then this necessity must rest upon a constant law, according to which B always and invariably follows from A ; and only in so far is the necessity knowable; the presence of A however is pure matter of fact, without which the necessity cannot take effect. Thus the consciousness of the ground resolves itself into consciousness of the law according to which B follows from its premises, and consciousness of these premises.

It may be that these premises are not themselves judgments, but objects of consciousness of another kind, of which nothing more is known than that they are at the moment present; such as sensations, reproduced ideas of all kinds, or concepts which are present to consciousness. Thus so far as concerns logical necessity we have arrived at an ultimate fact which must be regarded as pure matter of fact; all we can ask is: what is its necessary consequence? The judgment that the diameters of a circle are equal is based upon our concept of a circle; this concept however, or the intuition which gave rise to it, is in the last instance matter of fact. No universal logical necessity can be shown why this geometrical image should appear in my consciousness at all, either from intuition of external objects, or as the result of inventive construction. Every judgment of perception includes amongst its premises the immediate consciousness of a sensation, and this again is purely matter of fact. It is true that the question may be raised as to whether its conditions are normal, and whether, therefore, it justifies a judgment concerning the existent—*i.e.* it may be asked what necessary and universally valid inference can be drawn from the simple act of a subjective sensation. But that the sensation is there has nothing to do with logical necessity, we know it by our immediate consciousness of a simple fact.

When, on the other hand, the premises are themselves judgments, then the consciousness of necessity resolves itself into the consciousness of the laws according to which from certain judgments other judgments follow (*i.e.* the rules of inference), and the consciousness of the validity of the premises. Here again the same conditions apply; we must be conscious of the grounds of these judgments. No judgments are exempt from these conditions unless their certainty is so self-evident that it must be regarded as immediate matter of fact, no less than the "I think," or the presence of definite ideas; the certainty of these judgments cannot be further analysed by a consciousness of their necessity. To them we must add the judgments whose contents constitute the fundamental laws of all necessity,

according to which anything is necessary, and the validity of which therefore cannot be seen to follow necessarily from some other judgment, but can only be recognised—judgments, that is, which are as necessary as the proposition “I am” itself, or which can be shown to have a certainty which is necessarily involved in the certainty of this proposition.

Thus no logic which is to lay down rules for thought is possible unless such ultimate laws can be known by us, and shown to be absolutely certain and self-evident. Our first task is not to trace the inexhaustible phenomena of matter of fact and of individual experience which constitute the given premises of our judgments in particular cases, but to lay down the laws according to which certain ideas and judgments make other judgments logically necessary, and constitute a ground for their certainty. This involves what we have already claimed as a postulate in the Introduction, § 3: that we are able to distinguish objectively necessary thought by the evident certainty with which it manifests itself, and by analysing the conditions of this certainty to discover such universal laws. Our success in carrying out this task must show whether or not there is any ground for the postulate.

4. In order that a judgment may be always valid and immutably certain, other conditions must be fulfilled which apply to an earlier stage in thought, and are not fulfilled in its natural course. The ideas denoted by subject and predicate terms must be constant and completely determined. The consciousness of the identity of a judgment attaches in the first instance to its verbal expression, to the fact that the same statement is made about the same thing in words, and the predicate must always have this verbal expression, the subject at least in explicative and universal judgments. Unless every word always keeps exactly the same meaning, the meaning being thus perfectly definite and fixed, we cannot possibly be certain that the repetition of the same proposition is also the repetition of the same judgment, and the meaning of the judgment itself becomes uncertain. Indeed, the very act of judgment tends to bring about this confusion by a sort of shifting amongst our predicate ideas, for on ordinary occasions we are often guided by what is no more than a vague similarity between the new and the familiar (see § 7, 8, p. 44). When the Marcomanni mistook the lions which were let loose upon them by Marcus Aurelius for dogs, and slew them without ceremony, their judgment “these are dogs” meant only that the lions were more like dogs than any other animals known to them. Even while they were judging, their idea of the

dog and the meaning of the word was changed, and a new image comprised in it.

Again, although the necessity of judgments is a sufficient guarantee for their universal validity, this only means that every one who starts from the same premises must synthesize in the same way. But if the primary data, the ideal elements between which the synthesis takes place, were different for every individual and absolutely incommensurable, so that every one connected a different meaning with a given word, then, however small the difference, our judgments would never actually attain to universal validity, they would at the most approximate thereto. That community of thought which we endeavour to reach by means of language, and which is the condition of higher mental development, more especially of all science, would never be fully realized.

We saw, in § 7, that in the natural course of thought the ideas of the individual are not constant nor completely determined, and that different individuals do not have the same ideas nor denote them by the same words. On the contrary, the very laws which govern the natural formation of ideas necessitate both their variability in the individual and their difference in different individuals, thus prohibiting a completely fixed verbal signification.

Hence we cannot speak of the complete logical certainty of a judgment nor of its invariable validity unless we assume that whenever a judgment seems to be the same as another, because clothed in the same language, it is really the same; that the same statement is made about the same subject. And before we can say that any given judgment is universally valid—*in concreto*, therefore, comprehensible and convincing for all—we must assume that the ideas contained in it are common to all and the same for all. The anarchy of natural thought is completely excluded by the ideal of perfect thought, and a logic which is to contain the rules of perfect thought must begin by determining what are the conditions to be fulfilled by ideas themselves as elements of the judgment.

5. This brings us to the two main problems of this part of our work.

(a) The conditions upon which perfect judgments are possible are, that the ideas which enter into the judgment as subject or predicate should be absolutely constant, completely determined, the same for every one, and denoted by unambiguous terms. An idea which fulfils these conditions we call a concept in the logical sense of the word. One part of our task,

therefore, will be to investigate the conditions necessary in order that our ideas may be logical concepts.

(*b*) The condition of the logical necessity and universal validity of judgments is that they should have a ground. A second investigation must therefore discover the rules according to which a judgment follows necessarily from its premises.

One part of this investigation will be concerned with the laws according to which immediate judgments find their ground in the ideas of which they are composed ; the other part with the laws according to which mediated judgments are grounded upon other judgments.

CHAPTER I.

THE CONCEPT.

§ 40.

THE LOGICAL CONCEPT differs from the general idea which has developed in the natural course of thought, and is denoted by a word, in its constancy, in the completeness and immutability of its determination, and in the certainty that every one will employ the same term to denote it. It differs from the concept in its metaphysical meaning as the adequate thought of the essence of an object in that its only aim is the complete determination of our predicate ideas, and this aim has no direct connection with the question as to whether the concept corresponds to any real object, or whether it adequately represents such an object. The quality of generality it has in common with every idea. It is the fact that it is strictly defined, and clearly distinguished from all other ideas, which forms the distinguishing characteristic of the concept, and in the construction of logical concepts our aim is to establish one mode of arranging their manifold ideal contents for all thinking beings, and so to complete systematically the process which begins without conscious purpose in language.

1. In speaking of "concepts," we must distinguish three meanings of the word. It may denote a natural psychological production, and here it is the simple correlative of the "term" as used in ordinary natural language. It is the idea at the stage where it has become a mental possession, and has gained the generality explained in § 7, which belongs to every idea as such. It is now qualified to become an element of the judgment, more especially a predicate. Such ideas, as we have already seen, are different for different people, and are still in the process of formation; even for the individual himself they change, so that a word does not always keep the same meaning even for the same person. Strictly speaking, it is only by a fiction which neglects individual

peculiarities that we can speak of the concepts denoted by the terms used in ordinary language.

2. In contrast with this empirical meaning the concept may be viewed as an ideal: it is then the mark at which we aim in our endeavour to attain knowledge, for we seek to find in it an *adequate copy of the essence of things*. The concept in this sense must enable us to penetrate into the very heart of the thing, to understand it; *i.e.*, to see that its particular determinations as connected in it are the necessary consequence of its nature as a unity. The concept of life, in this sense, would be the keystone to physiology; that of matter, to chemistry and physics; that of mind, to psychology; and all knowledge in this direction would be complete when we could exhibit a system of concepts comprising the true counterpart of every existing thing. When we try to think of an absolute and divine knowledge, we define it by saying that in the absolute intelligence, concept and being are one. In this sense we may indeed speak of the truth of our concepts; they are true when they are in themselves the exhaustive expression of the nature of things. The true concept of God would comprise in its determinations the thought of the real nature of God in all its aspects.

3. Between these two meanings of the word, which may be called the empirical and the metaphysical, there lies the *logical*, which alone concerns us here. This has its origin in the logical demand for certainty and universal validity in our judgments. All that is required is, that our ideas should be absolutely fixed and determined, and that all who make use of the same system of denotation should have the same ideas. In what relation our thought stands to the existent, and whether there is absolute agreement or not, is not determined here—or at least not directly. We must indeed suppose that as knowledge is always growing, less is contained in our ideas at any given time than in the existent; at the best our ideas are representations which correspond to the existent so far as they go, they are never exhaustive. If our judgments could never be universally valid unless their elements were perfect concepts in the metaphysical sense, and if it were as difficult to get rid of individual differences and indefiniteness in our ideas as it is to overcome their inadequacy to the existent, then we should be debarred even from approximating to the goal of knowledge by the gradual progress of science, for science always presupposes uniformity in the construction of concepts. It is necessary, therefore, to distinguish between the formal sufficiency

of concepts for the purpose of judgment, and their metaphysical adequacy; and we must assume that it is at least possible to attain the former before the latter has been reached.

4. Finally, we must distinguish between the logical perfection of the concept and the appropriateness of its construction to some end, the latter being connected with the problems of classification within some given sphere of things (objects, actions, crimes, etc.). A concept may be completely determined, and hence logically perfect, and yet not so well adapted as some other to meet the requirements of science. For the aim of science is to attain, by means of concepts and the terms which denote them, the greatest possible simplicity and brevity in our knowledge; and it therefore asks, "How must our concepts be formed to enable us to give the simplest expression to the most important and comprehensive universal judgments?" By this aim we must be guided in methodology, which has its origin in the problems arising out of the nature of the conditions of our knowledge.

On the other hand, if we are really to attain logical perfection in our judgments, it is necessary that we should always have a sufficient supply of logically perfect concepts to enable us to express and determine everything which becomes subject to our judgment. In judging we do not merely repeat what is already known; we are constantly appropriating something new. In this sense the possibility of perfect judgments is determined by the extent to which the raw material of all human ideas has taken permanent form in concepts. We must always be prepared with concepts in which to express our knowledge; or we must at least be sure of being able to create them from elements which are already conceptually determined. We may compare this ideal with that of a universal alphabet which comprises generally accepted signs for all the distinguishable simple sounds possible to the human organs of speech. From this point of view Leibnitz's idea of the *characteristica universalis* was a most appropriate expression for that at which we should aim in the logical construction of concepts.¹

¹ Cf. Trendelenburg: "Ueber Leibnizens Entwurf einer allgemeinen Charakteristik." *Histor. Beiträge zur Phil.*, iii. p. 1 sq. And Descartes, *Ep.*, I, III, where a similar thought is developed: "Ejusmodi linguæ inventio a vera Philosophia pendet. Absque illa enim impossibile est omnes hominum cogitationes enumerare, aut ordine digerere; imo neque illas distinguere, ita ut perspicuæ sint et simplices . . . Et si quis clare explicuisset, quales sint ideæ illæ simplices, quæ in hominum imaginatione versantur, et ex quibus componitur quidquid illi cogitant, essetque hoc per universum orbem receptum, auferem demum sperare linguam aliquam universalem," etc.

5. The essential characteristic of the concept is generally said to be its *generality*.¹ Connected with this is the doctrine that concepts are gained by *abstraction*, *i.e.*, by a process which separates the common characteristics of particular objects from those by which they are distinguished from each other, and gathers together the former into a unity. But the supporters of this view forget that, in order to resolve an object of thought into its particular characteristics, judgments are necessary which have for their predicates general ideas (or, as they are commonly called, concepts), and as these concepts make the process of abstraction possible, they must have been originally obtained in some other way. They forget, moreover, that the process presupposes that the series of objects to be compared is already in some way determined, and a tacit assumption is made of some motive by reason of which we group together just these objects, and seek their common elements. If we are guided by any motive, and not by mere caprice,² it must be found ultimately in the fact that the objects are from the first recognised as similar, because containing certain elements common to all ; that is, a general idea is already there, by means of which these objects are selected from amongst all others. There is only one connection in which the doctrine of the construction of concepts by comparison and abstraction is at all appropriate, and that is when we elucidate the actual meaning of a word by enumerating the common characteristics of the things which, as a matter of fact, are always denoted by the word. In explaining the concept of animal, gas, theft, etc., we might

¹ See Kant in the *Transcendental Aesthetic*, § 2, 4. Every concept must be thought of as an idea which is contained in an infinite number of different possible ideas (as the element common to all), and therefore as comprising these ideas under it.

² Drobisch is quite consistent in admitting that this selection is arbitrary (*Logik*, edn. 3, § 18, p. 20). It is in itself a completely arbitrary matter what objects we choose to compare with each other ; we may compare a raspberry cane with a bramble, but we may also compare it with a penknife or a turtle. When, however, the Linnean system, which classifies together widely different plants, is cited as an example of this "far-fetched comparisons," it is overlooked that the concepts by which the Linnean classes are determined are not due to this simple and direct method of comparison. In this method it is the common element of objects which are chosen at random which is selected, while the Linnean classes, on the contrary, are the result of an endeavour to find simple distinguishing characteristics whereby to divide into definite groups the incalculable varieties of plants. First it was seen that plants differ in the number of stamens, etc. ; then followed the methodical grouping together of those which agree on these points. Of course, this distinction was itself based upon a comparison in the wider sense, but it was first employed to discover differences, and not the common element, of the objects compared. (The last two sentences refer to the contrary view expressed by Drobisch, edn. 5, p. 21.)

attempt to select the common characteristics of all the things generally called "animals," or of all the bodies called "gas," or of all the actions called "theft."¹ Whether or not we should succeed, whether it is possible to carry out such a recipe for the construction of a concept—that is another question. There might be some chance of success if we could assume that there is never any doubt as to what we ought to call animal, gas, theft, etc.; if, that is, we already possessed the concept we are seeking. But to form a concept by abstraction in this way is to look for the spectacles we are wearing by the aid of the spectacles themselves.

6. Nevertheless, there is some truth in this doctrine. So far as concerns the generality of the concept, we may find it first in the fact that as a rule logical concepts merely complete ideas which have arisen naturally; they do not supersede them; we cannot change the nature of ideation, and the concept as a work of art must always be preceded by the idea as a natural development. Now every idea naturally possesses generality, inasmuch as it is detached from the original particular intuition or particular function, and has been made our own as a reproducible object; nor can any arbitrary action on our part deprive the idea of this natural generality. But this generality in no way depends upon whether an idea has been formed from one intuition, or from many similar or dissimilar intuitions (§ 7); all that we learn from it is, that the idea—as Kant guardedly expresses it—is contained in an infinite number of *possible* ideas.

¹ The Socratic method of determining concepts is in the main the same as this. It always starts by assuming that definite concepts correspond to the current meanings of words, and it proceeds to find the right explanation of the word by comparing particular examples of that which it denotes, and contrasting with these examples of that which it does not denote. The only difference is that Socrates does not aim at surveying every particular thing, but contents himself with a few examples. To the present day the doctrine of the concept is really based upon this Socratic method of always presupposing that there must be concepts corresponding to words; and to this method is due the general failure to distinguish between the concept of psychology and the concept of logic. The method finds its justification and importance in the fact that in language as learnt by tradition we find it already established what concrete things and events are customarily named by certain words, and as we come gradually to understand a word, the idea of a number of particular objects is connected with it before we are conscious of its general meaning as such. A typical instance of this is to be found in the answer of Theætetus to the question: What is *ἐπιστήμη*?—it is mathematics, etc. Children and those without scientific training will always answer by giving an instance instead of a definition. The primary service rendered by the Socratic method was that it pointed out the meaning of the word as such, that meaning upon which particular acts of naming are based.

Whether or not it is contained in many actual ideas, is immaterial to the nature of the idea or concept, and it makes no difference whether it has been derived from one or from many.

But we may find a further justification for the importance attached to the generality of the concept in the fact that it necessitates a complete separation of the meaning of the word from particular intuitions, thus securing for the judgment a clear and definite meaning, and substituting for a vague comparison a real statement of unity between subject and predicate. When we see a palm for the first time, and call it a tree, we are led to do so by its general similarity to the firs and beeches, which we know, and the images of which are connected with the word "tree," although we may not have noticed exactly wherein the similarity consists. "The palm is a tree" is not strictly justified as a proper judgment unless the word "tree" means nothing but what the palm has in common with firs, beeches, etc., then only can we take the judgment in its strict and proper sense, "I find in the palm everything which is contained in my thought of the tree," instead of the vague and inaccurate meaning, "The palm is like a tree." This process certainly involves a conscious selection of all the characteristics common to the objects which I call trees. But the chief object of the selection is not to find a general idea comprising the particular; it is to obtain a firm grasp of the general idea which is already indefinitely and confusedly contained in the thought of the particular, and, by sharply defining it, to give a definite meaning to the judgment, thus completing a process which always begins unconsciously. The involuntary action of psychological laws is sufficient to give rise to generic images formed from manifold similar intuitions, the differences of the particular images being lost sight of, and leaving shifting schemata to correspond to our words. Thus we do, no doubt, drop the distinguishing elements, and retain those which are common, but the process is incomplete because it is not grounded upon any conscious comparison and distinction of the particular characteristics. To supplement this process is the aim of conscious comparisons (§ 7, 11, p. 47). It is true again that in the involuntary formation of our ideas we find what alone should be called abstraction, the abstraction which separates. It is by means of this that we break up the undivided whole of intuition into thing, attribute, and activity, and that by detaching them from this unity we form those abstract ideas which enable us to compare different things, finding them alike in some respects, different in others, and which alone can supply predicates for the

judgments in which conscious comparison and distinction take place. It is true, also, that a comparison of partially resembling objects taking place under these conditions may become the more or less fortuitous occasion for the formation of new concepts. If visible objects always presented the same combination of colour and form, we should have much more difficulty in constructing separate ideas of colour and form—that is, in abstracting them from the given whole ; but a conscious comparison of red things as to their colour is only possible after such an abstraction has taken place, or, at least, simultaneously with it. A comparison of the horse, the dog, and the lizard, may perchance lead us to form the concept of four-footed animal, if we happen to be struck by their similarity in having four feet (though more probably we should proceed by differentiation, finding that quadrupeds differed from men and birds, beetles and flies, on the one hand, and from snakes and snails on the other) ; and many generalizations arise in this way. But the processes, when thus carried out, are not the result of intentional skill, nor is their product such as to satisfy logical requirements. The characteristics found upon comparison to agree are always subject, when selected in this casual way, to that natural indefiniteness and absence of limitation which arises from the elasticity of our ideas and their tendency to annex what is similar to themselves and place it under the same denotation ; and the whole process lacks a solid basis when the characteristics which form the predicates of judgments of comparison are not themselves perfectly definite and determined by every one in the same way. It is one of the chief failures of the ordinary doctrine of the concept that it proceeds as if these characteristics were given ready made, and such as to need no further elaboration. Really, the enormous difficulty of passing beyond the natural state in which every one speaks his own language consists much less in the processes of comparison itself than in establishing accurate and generally accepted standards of comparison ; *i.e.*, the difficulty is to determine conceptually the attributes which are to be used as characteristics.

7. The logically perfect idea is distinguished from the idea which has developed naturally and is the basis of ordinary speech by the fact that in it the natural tendency of ideas to expand in the process of formation has been counteracted by an activity which negates and limits, and thus gives form and consistency. If we disregard for the moment the demand that ideas should be the same for all, we may say that the essential char-

acteristic of the concept is the constancy and complete distinction of the ideal content which is denoted by any given term.

This constancy presupposes that a definite ideal content and its appropriate verbal sign have been consciously fixed by us in order that we may always be able to reproduce it as the same, and be conscious every time of its strict identity. Complete distinction requires a comprehensive survey, in the first place, of those objects which most resemble each other and are most liable to confusion, and further of the whole range of the possible objects of our ideas. It depends also upon conscious acts by which we become aware of the differences between the ideas *A, B, C, D*, etc., retaining as clear an idea of the intervals between them as of the determinations themselves. This act, which first makes us expressly conscious of the identity of the same content by its negation of a different one, aids and completes the determination of our ideas,¹ while at the same time we are enabled to arrange them according to the degrees of difference.

8. Suppose that all the ideas which we have occasion in the course of thought to consider and manipulate as units, and which are destined to become elements of judgments, were capable of being produced by an indivisible act, either of intuition or of relating thought; suppose, further, that the objects of which we form ideas were limited in number and easily surveyed, with such marked differences between them that we should be as conscious of the transition in passing from one to the other as in passing

¹ The view that an idea owes its first determination to distinction overlooks the fact that distinction itself is only possible when different ideas are present, and that distinction does not therefore create the contents which are different. Ulrici, for instance (*Compendium der Logik*, edn. 2, p. 60), says, "Red is the particular colour we call red only because, being red, it cannot also be blue, yellow, etc.—without its difference from blue, etc., it would be quite indefinite, mere colour—in general, an absolutely indeterminate quality of which we should know nothing because, as we have shown, we are aware of colour as colour only by means of distinctions in colour." This I cannot allow. The sensation of red—more accurately, of a particular shade of red—is a positive *datum* with a content peculiar to itself, and it would continue to be so even though it should be accompanied by the sensations of fewer colours than are perceived by all normal eyes. Colour sensations are not less definitely determined for any one because it may happen that many colours have never been seen by him at all. The variety of his colour sensations would be the smaller, and he would have a poorer supply of ideas, but that would be all; no doubt to any one feeling red only, red would mean the same as colour in general, but this is only equivalent to saying that the idea colour would comprise no plurality of distinguishable qualities, not that it would be absolutely indefinite. The conditions which enable us to retain a plurality of sensations in consciousness are not those upon which the definiteness of particular sensations depends; on the contrary, this definiteness is rather presupposed by such ability. Cf. Lotze, *Logik*, edn. 2, p. 26.

from one to two, from two to three, then the logical task of constructing concepts would come to an end with the functions we have mentioned and with the establishment of a generally accepted system of names; no more would be needed but a memory able to retain what had been gained from the general survey. If, for instance, our world of ideas were limited to the twelve simple notes of the octave, then everything necessary to give the definiteness of concepts to our ideas would be that we should remember each particular note and distinguish it from the others with sufficient certainty to guard against confusion. The ideas of the particular notes and the consciousness of their differences would yield us the whole material for our concepts arranged in a fixed order.

But both assumptions are incorrect. The first, because the ideas which we treat as units and denote by one term can generally be resolved into a plurality of distinguishable elements; they prove to be products compounded from simpler ideas which the mind can grasp by themselves. This makes them more difficult to retain in thought, for in a compound idea both the particular elements and the manner of their composition must be retained; while, on the other hand, the difficulties of distinguishing are greater and more special in nature, inasmuch as a compound idea may resemble another in some of its elements and differ from it in others. When, for instance, I try to keep before my mind the idea of a horse, I can do it only by a process of mental construction, piecing together the parts of the figure in a given manner; when, again, I wish to distinguish it from some other idea, such as that of the donkey, I find that while it agrees with it in most respects it is distinctly different only in a few.

The second assumption is also incorrect. Throughout the materials collected in memory we come upon numbers of imperceptible differences which obliterate the marked intervals for which we look when trying to give definite form to our ideas. This continuity, moreover, applies to the simpler elements of our ideas as well as to compound images. In the sphere of colours, red passes by imperceptible degrees through violet into blue, through orange into yellow, through pink into white, through dark red into brown. A similar continuity is found in the sphere of spatial magnitudes and forms, and hence there arises such an unlimited number of scarcely distinguishable objects that it is impossible to differentiate them and to retain them in their differences. It is the same with intuitable things themselves. Intermediate terms intervene everywhere as know-

ledge extends between objects which were at first sharply distinguished; between snow and hail, tree and shrub, horse and ass, negro and European.

§ 41.

Since the greater number of our ideas are composite—*i.e.*, have their origin in distinguishable acts—we cannot determine their content except by a conscious determination of their elements (characteristics, or component ideas) and of their manner of synthesis. Thus the conceptual determination of the content of an idea presupposes first of all its analysis into simpler and unanalysable elements, the same analysis determining also the form of their synthesis.

This analysis could not be completely carried out unless based upon a perfect knowledge of the laws which govern the formation of ideas, and such a knowledge alone could assure us that the elements were the same for all thinking beings. But our analysis can never arrive at elements which are completely isolated as products of independent functions; it always ends in a system of connected functions which are related to each other, and which comprise the various forms in which the manifold is synthesized. The functions by which we think the logical categories (unity, identity, difference) are connected with the spatial and temporal forms of intuition; and when referring to that which is thought of as existing both are connected with the real categories (thing, attribute, activity, relation); and all are again connected with the intuitively given contents of immediate inner or sense perception. The perfection of our ideas as concepts presupposes a complete system of these elements.

As we have in intuition an unlimited multiplicity of ideas which are separated by imperceptible differences, our conceptual determination of them must be limited to establishing definite boundaries within the gradual fusion of differences.

1. The problem to which the first of the facts mentioned in § 40, 8—the composite nature of ideal objects—gives rise was familiar to the traditional logic. Here we are taught that the thought which is contained in a single idea and denoted by one term must be defined by its characteristics, that a concept must be resolved into its component ideas or concepts. These are thought in the concept and form its content. Thus the characteristics heavy, yellow, bright, metallic, etc., are thought in the

concept gold ; the attributes bounded, four-sided, equilateral, rectangular, plane surface in the concept square ; and in the concept murder the illegal, intentional, deliberate slaughter of a human being. This content is represented as the sum or product of the particular characteristics. It is generally held, moreover, that this analysis into characteristics solves also the further problem of distinction, these characteristics constituting just that whereby different ideas are distinguished.¹

The question as to how it is possible to distinguish different characteristics in the totality of an idea is generally treated as if it were already settled, and were evident from the examples cited. Attention has been repeatedly called—especially by Trendelenburg—to the need of a more exact determination of the relation between the characteristics. Are they all homogeneous? and if not, how do they differ? are they indifferent to each other, or are they mutually dependent? and finally, in what relation do the component parts stand to the whole? For in borrowing terms from spatial or temporal relations, and speaking of concepts or ideas as component parts, we are only using figurative language. We do not mean that the component ideas are ideas of the parts of a whole (as head, neck, trunk, are ideas of parts of an animal), standing in the same relation to the idea of the whole as the parts do to the whole ; we mean that they are elements of the idea in the same way that the particular attributes are of a thing.

2. We cannot analyse a given idea into its parts or characteristics unless it has been in the first instance developed by means of distinguishable functions out of different elements. If the idea was originally simple and produced without a succession of acts, it would show no joints into which the analysis could penetrate, nor would there be any justification for such an analysis ; it would at best be no more than an arbitrary breaking up of the idea.

As a matter of fact the ideas which most naturally occur to us in this connection, the ideas of intuitable things, have arisen through an unconscious synthesis. When we become conscious of them they are complete as wholes ; but psychological analysis gives irresistible evidence of the processes by which the whole has been developed from particular elements. The image of an apple does not penetrate through the doors of the senses to the tablet upon which our ideas are depicted at one flash, as

¹ Ueberweg, *e.g.*, writes, § 49, p. 103: "The characteristics of an object consist in such of its elements as distinguish it from other objects."

if conveyed by a kind of enchantment, or by the mechanism of a psychical photography. Analysis of sense-perception shows how the sensation of colour must be connected with the movements of the eye in following the outline; how one perspective view must be combined with others, and these again with particular tactile sensations of the hand, mentally grouped into the form of a stereometrical image; how one psychical function must construct from the sensations the idea of an external object, and another assign to this object its position in space. It shows, again, how the idea of the visible and tangible thing is enriched by sensations of smell and taste, the reference of these to the visible and tangible object presupposing fresh functions appropriate to the combinations of the impressions of different senses. Finally, it shows how such impressions when partially repeated are continually supplemented by reproductive imagination, and become associated with the word *apple*, in connection with which we easily reproduce a kind of mental abbreviation of these processes; and all this takes place so rapidly and unhesitatingly that the result appears in the mind without our being conscious of the steps by which it has been formed.

3. That which is true of ideas of things applies also to the ideas of attributes, activities, and relations. Equilateral is a composite idea, for it presupposes, first, the apprehension of the particular sides—and for the recognition of a line as a side we need a relational idea—and then a measurement of the sides and the judgment that they are equal. In the same way the idea of movement, the simplest activity, requires the apprehension of different positions and of the transition from one to the other. The idea of murder is a relational idea, and includes in addition to its two points of reference—the murderer and his victim—a whole series of determinations, such as the conscious and deliberate intention of the one, his action, and its effect in destroying the life of the other; so that the idea as a whole can only be produced by a series of acts. This is doubly true of ideas which contain the thought of a plurality of independent objects connected by one or more relations, the so-called collective concepts in the widest sense: people, family, etc.

4. So far as an idea is composite we can only determine it by consciously attending to its particular elements and their manner of synthesis. Thus the formation of concepts always presupposes, on the one hand, analysis into simple, indecomposable elements; and, on the other, a reconstructive synthesis of these elements. Here the form of the synthesis

may itself be called an element and characteristic of the concept in the wider sense of the term, and we shall in future denote it as such.

The concept thus is related to the naturally developed idea as the conscious construction of an object is related to its unconscious and involuntary formation; it presupposes that we are able to bring into consciousness the complete process of the formation of the idea. This takes place by means of judgments, which assign particular characteristics to the object as its predicates, and the concept therefore presupposes these predicates—*i.e.* the ideas of the characteristics—which must themselves be conceptually determined if the composite idea is to be so. We thus find that we need a number of simple characteristics, *i.e.* of ideal elements which cannot be further analysed, but which are nevertheless perfectly definite, fixed, and distinct.

5. But the concept must fulfil yet another condition; it must be such as to serve for universally valid judgments. That is, every one who shares in the community of thought must connect the same ideas with the same words, and be able therefore to analyse them in the same way, and reduce them to the same simple elements. We can communicate a composite concept by enumerating its elements, and the manner of their synthesis; but these elements must be the same for every one, and be combined in the same way if our concepts are to agree. There is presupposed, therefore, a store of ideas formed by every one according to exactly the same laws; and we can be certain that our concepts agree only to the extent in which we are certain that our ideas are formed according to laws which are the same for all. Thus completeness and perfection in the formation of our concepts depends upon our having complete insight into the processes by which our ideas are formed, and upon our consequent ability to arouse the same idea in every one else. If we could assume that all our ideas are innate and possessed by every one alike, as was assumed by a former theory of knowledge, at any rate with reference to a part of our concepts; or if we could assume that the same world when presented always produced the same system of ideas, with a mechanical certainty like that with which equally tense strings give out the same note when struck with equal force; then the assumption of the traditional logic that the characteristics of concepts present themselves, as it were, spontaneously might be justified. But in proportion as the process by which our ideas are formed is complicated and dependent both upon external conditions which must differ with the individual, and upon internal laws, it becomes more difficult

to recognise and lay down the conditions upon which every one will form exactly the same ideas, and to recognise what elements agree or differ in the ideas of different people. The difficulty, which is often great, of ascertaining whether two people understand exactly the same thing by the same word, is due to the difficulty of finding ideas which are the same for every one and are called by the same name.

6. Since we are only laying down the conditions of an ideal perfection in the logical formation of concepts, we cannot undertake to present a complete theory of the formation of our ideas. Such a theory belongs to the work of the future.¹ But the results of our investigation so far show that the problem of resolving a given idea into simple characteristics which are the same for every one is much more complicated than would appear from the formulæ which tell us that a concept *A* contains the characteristics *a b c d*, and that these are its constituent ideas. As if *A* were a kind of mechanical or chemical compound of familiar elements, all different, isolated, and of equal importance, in the same way that a syllable, to take the example given in the *Theætetus*, is a compound of letters.

7. The first question which arises, then, is whether we may assume any such simple ideas as isolated elements, which, like the letters of the alphabet, might each be expressed and retained by itself. If we return to our fiction of a world of ideas consisting solely of twelve notes, which only need to be fixed, distinguished and named in order to be conceptually determined, then we might say that composite ideas correspond to the various chords. But it was a fiction even to assume that the idea of a simple note is really simple, homogeneous and incapable of being further resolved or differentiated. To have the idea of any given note as such, we must think of it as *one*, identical with itself and differing from others; in no other way can it enter into consciousness at all, for consciousness itself is inconceivable without a plurality of distinct objects. The thought of the note *A* therefore is inseparable from the idea of *unity* and of *identity* with self; it involves moreover the idea of *difference* from others, and hence of a *plurality* of these others. All this points to functions by which we think of something as one, identical with itself and different from others; thus thinking also of plurality as distinct from and related to unity. A clear consciousness, therefore, of all that is involved in forming the idea of

¹ Here our views coincide with those enunciated by E. Zeller in his *Berliner Antrittsrede*.

A shows us that, in addition to the audible note, it comprises these various determinations ; and the idea proves to be already complex as it comes into consciousness.¹

We cannot, however, regard these determinations—unity, identity and difference, on the one hand, and the sense-given note on the other—as the ultimate and isolated elements for which we are looking. Unity, identity and difference cannot be thought in absolute independence. Not only is it impossible to think of identity without unity and negation—for these determinations are inseparably interwoven—but they necessitate also the thought of a *something to which the unity, identity and difference belong*. Nay, so soon as we attempt to think of these determinations each by itself the same story repeats itself ; we can only keep these concepts themselves before the mind by placing them under the determinations of unity, identity and difference. Thus our analysis never arrives at the absolutely simple, inasmuch as it finds certain elements included in every idea, even the simplest, which are due to the mere fact that it is thought at all, and that any judgment is to be made concerning it. These elements are therefore the necessary and constantly recurring products of the various functions by means of which we are able to grasp an idea and to turn it to account as the subject or predicate of a judgment. Thus, instead of the isolated letters we sought, we find a complex of functions, dependent upon, and mutually conditioning, each other, and manifesting their activity in what we may briefly call the FORMAL LOGICAL CATEGORIES. They are related in the same way to all objects of thought, being the conditions upon which any idea can be consciously retained in the mind.²

8. But more yet lies hidden in the note of our hypothesis. We can-

¹ Cf. Lotze, *Logik*, ed. 2, p. 26.

² Amongst these formal categories, without which nothing whatever can be retained in thought, we include also *number*, in the sense that these most general conditions of thought involve the functions upon which all counting is based. That is, they involve the positing and distinguishing of unities, the consciousness of progress from one unity to another, and from that to a third, and the unity of the consciousness of these steps. We allow that further development of counting, and the more complicated operations of arithmetic are due to the relations of intuitable things in space and time, and that fractions especially presuppose that divisibility of a whole which is originally found in space or time only ; but it does not follow from this that number is altogether dependent upon the conditions of intuition. Counting stands in no other relation to time than that in which all our activities stand, that is, a succession of them can take place only in time ; but it is in no way essential that in counting we should be aware of time. The idea of time is even dependent upon the idea of number, of a plurality of distinguishable acts. Cf. § 6, 3 b, p. 36.

not have an idea of a note or of a number of notes, except as in *time*; just as we cannot have an idea of a colour except as in space. Full consciousness, therefore, of what is present to our minds when we form the idea of a note includes the idea of time. And, again, when we try to set aside time as a simple element, not to be further analysed, we find that we can form no idea of time as absolutely isolated. We must combine it with the idea of *something in time*, something, moreover, which is various and manifold; just as we must think of space in connection with different things in space. Thus here again the nature of our ideas will not admit of our finding the absolutely simple and isolated; we can indeed find elements which are distinct, but they always involve each other. Moreover, the relation in which time and space stand to their intuitable contents is essentially different from that in which identity, etc., stand to their objects. Hence we get fundamentally different syntheses of the different elements of an idea, and we may express this difference, as Kant did, by speaking of space and time as FORMS OF INTUITION, in contrast with the formal categories.

9. When the concepts we are forming refer to the existing, then, in so far as we think of it as actually or possibly existing, there appear new elements again. Our only idea of the existent is of a *thing with attributes and activities*, and we can have no idea of particular existence which is not *related to something else*; at the very least it must be related to us as our object. Thus all our ideas of the existent, or possibly existent, include this cycle of inseparable determinations, which refuse to be resolved into isolated characteristics, and which involve a third kind of synthesis of the manifold, that of the thing with its attributes and activities. We call these elements the REAL CATEGORIES.

Traditional Logic generally chooses for its examples the concepts of things, and then the characteristics of the concepts are attributes (*e.g.* the characteristics of the concept "gold" are heavy, yellow, shining, etc.). But in addition to these, there is a distinct kind of synthesis among the characteristics—that of attributes in a thing. This synthesis is essentially different in meaning from that between the characteristics of a composite concept of attributes or activities, and it differs again from the synthesis of particular things to a whole by means of the definite relations connecting them. It can only lead to confusion if we express everything—three-sided figure, dark red, rotatory motion, yellow body, a kernel surrounded by a shell, etc.—all without distinction by the same formula $A =$

a b c d; as if this juxtaposition were the expression of a mode of connection which is always the same.

If these real categories are indubitably elements of our ideas of the existent, then it follows that in conceptual determination we must, first of all, determine these categories themselves as concepts. The popular distinctions between thing, attribute and activity, guided as they are by the forms of words, and vaguely and waveringly applied, must be developed into clearness. Thus the determination of any concept which refers to the existent presupposes a recognised theory concerning the nature of these categories. It is logically complete only in so far as this theory is logically complete, and will hold good only to the extent in which it is generally accepted. And the possibility of such a theory depends again upon the possibility of producing generally accepted concepts of the categories themselves. It depends, therefore, upon an analysis of our mental processes which shall make us conscious of what it is which everyone must think in accordance with necessary laws, when thinking of anything as existent.

10. The universality of such elements of our ideas as we have noticed so far is ultimately due to their derivation from functions which always recur in the same manner, however varied the matter of thought or intuition to which they refer. The nature of our spatial and temporal ideas is the same, whatever the particular objects thought of as in space or time may be. No matter how varied the ways in which our senses are affected, and the particular affections combined amongst themselves, the process of referring the sense-given to things with attributes and activities remains the same. The possibility of presenting a complete system of these elements depends upon whether, as Kant assumes, they are given entirely *a priori*, as forms which lie ready in the mind, and are therefore discoverable by a complete analysis; or whether the nature of our sensuous affections themselves determines what formal elements are developed. In the former case an organized and unalterable system meets the affections of the senses as they successively appear; in the latter, the categories would be the result of a development determined by the particular kind and order of our sensations. We need only mention these possible alternatives to show that the final determination of our concepts depends upon a clear insight into the genesis of our ideas themselves.

11. Different from these elements of our ideas are the elements which are given in intuition by immediate sensation or inner perception. From

the subjective psychological point of view there is no doubt that we have something simple and ultimate, really elementary, in particular colours, notes, odours ; and again in the immediate consciousness of inner events—of pleasure, pain, desire. The white of this paper, the black of these letters, cannot be further analysed ; it is given immediately by the affection of our sense-organs. It recurs in the most varied combinations and spatial forms ; but it is always the same and cannot be resolved into simpler elements. Here, then, it seems easy to set forth elementary characteristics, which cannot indeed be thought of as isolated—colour is never without space, and so on—but can at least be easily retained in the mind as distinct from their form and from each other ; odours from colours, colours from notes. Here, if anywhere, we have something which can only be named and not explained, something analogous to the letters of the alphabet ; and if it could be shown that all our ideas were formed from these elements given in immediate intuition, from the forms of intuition, and from the real and formal categories, then the circle of ultimate characteristics would be complete.

But here we come upon the second of the difficulties noticed in § 40, 8. Every definite sensation, every particular feeling of pain, is something simple and elementary ; but the number of these distinguishable simple sensations is infinite. It is absolutely impossible that every particular degree of light and warmth, each of which comes into consciousness as a simple presentation, should be fixed in memory and kept distinct from all others ; by no expedient could language be made to suffice for all this multiplicity. Language takes advantage of the fact that similar sensations are separated by imperceptible differences, and denotes a whole series of approximating degrees by one word. But similarity by itself is indefinite, it is of no use for conceptual determination, for it implies difference without stating its degree. The only way of attaining the definiteness of a concept here is to start from a comprehensive survey of the whole series formed by these imperceptible differences, and in this continuum to draw boundaries within which a given name shall hold good. Thus we get what we have called the generality of the *word* in distinction from the generality of the *idea* (§ 7). The names of the colours, for example, are not conceptually determined until the whole series of shades has been exhibited, and it has been settled within what limits the names green, red, etc., shall apply. Our means of carrying out this determination must be left for Part III. Here it is sufficient to point out that “red” is

not general in respect to dark-red and rose-red, in the same sense as "extended" is general in respect to different bodies. It is not the same red in different combinations which we think of in dark-red and rose-red; each sensation is absolutely simple, and cannot be resolved into one element which is the same in all and one which is different.¹

From this we may learn also what is the import of such words as colour, note, odour, etc. According to the ordinary theory, as colour is the general idea corresponding to red, blue, and yellow, the concept of colour should be one element of the concepts red, etc. But red, blue, yellow, are simple, and we cannot say what colour is except by an enumeration of the particular colours. If the word colour is to have over and above this enumeration any definite conceptual meaning, it must be in the fact that by grouping together a whole series of ideas it presents them as clearly marked off from others which are incomparable with them, such as notes and odour. But if we try to express the common element of these ideas, we can do so only by means of a relation, which does not directly denote the ideal content, but only a reference which red, blue, yellow, all contain, and by which they are distinguished from other simple sensations—reference, that is, to sight and to the eye. In such relations alone is a common element to be found, but these relations are not elements of the ideas themselves. This distinction between words which are merely common names of simple characteristics, and words which really denote simple ideal elements, must be constantly borne in mind; otherwise, the

¹ Cf. Werner Luthe, *Beiträge zur Logik*, p. 2: "We cannot distinguish in any given shade of red that element which is common to all red." In Lotze (*Logik*, ed. 2, p. 27 sq.) there is only apparent contradiction to what is said above. True, he says at first (p. 28), that in several distinct impressions there is something common to all which is conceivable apart from their differences, and to which (p. 29) the differences peculiar to the particular members of a series attach (light-blue, dark-blue, etc.). But he recognises that the general blue cannot be realized in the same way as the elements of other concepts which we compound from known particular ideas, and he adds that the presence of a common element can only be felt or experienced, that the common element does not form the content of a third idea of a like nature and species with those compared, and that it cannot be independently intuited.

This is what we wished to express above, and it would be more correct not to speak of a common element at all, but only of an unanalysable impression of similarity. This impression is present in very different degrees, and guided by it, we arrange our simple sensations in a series of diminishing similarity, so that we may fix limits in the series within which to apply a definite denotation (red, yellow, etc.). It is different with differences of intensity, e.g., of warmth, or of notes of the same pitch; here we can form an idea of the common element, because the differences are based upon the excitation of feeling, and are not differences of an ideal content.

doctrine of the characteristics of concepts, together with the accompanying doctrine of their subordination to each other, will fall into confusion. Nevertheless, even common names such as these may be looked upon as the signs of characteristics inasmuch as they indicate a common element which is the ground of the reference which they all contain.

Intensity of sensation and its differences, however, are true general concepts; for they refer to the excitation of feeling which accompanies the sensation, and which changes in the same way, although the objective elements differ.

12. What we have said of sensuous qualities seems to be true also of forms and movements, since these, too, appear as immediately intuitable. Here again we have infinite multiplicity and imperceptible degrees of difference; here again we seem to start from the particular sense-intuition, and the general idea (form or motion) appears to possess a generality of the word alone. But it only appears to be so. The idea of a given form—a triangle, square or circle—is far from being given so immediately and directly as the sensation of a sound or a smell. Perception of form necessitates movement of the eye or of the hand; and this movement, returning into itself and thus limiting a body in space in a definite manner, is really from one point of view, as this kind of action, the same in every perception of form; from another point of view it is differently modified according to the course it takes. It is the same with the idea of objective movement. The process by which it is perceived, the comparison of two positions, recognition of their difference and the idea of continuous transition from one to the other, is always the same; but the path, velocity, etc., are variously modified. Movement and form are true general concepts; colour and sound (as the expressions of something immediately given, not in the physical sense) are general words or common names. For this reason one example suffices to show what movement is, but not so with colour. This explains also why every theory which starts from sensations as the only primitive elements of our ideas must tend to regard all generality as only that of common names; and why it extends this view to all things which it regards as sense-given, ignoring the processes by which we form ideas of them. Sensationalism and nominalism always accompany each other.

§ 42.

The analysis of objects into their final elements gives rise to series of concepts in which each successive member is determined by a new distinguishing characteristic, and thus has a fuller intension than the preceding member. The analysis of a single object may form as good a ground for such a series as comparison between different analysed objects. The poorer concept with fewer determinations, which is included in the thought of the subsequent concept, is called the super-ordinate, higher, or generic concept; the richer concept, with more determinations, is called the subordinate, lower, or specific concept; their relation is that of SUBORDINATION.

The relation of subordination exists only between concepts of the same category; for it is the category which determines the sense in which their characteristics are synthesized, and through it alone they are comparable.

The extension of a concept is the sum-total of lower concepts which are subordinated to it. Within the same series of subordinated concepts the extension is greater as the intension is less, and *vice versâ*. We must distinguish between the logical extension of a concept and its empirical extension; and again between the empirical extension of the concept and the extension of the name.

We can speak of essential and unessential characteristics only in reference to objects in their relation to a given concept.

1. Let us suppose that the most important part of conceptual determination—the survey of characteristics in their different classes—has been carried out in accordance with a complete and universally valid theory of the formation of ideas; let us suppose further that we have thus discovered what characteristics presuppose and are dependent upon others (*e.g.* colour upon extended surface, a point which is generally overlooked), and that we are also clear as to which terms denote definite ideal contents, and which are only common names. Then the question arises how our universe of concepts will shape itself under these conditions.

In all conceptual determination we have to work upon ideas which are already given as material, and our first task is to reconstruct and determine these. Moreover, the constant supply of ideas which arise naturally and without reflection always has its origin in the particular, and we are constantly called upon to form judgments determining the particular by predicates. For this reason our explanation of the further relations of our

concepts will be facilitated if we begin by the conceptual determination of some idea excited by a particular thing.

2. When we wish to retain the idea gained from any particular thing—that is, when we wish it to be securely committed to memory, and recognised as the same when reproduced—the merely involuntary function of reproduction is not sufficient. This function, which we find aimlessly active in, *e.g.*, dreams, and upon which, in the earlier stages of judgment, our denominative judgments are generally based, simply repeats an image as a whole ; this involves no consciousness of the particular elements of the idea, and, therefore, no consciousness of its identity is necessarily connected with it, and it is in danger of being confused with others. To secure its exactly identical repetition, it is necessary above all that the idea should be decomposed into its particular elements, and this decomposition is again the condition which enables us to distinguish a thing from all others. It consists in going back to absolutely simple and perfectly definite characteristics, and the condition most essential to this is that fluctuating differences should be fixed ; *e.g.*, differences of colour must be fixed by a generally accepted denotation, of magnitude by a fixed standard.

The result of such an attempt is a description as it takes place in a conjunctive judgment. I might describe the wafer before me by saying it is an orbicular, circular thing, two centimetres in diameter, one millimetre thick, red, light, and smooth ; that is, I should describe it by enumerating all the predicates perceived through the different senses, and consciously recombining them into one whole ; the import of the synthesis is stated by the category of the *thing*, and the nature of the characteristics red, smooth, etc., determines their dependence upon the spatial characteristics. Any one hearing such a description is called upon to carry out, step by step, the synthesis which in intuition was involuntary and, until completed, unconscious ; it is supposed that the hearer thus obtains the same idea from the description which I had myself, always assuming that he thinks each characteristic in just the same way.

But it is obvious that when I describe something in this way the result is not what I intended ; it is seldom that the description is equivalent to the particular image, and it cannot replace intuition itself. The words “an orbicular, circular, smooth thing” constitute a formula in general expressions ; to the hearer it is like a riddle to be guessed, a problem for his imagination ; how may he form an image of a thing which shall satisfy the conditions of the problem ? It is true that every additional characteristic

distinguishes my idea from others which share the remaining characteristics, but by the nature of the predicates it is left open to each individual to form the idea in different ways. Predicates such as red, light, smooth, etc., even when accurately limited in meaning (*e.g.* light defined as of less specific gravity than water), admit of many degrees of difference, amongst which the hearer must choose before he can form his intuitable image. The description gives us a sketch which fits not only an indefinite number of things which are exactly alike, but also many distinguishable things ; it is, therefore, a formula having a generality which is not merely numerical, but also generic.

We see further that this generality does not result merely from the breadth of meaning of particular determinations, such as red, etc., it frequently happens that the enumerated characteristics do not exhaust all the attributes either directly perceived or inferable of my object. As the above formula does not state the material nor the attributes dependent upon it, it would apply just as well to a round piece of cardboard or a red counter. In this case the incompleteness of the description is easily corrected ; but the same deficiency may occur where differences are present which are unrevealed to our knowledge, or such as we cannot recognise. The most exact description of the germinal cell of a mammal would apply without further modification to those of many other animals, although we are bound to assume that there are hidden differences which manifest themselves in the course of development.¹ Nor again can the description of any real thing claim to be so exhaustive that it may not possibly apply in every respect to some other thing, which nevertheless differs in some way not known.

In a formula thus reduced to characteristics we have, then, not the full expression of a thing, but what is in the first place a subjective creation, expressing the idea we have formed from the intuition of a thing so far as we can retain it in characteristics which every one determines in the same way. It is a rule for the formation of the idea which we must observe, but may observe in various ways. Its generality is due partly to the wide meaning of the particular characteristics, partly to the possibility of adding new characteristics to those already given. It matters little for the present whether or not ordinary language has a special term for such an idea ; if it were worth while, one could be made.

¹ We shall speak in Part III. of the necessity to which this gives rise of employing in our determination of concepts characteristics based upon relations.

If our description were less complete—if, for instance, it omitted to state the size—a difference would be neglected which distinguishes this object from others which are larger and smaller ; the formula would be applicable to many other objects, inasmuch as we could complete it by supplying any possible magnitude. If it were more definite, if, *e.g.*, rose-red were substituted for red, then a number of distinguishable objects which were previously comprehended would now be excluded. We should, however, still have a formula expressing a synthesis of characteristics to which others may be added—a formula which the hearer can supplement in many ways.

3. It may thus happen that even the analysis of the idea of a single object may give rise to a series of formulæ, including successively more and more characteristics. As each characteristic is added the idea to be formed becomes more definite, and by each additional characteristic objects are excluded to which the previous ones by themselves applied. From each of these formulæ we may obtain the preceding one by omitting, the subsequent one by adding, a characteristic. The fewer the characteristics comprehended, the greater the number of different objects of which the formula can be predicated if we assume the possible differences to be actual, and *vice versâ*. The formulæ are related to each other as more and less general concepts. Even the most specialized is still general in so far as its characteristics admit of a certain width of interpretation ; only if all the characteristics were perfectly definite would the generality of the concept be merely numerical (*e.g.*, “ a cube of pure gold having sides measuring one centimetre ” is a perfectly definite concept).

This process is expressed by saying that we ascend from a given concept to one more general by abstrâction, *i.e.*, by omission of characteristics ; and that we descend to one more special by determination, *i.e.*, by the addition of characteristics. Abstrâction diminishes the intension, but widens the extension ; determination increases the intension, but narrows the extension. Intension and extension stand in inverse relation. The more general concept is called the higher or wider ; the more special the lower or narrower ; the relation between them is that of subordination.

We should reach the same result if we were to start, not from one single object, but from several ; the problem being to state what characteristics are common to various objects. The greater the number of different objects to be comprehended, the fewer will be the characteristics they have in common, and the smaller the intension of the concept ; the fewer the objects, the greater the intension.

4. These propositions, simple and obvious as they appear, nevertheless conceal several questions and difficulties which do not generally receive sufficient attention. These refer partly to the processes of abstraction and determination, partly to the relation between intension and extension.

In the first place, the omission and addition of characteristics is by no means so much a matter of choice as would appear from these propositions. Amongst the characteristics there is always one which determines the nature of the synthesis by giving the category; should this be omitted, the remaining characteristics would lose their common support, and the meaning of their synthesis would be uncertain. It is only concepts within the same category which can be subordinated to each other; and to speak of such a concept as red as super-ordinate to rose, or reasonable as the super-ordinate concept to man, or intentional as the super-ordinate concept to murder, is only confusing.

Moreover, the characteristics are not all independent of one another; to some extent they presuppose each other. It would be useless to omit the characteristic extended and retain red; the latter presupposes the former. Thus the course of generalizing abstraction is prescribed to it within certain limits.

It is the same with determination. In the first place, it is a matter of course that incompatible characteristics cannot be added without giving rise to contradiction. But by what is the determination to be guided? A twofold ground for the determination here presents itself. If the given concept-formula contains characteristics which naturally admit of a further series of distinctions—as red admits of a series of shades, circular of all possible magnitudes of the diameter, etc.—then the addition of one of these distinctions naturally presents itself as the next step in determination, and finds its justification in the given concept itself. But even here we must take care that other characteristics do not exclude some of these possible characteristics and thus limit the determination. The concept of a plane figure bounded by three straight lines contains nothing about the magnitude of the figure, nor about the magnitude of the straight lines and their relation to each other. Some magnitude of the straight lines is necessarily implied in the concept, but it is left for the determination to settle which. I cannot, however, fix upon any determination I like for each straight line; my choice is limited by the law that any two sides taken together must be greater than the third, this law being prescribed by the remaining characteristics and the nature of the synthesis demanded. Thus determination

cannot proceed from one characteristic alone, but only from the whole complex.

But determination of this kind is accompanied by another, which adds new and independent characteristics having no special connection with anything in the given concept. Matter, for instance, may be determined as extended and heavy substance; but at our present stage of knowledge we cannot regard the specific attributes of particular materials as in any way modifications of extension and weight. In such cases, however, we find that determination is guided by our purely empirical knowledge of objects which fall under the concept of matter; we add those characteristics which, in our experience, we find combined with the more general ones. Only if we had full insight into the nature of things could this determination be guided by the contents of our ideas.

5. Because there are these two modes of determination it is uncertain what we are to understand by the extension of a concept. The logical point of view, where the first demand is for conceptually determined predicates, is concerned only with the ideas with which we approach actual things. Hence the relation of subordination can never be found except between concepts, and the generality of the concept consists in its being contained in the thought of a number of ideas distinguished conceptually, *i.e.*, by different characteristics according to their intension. The mere numerical generality, through which the same idea is found in an indefinite number of particular intuited things, has nothing to do with the nature of the concept; it is one and the same concept which is thought in all the instances, and whether it can be predicated of one or of a hundred things, its nature is the same.

Hence the extension of a concept can never be measured by the empirical number of similar things which fall under it, if, in opposition to the *principium identitatis indiscernibilium*, we recognise the possibility of objects which for our knowledge are not distinguished by their attributes, but only by difference in time or space.

On the contrary, it must be laid down that a concept which does not admit of further determination has no longer any extension; it represents the final limitation of extension—the point, even though that which corresponds to it may be empirically forthcoming in millions of instances. Assuming all cast-iron to be the same, then a cast-iron ball having a diameter of 10 centimetres is such a concept.

There are certain characteristics which can be conceptually determined

only by fixing limits within a continuum of imperceptibly small differences. In connection with these alone is it true that even the lowest conceptually determined formula still has an extension, which, however, cannot be further broken up into discrete concepts.

The question of singular concepts partakes of this difficulty. A concept can never be called singular because there happens to be only one thing corresponding to it in empirical reality; any more than the logical nature of the concept would be affected if there were no object whatever corresponding to it. Only concepts whose characteristics involve the uniqueness of the corresponding object can be called singular; in this sense the centre of the material universe is a singular concept. The question, on the other hand, as to whether all individuals which actually fall under a given concept are distinguished otherwise than spatially and temporally, and whether only one, or whether several, particular things can be comprehended in a concept of the smallest extension, has nothing to do with logic, but belongs to the science of reality.

For this reason also it is purely a matter of chance when the same number of things fall under two concepts of different intension. Hence as concepts they must not be called equivalent or interchangeable ideas, but only in so far as, when used as names, they denote things which are the same within the range of our knowledge. They are really different, and logically considered they have a different extension. Two-footed unfeathered animal is a different concept from that of man; only when used as names do they denote the same beings. We must, then, distinguish between the *logical extension of the concept* and the *empirical extension of the name*.

At most there is room for doubt as to whether such concepts as "equilateral triangle" and "equiangular triangle" are identical or different. They differ in the formula; but as the characteristic equilateral, taken together with those comprehended by the word triangle, necessarily contains the characteristic equiangular, and *vice versa*, they are absolutely equivalent. Only by insisting upon the verbal expression can we maintain that they are different; and in that case we must also say that an equilateral rectangle and a rectangular rhombus are different concepts.

Again, it is only the extensions of subordinated concepts which are directly comparable. We cannot compare the extensions of concepts which are independent of each other, except in so far as every concept which admits of many additional determinations may be generally called wider,

while those which admit of few may be called narrower. There can, however, be no definite and universal standard of measurement for the extension of concepts.

There is a further distinction between the *logical* and the *empirical extension of a concept*. The logical extension consists in all the concepts which are gained by the further determination of its characteristics when this determination is involved in the characteristics themselves. But we may be guided in our task merely by our knowledge of actually existing things, and thus fail to work out any series of determinations or combinations of characteristics because no empirical occasion presents itself for so doing. When this is the case, because we do not see any necessity for selecting these, and only these determinations, we can only speak of an empirical extension. No one could infer from the concept of metal that there are just so many and no more different metals, nor could we even discover the number by attempting all the possible combinations of characteristics; for us the extension of the concept metal consists in the concepts of the metals we know. Just for this reason, however, the empirical extension of a concept can never be regarded as complete.

6. The expressions Genus and Species are often used with reference to the relation of subordination between ideas; each concept is genus to a lower, species to a higher concept. Of these terms it is again true that they have no fixed meaning except within the same category. Red is not the generic concept of rose, but only of the various shades of red. The highest genera, the *πρῶτα γένη*, are therefore the categories; and their common element is again nothing but the relation which consists in being objects of thought. Unless we accept this limitation, there would be as many highest genera as there are independent characteristics of any kind.

We must of course distinguish between the generic concept and the genus in its concrete sense as the sum-total of all the things falling under a generic concept, between the generic concept "human being" and the human race or genus.

7. When a concept contains different characteristics which are independent of each other, the higher concepts to which we may pass are various. From the concept of the square we may ascend to that of the equilateral quadrangle, or the equiangular quadrangle or the regular figure, according as we omit one or the other of the independent characteristics equiangular, equilateral, four-sided. All these higher concepts are equally

related as generic concepts to that of the square. In like manner the determination may take place in different order, according as one or the other of a number of independent characteristics is first to be added. We may proceed from the concept plane figure bounded by straight lines in the order *figura plana rectilinea quadrilatera—figura plana rectilinea quadrilatera æquilatera—figura plana rectilinea quadrilatera, æquilatera, æquiangula*; but we may also proceed in this order: *figura plana rectilinea æquiangula—figura plana rectilinea æquiangula, æquilatera—figura plana rectilinea æquiangula, æquilatera, quadrilatera*, etc. Thus every concept containing characteristics independent of each other may find a place in different series of subordinated concepts, and to exhaust all possible variations would need an arithmetical calculation of combinations.

There is therefore no order of succession in the subordination necessarily given by the nature of the concepts; there is no settled order of precedence according to which all concepts which have a logical possibility and justification may be arranged in one way. Just because concepts, as we understand them, are subjective creations, formulæ whose chief purpose is merely to fix our ideas and mould them into commonly accepted and unambiguous predicates, they are capable of unlimited variation by the many ways in which they may be combined.¹

8. As the primary function of concepts is to serve as predicates in our manifold judgments, we cannot regard it as an imperfection that, generally speaking, they are poorer in determinations than the concrete and fully determined subjects of which they are predicated, and that they are more or less deficient when compared with the intuitable reality of particular things and events. The fact that no one can eat "fruit" in general, but only apples, pears, etc., each, moreover, of an absolutely definite kind, with its own individual size and shape, in no way diminishes the value of the concept "fruit." Nor is the value of the concept "clock" any the less because no one can have a clock in general, but only a clock with a pen-

¹ The idea of an arrangement of concepts such that the more specialized concepts branch off in increasing numbers from one point, the concept of the *ὅν* or something, is thoroughly vicious. It presupposes that there must be a much smaller number of higher generic concepts than of the more specialized; but if we regard concepts as being combinations of a limited number of characteristics it depends entirely upon their interrelations whether combinations of greater or less generality are the most numerous. The necessity of any fixed order, and hence this imaginary pyramid of concepts, can be grounded only upon the metaphysical view which regards the higher concept as the real cause of producing the lower.

dulum, or a clock with a spring. It is a necessary part of the purpose and function of the concept that it should thus differ from the real thing. For this reason, when logical theory aims at supplying a supposed deficiency by making the statement or condition that the concept of a thing should contain its *essential* characteristics, it fails to understand the most important and universal purpose of the formation of concepts. Moreover, according to this theory, that which is left undetermined by the concept is represented as unessential, as accidental. Now, apart from the fact that a complete knowledge of the whole universe would be necessary before we could know what are the essential characteristics of a thing and what not, this view, when taken in connection with the subordination of concepts, would lead to the pantheistic conclusion that there is only one essential nature of all things, and that all differences are merely accidental, having their ultimate ground only in the subjective view of things. There is no absolute and fixed line to be drawn between differences which must be neglected in our determination of concepts, unless these are to be multiplied to an indefinite extent, and the differences which are fixed and formulated in concepts. Hence there is as much reason for saying that the differences between the last species are accidental in respect to the genus as for saying that the merely individual differences of a thing which falls under a lowest concept are accidental; and, finally, since a higher genus is always related to its species in the same way as the most specialized idea is related to distinguishable individuals, the essential nature, properly speaking, can only be expressed by the highest concept. This is, in fact, the origin of Spinoza's doctrine, that there is only one substance, and that all differences are merely modifications of this.

The distinction between essential and unessential characteristics first obtains a meaning and justification when we come to concepts of purpose. When we desire something actual as means to an end, these means are generally so constituted as to possess also a number of attributes which are not desired, and therefore not determined by the end; with reference to it they are accidental. The necessity of guarding against the cold gives rise to the concept of a covering which will prevent the loss of heat; and this necessitates that the material which is to serve the purpose shall be flexible and a bad conductor of heat. But any available material has many other attributes besides those of being flexible and a bad conductor of heat, and these other attributes contribute nothing towards the purpose. With reference to the concept of clothing, they are accidental. In the

same way the concept of a clock is originally a concept of purpose—the concept of an apparatus which measures time by spatial changes. So far as concerns the concept of the clock, the mode of its construction is accidental so long as it fulfils its purpose. Here then the subjective concept with its characteristics does actually precede the reality. It follows from the nature of the things which must be used as means that we cannot realize those determinations alone which are included in the concept ; its species are determined by the variety of the means. Only when nature itself is placed under the concept of purpose, and regarded as aiming at the realization of certain ideas or forms which, like human purposes, take the shape of indeterminate and varying thoughts, is there any meaning in distinguishing between the essential and unessential characteristics in the idea of an existing thing. If nature is concerned merely with creating the form, structure, and organization of a horse, and if colour has nothing to do with her purpose, then it is an unessential characteristic which is only present because the horse must have some colour. The fact that such characteristics vary in individuals otherwise similar is then taken as a sign that they are immaterial. Nevertheless, from the scientific point of view the colour of the white or of the black horse follows as necessarily from the constitution of the particular individual as the structure of its skeleton and muscles.

Thus the distinction of essential and unessential is always found when we compare an idea with a previously given concept in which it is contained, and try to find in it the realization of the concept. The penal law sets forth certain concepts of crimes, and the judge endeavours to find in particular concrete actions the characteristics determined by the law ; for the subsumption of the crime, and the meting out of punishment, these are essential, while the special circumstances of the deed which are unprovided for are unessential. If one man has killed another, it is essential to know whether he did it intentionally or unintentionally, with or without deliberation ; it is unessential whether he did it with a round or a conical bullet.

The distinction recurs again when the problem before us is to give conceptual determination to the meaning of some word in ordinary use ; here the unessential characteristics are those which belong, not to the general meaning, but only to the particular things included by it, or to the more specialized ideas. In this sense it is unessential to the concept of the house whether it is roofed with tiles or straw, but it is essential that it

should have some roof. The actual meaning of the word "house" includes the characteristic of having a roof, but does not specify the material.

9. This logical treatment of the difference between the essential and unessential characteristics of a thing must be carefully distinguished from the question as to what belongs to the real nature of a thing, what is essential to it or not (cf. § 33, 4, p. 198). We may be called upon to form our concepts of things in such a way that they shall express the essential nature of things, *i.e.* those determinations which belong to them independently of all other things, and proceed entirely from their own nature. Then the characteristics of the concept must include the essential determinations of the things, and all things having the same essential nature must fall under the same concept. But it is clear that unless we are to fall in with the pantheistic tendency, this condition can only be fulfilled by the *infimæ species*; it is inapplicable to any higher concept. In the same way it is clear that these essential concepts, if attainable at all, can form but a small part of the concepts of which we have need. Knowledge is concerned not only with the essential nature of things, which always remains one and the same, but also with the many manifestations, phenomena and effects due to this nature; and for this also judgments are needed which have concepts for their predicates.

From one point of view there is certainly a difference between the permanent and lasting states or attributes of a thing and those which change and vary, the difference which Descartes meant to denote by his distinction between *attributa* and *modi*. As the concept must be a constant idea, and the concept of a thing means something enduring in time, only that which belongs permanently to the thing can be contained in the concept of it. Thus its changing aspects are unessential with respect to the thing, but only because they cannot be included in the concept, not as having no reference to the real nature of the thing. For it is just in these changes that the real nature unfolds itself, and if we wish to express the real nature of a thing, we must include in the concept the permanent ground of its changeable aspects, under some such name as faculty, or power.

10. The distinction between essential and unessential characteristics, which in reference to the concept as such is meaningless, must not be confused with the distinction between fundamental and derivative characteristics. When from a combination of elementary characteristics other

predicates necessarily follow, then the former are called fundamental, the latter derivative.¹ It is a fundamental attribute of the rectangle to have parallel sides and right angles, it is a derivative attribute to have equal diagonals. It is a fundamental characteristic of an odd number that when divided by two there is a remainder one ; derivative, that it is not divisible by even numbers, and so on. But here again we must guard against confusing the logical and the metaphysical. We must not look upon the fundamental characteristics as constituting the real essence of a thing ; in many cases nothing is known about this. All we can say is, that according to the way in which *we* consider the attributes to be dependent upon each other, these fundamental characteristics constitute the concept as a fully determined idea.

II. It follows from our doctrine of the negation that negative determinations can never be original elements in the idea, and therefore that they cannot be characteristics in the proper sense. Every negative determination presupposes a negative judgment, and the subject of this judgment must be capable of being distinctly thought before the negation which is founded upon it. How far negative determinations may nevertheless be necessary to enable us to arrange our concepts will appear from what follows.

§ 43.

We must not confuse the distinction of simple characteristics and the resulting distinction of composite concepts with the difference of the objects in which the concept is contained. Different concepts which are contained in, and can be predicated of, the same object, are called compatible ; they are generally cross concepts. Different concepts which are incompatible cannot be contained in the same things ; their extensions are mutually exclusive.

The determination of a generic concept by incompatible characteristics gives rise to its differentiation into disjunct co-ordinate concepts, and a complete enumeration of disjunct co-ordinate concepts constitutes a DIVISION.

Division may proceed either by a development of characteristics already

¹ Derivative are not the same as dependent characteristics. A characteristic is dependent when we cannot think of it without presupposing another, as colour presupposes extension ; it is derivative when it is also the necessary consequence of other characteristics.

given in the concept, or by the addition of new ones ; in the latter case sometimes by negative determinations. In division we are justified in including negative characteristics of the form non- B in a concept, but not in regarding non- B itself as an independent concept.

The distinction between so-called contradictory and contrary opposition, when rightly understood, coincides with the distinction between a division having two terms and one which has more than two.

The members of a division may be complete either in a merely empirical, or in a logical sense.

1. The distinction between the various characteristics must find its expression in the negation, which tells us that A is not B , not C , etc. (§ 21, 1 ; § 22, 6). Conceptual determination can only be complete so long as this negation remains obvious and indisputable, and is not made uncertain by the indefiniteness of ordinary language when dealing with gradual transitions.

The same is true of all composite concepts, which are not absolutely identical, *i.e.*, equivalent syntheses of the same characteristics. They necessarily differ in their contents, according to the difference of their characteristics, and this difference is again expressed by the negation of identity, which tells us that A is not the same as B . The only object of the negation is to confirm the fixed and immutable rule according to which different words signify different things, and which, if we are concerned merely with the contents of concepts denoted by different words, holds good even when the predicate denotes a concept to which the subject is subordinated : square is not parallelogram.

The attempt has been made to express a maximum of difference by speaking of disparate or incomparable concepts, which have no characteristic whatever in common (such are understanding and table, and the various simple characteristics themselves, red and sweet, etc.). These are distinguished from concepts which are comparable, which have one or more characteristics in common (hence, according to the ordinary doctrine, are included in the same higher concept) and differ only in the remaining characteristics. But this distinction is only relative. Nothing whatever is absolutely incomparable, to the extent at least that the formal logical determinations apply to everything which enters into thought. But if we disregard this fact then the most fundamental difference to be found amongst concepts is that which causes their characteristics to be synthesized in a different way, the difference of their categories. To this extent

it is correct to speak of concepts which belong to different categories as *fundamentally* different (e.g. man and virtue, man and movement) ; of those which stand within the same category as *relatively* different. But fundamentally different ideas may still have many characteristics in common, though they have them in a different way, as with iron and metallic, man and living, and yet not fall under the same higher concept in the ordinary sense, since subordination has no meaning except within the same category.

2. We must be careful to distinguish between the difference of concepts themselves according to their content, and difference of the objects in which they are contained, and of which they can therefore be predicated. Composite concepts would be impossible unless we could think of different characteristics as determinations of one and the same idea, whatever the form of their synthesis may be ; and our ability to form ideas of the incalculable number of different things is especially dependent upon the possibility of combining different attributes as determinations of the same thing. Every concept which admits of further determination by different characteristics is included by the addition of these in various other concepts ; while on the other hand many higher concepts may be included in one and the same lower concept.

Characteristics which can be combined in one concept, and concepts which can be thought as parts of one concept, are called compatible. When one species is comprehended under different generic concepts, these are called cross concepts, inasmuch as they have at least a part of their extensions in common ; when they are represented figuratively (say, as circles) the boundaries of their extensions intersect and include a portion which is common to all. Thus in the square, quadrangle and regular figure overlap. It is clear that the concept in which two higher concepts overlap arises from the combination of characteristics in which these two differ, and which thus appear as mutual determinations. The two concepts abc and abg overlap in the concept $abcg$, and this may be regarded as the determination of abc by g , or of abg by c .

3. Opposed to compatible characteristics are those which are irreconcilable or incompatible (cf. § 22, 8-13, p. 131 sq.). These cannot be thought together in the same concept, but are mutually exclusive as determinations of the same thing. There is no characteristic which would be incompatible with all others ; all must at least be compatible with the formal logical determinations. But incompatibility itself, when logical, is involved in the nature of our ideas (cf. § 22, 8, p. 131 sq.).

4. Upon the fact that characteristics which are incompatible among themselves may yet be all compatible with another, is based the differentiation of concepts, and their complete development¹ (Division).

When a concept A is determined by two incompatible characteristics b and c , these are called specific differences, and the resulting concepts are themselves incompatible; *i.e.*, they cannot both be thought as parts of the same lower concept, nor both be predicated of it (no Ab is Ac , no Ac is Ab). Hence their extensions are absolutely distinct, and all the more specialized concepts which are developed out of them are also incompatible, each of these extensions, meanwhile, forming a part of the extension of the higher concept (right-angled and acute-angled quadrangle, red and yellow rose, etc.). Such concepts are called disjunct, and when their relation of subordination to a higher concept is the same, disjunct-co-ordinate concepts.

When a concept A admits of only a limited number of mutually exclusive determinations $b\ c\ d$, there arises a series of disjunct concepts, whose extension covers the extension of the concept A , so that when A is developed into all the differences possible to it, each lower concept must possess one or the other of these determinations. The concept A is said to be divided into the concepts Ab , Ac , Ad , and these are called the members of the division.

The division itself is represented by a divisive judgment: A is partly Ab , partly Ac , partly Ad . Of every particular thing which falls under A the disjunctive judgment is true that it is either Ab or Ac or Ad . (See § 37, 6, 7.)

5. Every differentiation presupposes that a concept is still undetermined in one or more of its characteristics and admits of additional and mutually exclusive differences; or that the synthesis of its characteristics is incomplete and leaves room for more characteristics. Every division presupposes that the total number of possible determinations is limited and

¹ The prevailing logical terminology is inconvenient in that it employs the same expressions to denote two processes so different as the analysis of a concept into its characteristics and the development of opposed concepts from one higher concept; these expressions being derived from the act of dividing and signifying sometimes the division of the content into its elements, at others the division of its extension into mutually exclusive extensions. To this is due the paradox that by dividing a concept we do not get parts of the concept, but concepts which each contain the whole divided concept as a part. If we keep consistently to the content of the concept we are concerned with nothing but a development of the characteristics contained in it. The term division (Aristotelian *διαίρεσις*) is more applicable to the sum of the particular objects which fall under the concept; this sum is regarded as a whole to be broken up into different groups.

completely known. The concept of the rectilinear plane figure is undetermined in several points, both as to the number of the sides and their magnitude, and as to their relative as well as their absolute magnitude ; and again as to the relative magnitudes of the angles (the absolute magnitude of these is not a completely independent characteristic, but depends within certain limits upon the number of sides). According as determinations are added on one or the other of such points, the concept is developed in different directions into its differences. In the same way the concept of fluidity is as yet undetermined with respect to its transparency or capacity of reflecting light, with respect to its smell, taste, etc. Smell, taste, and colour are not differences in one of the characteristics forming the concept of fluidity ; but they can be added to the other characteristics because their general possibility is involved in the characteristics of the concept fluidity.

Strictly speaking, only the first form of differentiation can be called development or explication. If we call that characteristic which is differentiated the ground of division (*fundamentum divisionis*), then the ground of division here is in the given concept itself, and consists in the fact that a characteristic admits of mutually exclusive determinations. Thus the concept of the line is developed into the concepts of straight and curved lines. The idea of the line involves movement, and this cannot be thought without direction ; direction which remains the same forms the straight line, direction which is constantly changing the curved. The concept of the curved line develops into the concepts of those which return into themselves and are thus closed, and those which may be continued to infinity, for constant change of direction includes both possibilities.

The second form of differentiation adds determinations from without. The ground of division is in the first place nothing but the indefinite possibility of a further characteristic which is independent of those already present, or the possibility of various irreconcilable characteristics ; the question is asked whether there are any more characteristics which are reconcilable with *Ab*, and such determination might be called synthetic. The concept of fluidity contains no characteristics but those which are founded upon sensations of sight and touch, only the mere possibility of taste and of differences of taste are given with it ; they must be added as new elements. It is here then that there arises the possibility of negative distinguishing characteristics, which express mere privation. We divide the concept of organic being into feeling and not feeling, of flowers into scented and

scentless, of fluids into colourless and coloured ; the absence of a characteristic which is compatible with the remaining characteristics, but is not necessarily connected with them, forms here a specific difference. In such cases the negative formula loses its indefiniteness ; it has for its content the possibility of the positive characteristic which is included in the general concept and realized in one of its species. Its function is not the independent expression of some content ; it serves merely to mark a distinction and to fix the order of the concepts.

We must carefully distinguish between these privative characteristics as means of differentiation, and those in which the negative expression of characteristics is only an indirect statement of the positive differences contained in the same general characteristic. When I divide lines into straight and not straight, or men into white and not white, the negative expression has a definite positive meaning ; it signifies those characteristics which are excluded from the negated difference on the ground of the same *fundamentum divisionis*. The negation of the possible determination is limited to a definite area and thus states something positive ; it is grounded upon a disjunction (straight or crooked, white or coloured, and again white or yellow or red or brown or black). The negation of one member of the disjunction contains the assertion of the others.

This negative formula finds a double application. In the first place it serves to comprehend a number of co-ordinate disjunct members into one expression, because they are alike in some other respect in which they differ from the concept excluded by the formula. The meaning of dividing men into white and coloured (*i.e.*, not white) lies in the fact that in all coloured people the capacity for higher culture possessed by the white is wanting. Otherwise, if colour alone were taken into consideration, the difference between black and red, or red and yellow, is as great as that between yellow and white ; there would be no ground for expressing this series of equivalent differences by the negation of one alone.

The second application is where one amongst an infinite series of possible differences can be conceptually determined, this being impossible or less easy with the others because of their endless number ; they can be conceptually determined only by limiting them with reference to the one. This is the case with regular and irregular figures. Each of the latter has itself a definite relation between sides and angles, but opposed to the simple characteristic of having equal sides and equal angles there is an endless number of other relations, none of which can be reduced to so

simple an expression, and which it is absolutely impossible to determine one by one.

6. Here, then, in connection with the division of concepts, we find the value and significance of negative expressions, to which we were obliged (§ 22, 11, p. 134 sq.) to deny any justification when put forward as independent symbols of ideas, by themselves and without reference to the question before us. We can see now also in what sense the distinction between the so-called contrary and contradictory oppositions is justified. If we limit the expression "opposition" to the relation between disjunct co-ordinate concepts, then we find contradictory opposition between the disjunct members of a division having two members, contrary opposition between the disjunct members of a division having more than two. In the former case, one member can always be quite definitely and unambiguously denoted by the negation of the determination which constitutes the other member; not so in the latter case. In the former case, if Ab and Ac are the disjunct members, Ab is the same as $A \text{ non-}c$, and Ac is the same as $A \text{ non-}b$; in the latter case, if Ab , Ac , Ad are the members, Ac is indeed included in the formula $A \text{ non-}b$, but as this formula comprehends both Ac and Ad , it should be expressed as $A \text{ non-}b, c$.

7. When the number of differences is by nature unlimited, we cannot speak of a division in the proper sense; but only of the development of a higher concept into an infinite series of disjunct lower concepts. Thus the concept of the polygon may be developed into the species triangle, quadrangle, pentagon, and so on *in infinitum*.

8. Our attention is drawn by this example to another point. When a characteristic taken by itself implies a series of disjunct differences,—as the characteristic plurality implies numbers, and colour the particular colours,—then it depends upon the nature of the remaining characteristics whether any of these different determinations can be combined with the concept, or only certain of them. All numbers occur as disjunct characteristics of the concept of the spherical polygon; but the characteristics of the rectilineal plane figure exclude the number 2, and those of a body bounded by plane surfaces exclude the numbers 2 and 3.

This choice amongst the various determinations of a characteristic becomes of special importance when the process of division does not take place by the development of the contents of a given concept, thus circumscribing its logical extension, but starts from its empirical extension.

Then the problem arises of dividing a concept in such a manner that all the different determinations may be empirically forthcoming. The fact that the human body is not transparent involves its having some colour ; and if we were to develop the concept from this characteristic alone, we should have to introduce all the colours as members of the division. Starting from this characteristic and leaving other considerations out of the question, we are as much bound to assume a species of blue or green men as of black or white. In reality many of the colours are wanting, and when we divide the concept man on the ground of colour, we enumerate only those colours which are actually forthcoming, regarding the division as complete when it includes all these actual variations.

But there is no doubt that in this limitation we generally confuse two quite distinct problems : there is the problem of classifying a given number of particular beings, and this we shall afterwards examine more closely, and there is the problem of presenting a system of concepts to serve for our knowledge of the particular by means of Predicates which have a perfect logical determination. If it were merely a matter of chance that only some colours are actually found within the range of our experience as the colour of the human skin, then the so-called division of men would be no division of the concept, but merely a classification of actually given men ; we could never say that we had in this way exhaustively divided the concept. It would be a mere enumeration of disjunct species ; just as in chemistry we enumerate the metals without meaning to say that no more new kinds can be discovered.

If, indeed, we can regard the fact that no other complexions are forthcoming as a sign that a blue or green complexion is excluded by the other characteristics of the human being, then, and then only, would it be possible to accept the empirical classification of men as an exhaustive division of the concept man. That a mere classification of what is given in experience is frequently substituted for the division of the concept—the logical extension being thus confused with the empirical—is quite in accordance with the habit of neglecting to consider the concept according to its intension, and with the more popular and intuitable method of always starting from the empirical extension. In consideration of this it must always be remembered that logical completeness of division is never guaranteed by the fact that the extensions of certain members of a division are together equal to the empirical extension of the concept divided.

9. When the division has been carried out, it brings to light a dis-

tion amongst the characteristics which has not yet been noticed ; the distinction between the *notæ communes* and the *notæ propriæ*. Some of the characteristics, that is, may be common to a great number of concepts which are otherwise different ; while others presuppose a definite combination of other characteristics, and thus distinguish a given concept from all its higher or co-ordinate concepts. Thus the characteristic "possessing only plane right angles" can occur only in the quadrangle ; it is a *nota propria* of the right-angled quadrangle. But such a *nota propria* may also belong to a generic concept, hence characteristics which belong only to an *infima species* are called specific *notæ propriæ*. Where such characteristics are found they distinguish one concept from all others, and they are then called distinguishing characteristics.

10. The same concept may be divided according to different grounds, and because the resulting concepts generally overlap, these are called cross divisions. Thus the division of parallelograms into rectangular and oblique-angled crosses their division into equilateral and unequilateral ; the division of plants into phanerogams and cryptogams crosses their division into land and water plants. Such a combination of grounds is a means of breaking up a concept into a number of others which are not immediately subordinated.¹ The number of members resulting from several independent divisions, each of which by itself would yield *a*, *b*, *c*, etc., members, is equal to the product of these numbers.

11. We can imagine a logical system containing all the most simple combinations of characteristics which can be thought as independent and isolated concepts, in which these combinations should again be developed by division on all sides and according to all grounds into the most specific concepts possible. We should then have a systematic synopsis of all the concepts we could possibly construct ; both their relations of subordination and their differences would be fully determined, and we should be able to see at once how any given idea was subordinated and opposed to all others. The logical ideal of a perfect analysis of our ideas would then be attained, and we should have moreover a system of Predicates appropriate to all particular objects, and a means of classifying them in the most varied ways. For though every object would then fall under only one lowest concept, so that it would be distinguished from all other objects which did not agree with it in every characteristic, still its various

¹ There is no ground for any special treatment of what is called sub-division, for the process is exactly the same whether we divide a higher or a lower generic concept.

aspects would enable us to subsume it under different series of higher concepts.

§ 44.

A DEFINITION is a judgment which states the meaning of a term denoting a concept. This it may do, either by an expression which exhibits the concept broken up into its characteristics, and gives a complete account of its content, or by stating the genus next above it, and the specific difference, thus indicating its position in the ordered system of concepts.

All logical definitions are *nominal* definitions. The demand for a *real* definition arises from the confusion between metaphysical and logical problems.

Definitions are *analytical* or *explicative* when they refer to a concept which is already formed and denoted by a recognised term; they are *synthetical* or *determinative* when by synthesizing determinate characteristics they present us with a new concept, and introduce a term for it.

We must distinguish between definition proper and the attempt to discover the concepts upon which our ordinary use of language is grounded.

1. Suppose our logical ideal to be attained, and every concept to have, moreover, its established and unambiguous denotation in language: then the problem of stating the contents of a concept would be solved by the mere repetition of the analysis and synthesis through which it first became a concept, and we should only have to make sure that we could at any moment realize the meaning of such a word by unfolding the elementary characteristics contained in the concept denoted by it, and that we could recognise its position as subordinate and disjunct. The first is done by a formula stating the particular characteristics, from the synthesis of which the concept arises; the second by a formula naming the *genus proximum* and the *differentia specifica*, i.e., exhibiting the concept as member of a division. (Since the same concept may have different genera, and the order of determination may vary, there may be different formulæ for the second answer. The square is a quadrilateral regular figure, an equilateral rectangle, etc.; such formulæ differ only apparently, and cease to differ when the analysis is continued, and these higher concepts are also resolved into their characteristics.)

We may call the statement of all the characteristics of a concept, or of its *genus proximum* and *differentia specifica*, definition, and it is evident that in such definition we cannot be concerned with the explanation of a con-

cept, but that, so far as there is any explanation, it is that of a term. An idea is not a concept unless it is clear, *i.e.*, unless we are fully conscious of what is contained in our thought of it ; thus the definition is the concept itself, not something different. It is only the term, which in comparison with the concept is external and fortuitous, which secretes the wealth of a thought in one sound, and which, indeed, is frequently used like x and y in algebra, as a mere symbol of which we fail to realize the meaning at every step ; it is the word which, because its external form fails to show its relation to the words which stand for higher or lower concepts, as a chemical formula shows composition from elements, needs to be explained and to have its content constantly brought to mind. Such explanation is especially needed when the word is taken from popular language with its fluctuating boundaries, and when we have formed, or wish to form, a constant and unambiguous symbol, from one which was uncertain and ambiguous. If we were to carry out Leibniz' ideal of a *characteristica universalis*, then the symbol indissolubly connected in thought with every concept would also be its definition, and would enable us to see its relation to all others.

Definition in this sense, then, can never be anything more than nominal definition, stating the meaning of the word ; it can be a real definition ¹

¹ To insist that a definition should be real in any other sense is to confuse different problems. We cannot say whether any actual object corresponds to a concept of complete logical definition until we have our concept and can subsume under it the objects given. We cannot say whether the characteristics of a concept represent the essential nature of the things which fall under it, or whether they include the real causes of these things, until we have a perfect knowledge of the objects ; but this knowledge itself cannot be called a definition. This applies also to the example given by Lotze (*Logik*, 2nd ed., p. 202) : "When we call the soul the subject of consciousness, of ideation, of feeling and will, this may be appropriately called a nominal definition. The real definition of the soul could be established only by proving either that the subject of consciousness and its manifold phenomena could be nothing but a supersensuous and indivisible being, or that it could be nothing but a complete system of material elements." The knowledge of the nature of the being which we have first determined in our concept of the subject of consciousness is no definition, it merely establishes the dependence of the characteristics first thought of upon others which are not yet included in the concept. By the knowledge of this dependence the concept is enriched ; we now understand by "soul" an immaterial, indivisible being, which is the subject of consciousness. But this definition is nominal in the same sense that the first was, and in the same sense that the first was it is also real ; here again we "name the conditions which any thing real must fulfil, if it is to claim the name of soul" ; the only difference is that we name them more fully. The two concepts merely denote two stages on our way towards the goal of knowledge ; further investigation will teach us how the beings which fall under this enriched concept must, by their nature, be related to other beings, and from such increased knowledge will result still richer definitions. Knowledge always presupposes

only in the sense that it must analyse the contents of the concept in thought, and distinguish them from the contents of other concepts. Merely verbal explanations, such as "logic is the science of thought," "democracy is the rule of the people," or explanations of verbal abbreviations, such as "a perpendicular is a perpendicular straight line," cannot be called definitions at all (cf. § 5, 3, p. 27).

Thus a definition is a judgment, in which the meaning of a word representing a concept is equated to the meaning of a composite expression; and this expression states the particular characteristics of the concept, and the manner in which they are synthesized, by means of the particular words forming the expression, and the way in which they are grammatically connected. We have an equation between two symbols of the same concept, and the symbols may therefore be substituted for each other. It follows, as a matter of course, that anything which is covered by the one word is also covered by the expression, *i.e.*, that the extensions of subject and predicate are absolutely the same. The existence or material possibility of something corresponding to the concept may indeed be presupposed by the definition, but can never be stated by it; the definition is an explicative judgment, such as we dealt with in § 16.

From this it follows directly that the definition (*definiens*) *must not repeat the word to be defined* (the *definiendum*). We must not define *idem per idem*, for in so doing we fail to fulfil the condition that analysis must always break up the thought, which, when it appears in one word, is a unit, into elements which are necessarily denoted by different words. This is the source also of the objection to repeating in the *definiens* even a word of the same derivation (*e.g.*, freedom is the ability to act freely), an objection which is justified only when the etymological relation of the two words is unequivocal, and both are used in exactly the same sense (*e.g.*, redness is the quality of being red). The explanation of freedom given above may serve as it stands for a definition, because the meaning of "free" is limited

a definition of the word which it employs, to give clear determination to its object; the characteristics which are thus established may be found to be necessarily connected with others, and then these are taken into the definition and the same process repeated with the concept thus enriched. The demand for a real definition, containing the essential characteristics, is a return to the Aristotelian demand, that the concept should state the essence of the thing, according to his metaphysical theory. Now that we have left the Aristotelian metaphysics far behind, and confess our ignorance in most departments of knowledge of the *τί ἐστίν*, in the Aristotelian sense, it would be well also if logic were to relinquish the concept of the real definition. It has ceased to have any meaning in logic, and represents only a one-sided ideal of knowledge.

by the expression, "to act freely," and not every instance of being free, *e.g.*, free from pain, is to be called freedom. In such cases it is the meaning of the derivative syllable which is the chief object of the definition; and there is as much justification for this as for explaining only one portion of a composite term (*e.g.*, vital force is the inner ground of vitality).

It follows further from the nature of the analysis demanded that in defining we must go back to simpler elements, and that a correct definition must not move in a circle, so as to reintroduce the *definiendum* itself in its enumeration of characteristics.

On the other hand, the rule "*definitio ne fiat per negationem*" does not hold good under all circumstances. It is true that the statement of what a thing is not does not tell us what it is; but as a concept is often distinguished from co-ordinate concepts only by the privation of a characteristic, and as this task of distinguishing is included in definition, we cannot always avoid negative determinations.

That an enumeration of the species of a concept cannot be a definition is evident from the fact that the species contain the concept, and their enumeration would involve a circle.

The rule that definitions must be *precise* prohibits the statement of characteristics, which are already contained in, or necessarily connected with others (derivative characteristics), *e.g.*, to include in the definition of a parallelogram the statement that the opposite sides are equal as well as parallel. Still, a so-called "*superfluity in definition*" is not positively wrong, and is even preferable where there is not absolute certainty as to the connection between the characteristics.

For the names of ultimate elements there is no definition; we must assume that every one attaches the same meaning to these. They can be named, but not explained. Where they are not yet known they can only be shown by bringing about the conditions under which the idea will be aroused. This is possible in the case of colours, smells, and tastes, if we assume that different people are similarly constituted. The only case in which we find something analogous to definition is with a series of concepts bearing a common name, the statement of which recalls the various members of the series—*e.g.*, when we say "red is a colour." Here we can give something which corresponds to the *genus proximum*, but we cannot give the *differentia specifica*; this could be replaced at best only negatively, by denial of all other varieties.

If, then, in constructing our concepts we are obliged to fall back upon the laws of our simple mental functions and the forms of their synthesis, a perfect definition becomes one which enables us to build up the idea of the object from its elements ; this alone can be called a *genetic* definition.

3. Unless we may assume a complete and accepted analysis of our ideas into elements which are perfectly definite in their determination, and denoted by all in the same way, there can be no concepts in the logical sense, and this would make it generally impossible to solve any problem of definition—as impossible as from an equation of which every term is an unknown quantity to determine any one of them. Every definition presupposes a scientific terminology.

When such a terminology is not forthcoming we can form a good definition only in so far as it may be possible to find expressions in ordinary language which are unambiguous, and which may serve, at any rate in practice, for the unerring subsumption of actual objects. Such, for instance, is the case with jurisprudence in its application to the relations of daily life.

4. It may happen that although the elements of certain concepts are familiar to us, we have not ourselves as yet formed all the concepts which might be formed from them, nor completely learned the meaning of their denotations. Then the definition, when heard, serves both as a guide to the formation of our concepts and as an interpretation of a word we do not understand.

Moreover, since all that is included in the logical ideal is that all the elements and forms of combination should be conceptually established, the formation of composite concepts may be continually progressive. This is the more true because of the impracticability of attempting all the combinations of concepts possible in the sphere of reality, until we can discern the grounds of the real compatibility or incompatibility of particular characteristics, and of their combinations, or until the occasion for forming some definite combination presents itself. Hence the necessity of forming new concepts, and of coining new words for them, to which we must give conceptually determined meanings.

The definitions first mentioned set forth analytical equations, in which the import of a word is expressed by means of an equivalent formula. The equations, by means of which we first get definite expressions for new concepts, are determinative equations ; they give import to a symbol by equating it with an expression consisting of familiar elements. When we first form the mathematical concept of a function, we give a meaning to

the word by means of a formula, which has an external resemblance to a nominal definition, but which is really different. Definitions of words which stand for concepts already formed have been called analytical; those which introduce the term for a new concept, synthetical.¹

5. We must further distinguish between these two kinds of definitions and those explanations of words which merely serve to establish the way in which language is actually used. In their original purpose these are merely attempts to justify this actual use of language, and to give a reason for it by showing that it is grounded in each instance upon a definite concept, which is contained in the thought of all the objects named by the word and in no others, and they thus indicate the point of view from which language ranges many objects under the same name (§ 40, 5, note). When these attempts are successful, they *narrate* the meaning, which, as a matter of fact, always belongs to a certain word. It is only to explanations of this sort that the warning against making a definition too narrow or too wide originally applied, *i.e.*, its characteristics must exclude no objects which language continues to name by the word, and include none which language names by another word. But it is evident, first, that a definition is only possible when we presuppose characteristics which are conceptually determined, and then that many words do not admit of any definition in this sense. This is sometimes because their denotations have become arbitrarily extended, thus making it necessary to distinguish between their different meanings, sometimes because they are used only to denote certain individual phenomena, and their extension to others, even when they agree in possessing the common characteristics, is not authorized by the use of language. The greatest acuteness can find no simple definition of the word *people*, when we try to give the usual meaning of the word. Words such as *church*, *theocracy*, *feudalism*, are not symbols of concepts; they merely denote certain historical phenomena by their prominent features, and are thus names of single things. The dispute as to the concept belonging to them may continue for ever.

Here also, when the problem from which we start is that of discovering a concept from the objects to which ordinary language has given the same name, there is a meaning in the rule that the essential characteristics

¹ Drobisch, § 117 sqq., rightly notices that in a synthetical definition the *definiendum* really occupies the place of the predicate, this predicate being nothing more than the word as a name. In the case of the definitions at the beginning of Spinoza's *Ethics* the formula employed—*Per substantiam intelligo id quod*, etc.—shows them to be definitions of the second kind; that is, they introduce simple verbal denominations for definite ideas.

should be united in the concept (cf. § 42, 8). For here, no doubt, what we have to do is to determine the concept in such a way that it may contain the ground upon which the name was given, and that only those characteristics may be included by which we were guided in giving the name, and upon which it depends whether or not new things will be called by the same name. Starting from the empirical extension of the name "man," we should be obliged by the laws of abstraction to include in the concept the characteristic "without a tail," for this is a characteristic common to all known men. When, however, we have determined that, assuming complete similarity in all other respects, we should not be deterred by the external appearance of the rudiments of a tail, which are present in human beings, from continuing to call the bearers of this limb a man, then the characteristic "without a tail" is no longer a part of the concept "man"; it must not be included in the definition, because it is not taken into account in subsuming the particular under this concept. But it depends entirely upon the point of view taken in classifying objects by means of language what is in this sense essential, and what is immaterial. A characteristic may be immaterial in one aspect, while in another it is essential.

From this problem of determining the actual meaning of a word from the way in which it is used as a matter of fact, we must distinguish that of stating the meaning in which certain words ought to be used in definite scientific statements or laws, as opposed to the uncertainty of their ordinary use. Assuming the general concept under which they fall to be given and known, any determination may serve for this purpose which marks the limits within which the word is to be applied with certainty, and without ambiguity, even though the distinguishing marks employed may be only derivative and accidental. As an extreme instance, we may take section 1 of the German Penal Code, which distinguishes between crimes, misdemeanours, and transgressions, according to the measure of punishment attached to the actions. Taken as a definition in the ordinary sense, this would be a logical monstrosity; as merely marking the limits within which certain terms are meant to be employed, when it is assumed that the general meaning of these terms as a punishable violation of the law is known, it is justifiable.¹ We find a similar instance in the statement that the warm zone is that which lies between the tropics.

¹ Cf. what G. Rümelin has to say on this point in his *Juristische Begriffsbildung*, 1878, p. 22 sq.

6. It may be that our only aim is to denote certain objects in such a way that we may be able to distinguish them confidently from all other similar objects. Then it is not necessary to state the whole contents of the concept; a formula which states its distinguishing attributes, and which may be called a diagnostic definition, is sufficient. Chemical reactions which are peculiar to certain substances are instances of such characteristics; they make it superfluous to state the complete contents of the concept when the only object is to subsume given phenomena correctly, and to distinguish them from others. The property of colouring starch blue is characteristic of iodine; hence the manifestation of this property is sufficient to prove the presence of iodine, and is a means by which to distinguish iodine from all other similar elements. But only in this connection can this characteristic be substituted for the whole concept; its significance lies in the fact that its presence proves the presence of the remaining characteristics which constitute the concept of iodine. We find similar distinguishing attributes of a derivative kind in the spectral lines of the particular substances.

From one point of view it is impossible, as Kant has shown, to give any exhaustive definitions of natural products. We must be content to include in our formulæ a selection of characteristics which may state the most easily recognisable attributes so far as to enable us to distinguish the objects with confidence. For this reason, all definitions of such objects are diagnostic, inasmuch as they cannot enumerate all the characteristics which belong to the object, nor even all which are contained in our knowledge of it.¹ Nevertheless, there still remains a difference between

¹ In such definitions we see a particular aspect of the nature of concepts in their relation to concrete existence. So far we have not expressly considered a difficulty which has been lately revived by Volkelt (*Erfahrung und Denken*, p. 342 sq.) with penetration and acuteness, the question, that is, as to whether the general idea as such is in any way conceivable; can we form any actual idea which is general, or must we not rather agree with Berkeley that we have only particular intuitions, and that generality is to be found in the word alone? Referring to Lotze (*Logik*, edn. 2, p. 40 sq.), Volkelt says that generality cannot be obtained by simply omitting the distinguishing characteristics. "Must we not say that the thought of a triangle which is neither equilateral nor irregular, neither acute-angled, right-angled, nor obtuse-angled, is no thought at all?" Generality, therefore, can be conceived only with reference to the indeterminate totality of the particular; in the concept is implied the thought that the general idea is a conceivable something only when accompanied by distinguishing characteristics, only in the particular or as the particular. Hence it follows that the ideal expressed by the concept could only be realized in a consciousness which should combine with the general idea its appropriate intuition in one indivisible act, and should do so, moreover, as infinite, absolute timeless thought. These propositions are no doubt true in so far as an all-

CHAPTER II.

THE TRUTH OF IMMEDIATE JUDGMENTS.

IMMEDIATE JUDGMENTS are those which we can state with a consciousness of their objective validity, without anything being presupposed beyond the ideas of subject and predicate which are connected in them (§ 18, 1). The most obvious of these are (1) the merely explicative judgments, which only state in the predicate what is thought in the idea denoted by the subject word, and (2) judgments concerning the particular which rest upon immediate intuition, and which state what predicate belongs to a given particular idea. Amongst the latter we may distinguish between (*a*) statements concerning ourselves, and (*b*) judgments of perception concerning the external world.

§ 45.

For the truth of judgments which merely state something about the relations of our established concepts we find a ground in the Principle of Agreement, and in so far as the relations of concepts include the incompatibility of certain characteristics and concepts, in the Principle of Contradiction.

1. We have seen that we cannot speak in an unambiguous sense of the truth or falsehood of judgment unless the ideas presupposed in it are completely determined. Explicative judgments (§ 16) deal only with ideas which are assumed to be already possessed by every one. When those ideas are concepts in the logical sense, such judgments merely state the relations between concepts which are already fully determined; they are a repetition of the process of unification and differentiation which took place when the concept was formed.

2. Positive judgments containing definitions, judgments which predicate its characteristics of a concept, and judgments which predicate a higher concept of a lower one, all derive their necessary truth from the given

to be conscious that they are the same, so also it is possible for us to be assured that in our judgment we shall always proceed in the same way as certainly as we are ourselves the same. All consciousness of necessity depends finally upon our immediate certainty that our action is invariable. For this reason it may be urged that, after all, it is only an inner experience which yields us this certainty, and to this there is no answer. But then we must distinguish between the experience of the particular acts of our varying ideation and the experience which in each particular act is accompanied by the certainty that it is not dependent upon the momentary and changing conditions of the particular act, but will continue the same throughout all changing moments. It is this immediate certainty which gives us the immediate and unanalysable intuition of necessity, and though this is indeed the object of experience, *i.e.*, of an immediate consciousness taking place at a given point of time, yet it is not merely the result of a number of experiences.¹

4. The establishment of definite relations amongst concepts leads also, and as directly, to the necessity of all negative judgments which distinguish between different concepts as wholes, and, according to the Principle of Contradiction (§ 23, p. 139), of negative judgments which deny irreconcilable characteristics or concepts of a concept. To put the same thing in another way, it involves the necessary falsity of judgments which attempt to attribute a predicate to a concept contradicted by it (*contradictio in adjecto*). The establishment of conceptual relations involves the incompatibility of certain characteristics as an invariable relation; that is, it involves the necessity of denying *b* when *a* is affirmed. Thus to ascribe *b* to a concept containing *a*, is to say that the same thing is *a* and is not *a*.

5. The Principle of Contradiction as a normal law has just the same meaning as when it appears as a natural law, merely determining the meaning of the negation; but while as a natural law it states only the impossibility of consciously affirming at any moment both that *A* is *b* and that *A* is not *b*, as a normal law it is definitely applied to the whole range

¹ For this reason I cannot altogether agree with the view taken by Baumann (*Philosophie als Orientierung über die Welt*, p. 296 sq.) concerning mathematical necessity. The experience that when repeating our ideas we always form them in the same way cannot of itself yield necessity; the fact that anything actually happens cannot exclude the thought that it might possibly be otherwise, only the consciousness that the fact will always actually be as it now actually is—*i.e.*, the consciousness of its necessity—can do this.

of invariable concepts covered by the comprehending unity of consciousness. It is in this application that it becomes the ground for what is generally known as the *principium contradictionis*, a principle which then ceases to be co-ordinate with the principle of identity (in the sense of the formula A is A) and presupposes it as already satisfied by the absolute invariability of our concepts. And here again the absolute validity of the Principle of Contradiction, and consequently of those propositions which deny a *contradictio in adjecto*, rests upon the immediate consciousness that, so surely as we ourselves remain the same, our action in denial always is and always will be the same.¹

6. To the establishment of conceptual relations is also due the validity of judgments of possibility, which attribute to a concept as yet undetermined the possibility of receiving compatible determinations (§ 34, 5, p. 207), and of disjunctions which are based upon division.

A concept may cover a number of species, or be applicable to many particular things, and may therefore be predicated of different subjects. To the extent to which this is the case the hypothetical necessity of attributing the predicate of the concept to that which falls under it also follows immediately from the relations of concepts. If the statement " A is B " is true of the concepts, then the statement "if anything is A it is B ," or "all A 's are B 's," is true also. Such judgments have always been rightly regarded as analytical, and as owing their certainty to the principle of agreement. It is the same with negative judgments; if the statement " B is incompatible with A " is true of the concepts, then the statement "if anything is A it is not B " is also true.

7. In all these judgments, if we assume our concepts to be established, we have only to read off what we have ourselves put into them; we are dealing only with fixed ideas, and no one who has exactly the same ideas can possibly doubt the judgments. For this reason they are independent of time and unconditionally valid; according to Leibnitz they are eternal and necessary truths. But just for this reason they never make the direct

¹ J. S. Mill (*Logic*, Bk. ii. *fn.*) treats the principle of contradiction as one of our earliest and most familiar generalizations from experience, and takes it to mean that belief and unbelief are two different mental states which exclude each other; a fact which we learn from the simplest observation of our own minds. To some extent I agree with him, but then the puzzle is: how do we know that they are not merely different but mutually exclusive? If the certainty that they are mutually exclusive is to follow from a slight observation, still the necessity of this mutual exclusion must be a matter of immediate consciousness.

statement that anything is; neither do they refer to definite, particular things, nor to existing objects. An existential judgment can never be analytical in the Kantian sense; for in an existential judgment the question is—as Kant has irrefutably shown—whether anything exists which corresponds to the concept. We first think of the subject of the existential judgment as without existence, then we go on to say that it also exists just as it is thought. Thus the ground for affirming that something exists cannot lie in the process by which we think the contents of an idea, in conceptual thought. If there is any such ground, it must be a something present to consciousness which is distinct from conceptual thought.

8. But while it is easy to discern the truth of conceptual judgments when we assume a completed system of concepts, we are far from having exhausted the proper function of concepts. Conceptual judgments are valuable as constantly reanimating our concepts and keeping them before our minds, and as developing the abbreviation of the word into its fuller meaning. Nevertheless the whole value and significance of a system of concepts lies in its application, and in the assistance it gives, when used for predication, towards a knowledge of that which is not yet contained in the system as such. It is the organon of all knowledge, but it is not knowledge itself; the apparatus with which we work, but not the product. If all our thought were but a monotonous recurrence of what we already know, and all our judgments but repetitions of those by which we formed our concepts—as is tacitly implied in the school logic—then the human mind would be condemned to everlasting sterility. Its progress consists in constantly combining new elements with established concepts, or with new concepts formed from these. Nor does the ideal perfection of a universally accepted system of concepts complete the problem of knowledge, any more than a lexicon is the literature of a nation. The progress of thought and investigation gives rise to new intuitions and ideas, and our chief task is to become aware of the laws according to which our judgment in its constant progress has a claim to truth and universal validity.

9. This progress in thought and knowledge has its origin in the particular individuals who form judgments, and spread them abroad by communicating them to others. Since the predicate must always be assumed to have been previously given, the condition upon which these judgments will be formed is that new ideas shall arise to serve as subjects. When these are only new conceptual creations brought about by deter-

minative definitions (§ 48, 2) the question of their validity is covered by the laws given above. They serve only to effect a new abbreviation and to introduce a term ; any judgment concerning them is again nothing more than the explication of a concept.

It is different when the new subjects are ideas of particular objects. We must assume for the present that the distinction between the idea of a particular thing and the thought of the contents of a concept is accepted and known to every one (§ 7, p. 40 sq.); if we express it by the distinction between intuition and concept, we must still assume that it is immediately comprehensible. The most we can do is to determine it by derivative and external characteristics, to say that what we think in the form of a concept is purely mental, can be repeated at pleasure and without alteration, and is dependent upon nothing but our own mental activity ; while, on the other hand, that which is intuited is presented to us at a given moment, and our thinking of it depends upon conditions placing it in a relation to us which is independent of our own mental activity, and compels us to locate in a particular object contents which might be expressed in a general form.

It may happen that when a particular thing is given to me in intuition I am nevertheless aware that it is peculiar to me as an individual, and that if it is presented to others also it is so only by chance. Such is the case with a dream, or a vision appearing to me alone, or the mental production of creative imagination which for the moment appears as a particular object independent of voluntary thought. Then, according to the Principle of Agreement, I must, in so far as I consciously retain the idea of the object, describe it correctly, *i.e.* in a manner corresponding to its contents ; but beyond this we can have no motive in seeking a ground for such judgments, inasmuch as they are absolutely individual and incommunicable, and can be believed by any one to whom the object is not presented upon authority alone.

But the ideas may be such as can be produced in every one alike, and, indeed, under certain conditions, must be produced in all alike, so that they are naturally capable of being shared in by every one ; and then we have a motive for making our judgments concerning them in such a way that they may be recognised as universally valid. This, for instance, is the case with geometrical figures,¹ inasmuch as we suppose the idea of space to

¹ Geometrical constructions occupy a peculiar position, in so far as in them the distinction between the particular image and the concept disappears from one point of view. So

be the same for all, and the elementary data of geometry to be given as intuitions. But more especially is it the case with all which we regard as existing. When we affirm that anything exists, we think of it as a particular thing, or as the determination of a particular thing. It is further implied in the concept of being, that the existent is independent of individual thought, and the same for all. But if it exists, not through, but before thought, it may conceivably stand in different relations to different thinkers; the idea of it may be present to one person, and not to another; it may be complete in one person, incomplete in another. Inasmuch as the ground for affirming is not to be found in the conceptual thought which is common to every one, it may lie in data of consciousness which differ with the individual. On the other hand, a judgment concerning the existent can only be true when all agree to it, since the existent is the same for all who know it. It is this which makes us want to ascertain the ground upon which the necessity rests of our judgments concerning the existent.

When it is affirmed or assumed that something exists, we may generally distinguish between the idea of the particular thing as subject, and the judgment that it is,¹ whether the latter be only tacitly understood (as is generally the case), or expressly stated in an existential judgment.

§ 46.

Those judgments in which we give utterance to that immediate consciousness of our own action which is present in every moment of our waking life take the first place amongst immediate judgments concerning the existent. Their certainty cannot be analysed any further. They in-

far as we regard them merely as mental images, due to our constructive activity alone, images which are indeed particular intuitions at the moment, but which can be repeated at will in such a manner that their identity is purely an identity of the ideal content—such constructions possess the generality of the idea and of the concept; the particular as such is general. While, on the other hand, inasmuch as it is presupposed that the elements of such constructions are presented to all in the same way, and that they can be forced upon every one by external perception, they are allied by the fact that they are the same for all to the intuitable existent, and we can speak, in a certain sense, of their objective being. To avoid repetition, we will reserve a more careful investigation of the judgments arising out of them for Part III.

¹ Being in general cannot be regarded as the true generic concept to the particular things which are; conceptually regarded it is only a common name. For, inasmuch as "Being" is for us a relational predicate, it cannot be a common characteristic; we should have to show that this predicate had its origin in a determination common to all existing things.

clude not only the certainty of the judgment "I am," but also the certainty of the reality of the unity between substance and action.

In so far as they are subject to time, these judgments presuppose a universal necessity to think of our particular actions as occurring in a temporal series, and universal laws as assigning to each particular its position in this temporal series.

1. Judgments such as "I feel pain," "I see light," "I wish this or that," are so absolutely certain, and so obviously valid, that they seem to leave no room for a logical investigation as to their justification and the ground of their necessity. Nor, indeed, can any one doubt their immediate truth if we presuppose clearness of consciousness, and distinctness and complete development of the concepts which serve as predicates. Assuming that a person speaks the truth, no one can take upon himself the right of doubting his statements, and questioning whether or not he is to be believed in what he says about himself. It would seem then that at present we have only to determine what is the difference between these and conceptual judgments.

2. This difference is fundamental. Conceptual judgments have subjects which are assumed to be thought by every one in the same way. The judgment, "I see," has a subject which can be present to no one as it is to me. The conceptual judgment unfolds the contents of its subject, these contents being always contained in it in the same way; the contents of that which is denoted by "I" can never be exhaustively stated; it is given to us in a manner which is absolutely incomparable with all other objects of thought. The conceptual judgment says, "If I think of *A*, I must think of it as having the determination *B*"; in the judgment of self-consciousness there is no if. The subject is simply thought whenever anything is thought, and the fact that it is thought is the absolute presupposition of all other thought. The conceptual judgment tells us nothing as to the existence of its objects, but the judgment, "I see," always includes the judgment, "I am." We can always ask whether that which is contained in a concept exists; I can never ask whether I exist. The characteristics of the concept are unvarying; the predicates of the ego, with the exception of the ego itself, change from moment to moment, and yet every judgment as it is formed possesses an immediate certainty of truth, which can only be acknowledged, and never proved. The Principle of Agreement does indeed guarantee that the general concept of the predicate shall agree with the action immediately intuited, but it cannot guarantee

either the statement that the subject is just performing this action, or the statement involved in this, that the subject exists.¹

3. If we are thus obliged to acknowledge that the utterances of every self-consciousness possess a certainty which cannot be referred to something else upon which it is dependent, then all that remains for us to do is to ascertain how much is involved in this acknowledgment.

In the first place, we find that with reference to this subject it is impossible to separate the mere thought of it from its being. Thus the judgment, "I am," differs from all other existential judgments, in that its subject is not the mere idea of an ego to which being is ascribed; subject and predicate are inseparably connected.

Further, the immediate certainty of self-consciousness involves, at any rate upon this point, the reality of the synthesis between substance and action; and so far as actions may be referred to attributes, the reality also of the synthesis between substance and attribute.

Finally, we find that the most fundamental certainty with respect to being attaches to a judgment which cannot be repeated in the same way by any one else, and which depends upon a purely individual act. For the idea which *some one else* has of *me* is different from that which I have of myself; it refers to the same subject, but not in the same way. Thus the affirmation of being in its most original form is an act peculiar to the individual, and dependent upon conditions peculiar to the individual. Any judgment concerning another ego is necessarily mediated, whether it is the acknowledgment of his being or belief in his utterances.

4. This immediate certainty, however, belongs only to the self-consciousness of the moment; to the judgment which states what is present at the moment. Thus the judgment is true only for a given point of time. The way in which we are conscious of our particular states involves the idea of time, for we are never conscious of a particular act without a recollection of the act which preceded it in time, and our self-consciousness always includes the consciousness of a self which is identical in time. Now, in so far as every moment brings with it the thought of our being at a previous time, and thereby of our continuous existence, we find im-

¹ Kant's doctrine, that statements of the inner sense refer only to phenomena, because of the subjectivity of time, does not affect the logical character of such judgments; it merely affects the metaphysical import, and the meaning of the statement about reality, which they involve. Their immediate certainty as judgments is as unassailable under the Kantian hypothesis as under any other.

mediate certainty here also ; the "I am" involves an "I was before," which is as unquestionable as itself. But, strictly speaking, the certainty extends no further. It is true that, when we are dealing with the particulars of memory, the statement, that "I believe I once did so and so," cannot be questioned, but the belief itself cannot lay claim to the same certainty. The inference of the reality of an actual previous event, from the reality of a present recollection, could only be justified if there were an absolutely necessary law, according to which I must, under all circumstances, retain the conviction of having at one time acted as I now believe myself to have acted ; if, that is, there were no such thing as the discovery of an error in our recollection. Now it is true that we regard some of our recollections as absolutely certain, particularly those of more recent events. But it is just as certain that, occasionally at least, our memory deceives us, and that there is no sure criterion by which to distinguish between perfect and imperfect recollections ; in the last instance our only warrant for the truth and trustworthiness of our recollections is our ability to place them in a conscious connection, which is continuous and consistent in all aspects. Thus the judgment that I have actually performed a given action, because I believe that I remember doing so, cannot be regarded as immediately certain. It is a mediated judgment in so far as it infers from a present idea the reality of a previous action corresponding to it, and for judgment such as this there is no rule which is immediately certain and absolutely reliable.¹

¹ Compare W. Windelband's excellent pamphlet, *Ueber die Gewissheit der Erkenntniss*, a work which agrees in so many points with the position here laid down, that, with the exception of a few passages, I could subscribe to it almost word for word. With reference to the above question as to the source of our certainty that an idea is a recollection, he says (p. 87 sq.) that in the last instance it is nothing more than a distinct feeling accompanying the idea, which tells us that it has been presented before ; this feeling, however, is again due to the fact that with the idea there is associated the subconscious idea of its connection with, and reference to, the ego, which is excited together with it, and manifests itself in consciousness as a feeling of recollection. In this way he explains how it is that we are able to have ideas for the second time, without knowing them to be recollections, for this happens when we have not been clearly conscious of their connection with the thinking ego. This explanation is correct to the extent that there is a peculiar feeling by which we generally distinguish the known from the unknown. But this feeling can yield no certainty unless it enable us to recall some accompaniment of the particular idea, other than that in which it now appears, and thus to connect the past with the present. I may, on seeing some person, experience most strongly the feeling which generally distinguishes the known from the unknown, but I shall not have perfect certainty until I recollect the circumstances under which I previously saw him, and thus fit him into the complex of ideas, which constitutes my self-consciousness, and is always

Here, again, we are met by the psychological difficulty which we find in being certain that our concepts are invariable, and hence in being certain that the logical ideal is attained. For since our thought takes place in temporally differentiated acts, the uncertainty of memory affects also my consciousness that what I now think is what I thought before. For this reason the logical ideal can only be approximately reached, and needs not only unremitting practice, but also external aids. Amongst these writing takes the highest place, and so great is its importance that it is safe to say that without writing science is impossible.

5. On the other hand, we must note that every judgment concerning a present action, in so far as it places this action in a temporal series, also determines a particular point of time for its own validity; this "now" is an essential part of the judgment, because, if referred to another point of time, the validity of this judgment would be annulled by others. If, then, a judgment which thus includes the time of its validity is to be objectively valid, it is not sufficient to assume that there is a universal necessity for the particular moments of consciousness to appear to every one alike as moving in a temporal series. It is not sufficient, that is, that there should be a time which is the same for all. When such a judgment lays claim to universal validity, there must also be universal laws making it necessary to assign a definite time to the truth of every judgment. If, then, my judgment is to be valid and universally acknowledged, I must be able to determine the time for which it is valid in a universally valid way.

Thus the Kantian teaching that time in general is a necessary idea is not sufficient. We need also the determination in an objective time of a point of time which shall be the same for all; and we need a common measure of time according to which every particular fact of consciousness has its place assigned to it.

The question as to how these laws may be discovered cannot be answered by reference to any immediate certainty, for it depends upon a comparison between that which is immediately certain to me and the temporal ideas of others. Its investigation forms a problem to be dealt with in our third part.

present. In this consists that reference to the ego, rightly emphasized by Windelband. It is not a reference to the abstract unity of self-consciousness, but to the empirical ego, and knowledge in the sphere of recollection is nothing but the continual review and consistent combination of a number of acts of my past life.

§ 47.

Immediate judgments concerning external existence are JUDGMENTS OF PERCEPTION. As ordinarily used, they include the statement that their subject exists. The perception is in the first place subjectively certain (§ 46) in so far as it implies the statement that I have at the moment the idea of a definite existing thing; the condition upon which a judgment of perception becomes objectively valid is that there should be a necessity to refer this subjective fact to some existing thing, according to which we are compelled to interpret the contents of a perception as the real predicate of an existing thing. Especially must there be laws according to which my spatial intuitions may be translated into the spatial determinations of objects, my reference of attributes or changes to a thing into the real attributes and activities of substances, and my idea of its relations into real relations.

1. To ordinary consciousness the judgments of perception, in which we make statements concerning that which is immediately present and external, appear as immediately certain as the statements of immediate self-consciousness.

These judgments of perception include first of all the consciousness that an idea of a particular thing is at the moment present to me, and has the peculiar and indescribable character whereby perception is distinguished from memory and from merely inward ideation. Our possession of fixed concepts and a denotation for them next enables us to express the contents of what is thus presented in a universally valid form. This is done partly by subsuming¹ it as a whole under a generic concept (a process corresponding to the denominative judgment), partly by analysing it into its particular elements, and stating the predicates which correspond to these. When these predicates are simple the judgment continues to be immediate; one element of the perception is recognised as agreeing with a conceptually determined characteristic (what I see is red).² But the process may be one of subsumption under composite concepts; and then, instead of an immediate naming which unifies a whole with a whole, we

¹ Concerning the term subsumption, cf. § 8, 6, note.

² To the extent in which there is a difficulty in maintaining the conceptual limits of distinctions which melt into each other in the merely internal reproduction of concepts, the objective validity of such a judgment may no doubt depend upon other processes (measurement, etc.).

have the necessity of a comparison between the particular characteristics of the perception and the characteristics of the concept. The judgment then becomes mediate, in that it is the result of a number of particular judgments (cf. § 56 on the inference of subsumption).

2. If no more were meant by a judgment of perception than that my present sensation is of redness or sweetness, the Principle of Agreement would suffice here again to guarantee that the judgment is necessary, and such that every one having the same idea must affirm it.

But such a judgment does not compare mere ideas. It refers an idea to a particular object which is thought of as existing, and it states a predicate as objectively belonging to this definite thing. If the judgment is to be true, it is not sufficient that there should be a ground for the agreement of the particular idea with the general. The confident assumption of ordinary judgment that this particular idea refers to a definite existing object, and that this object possesses the predicates which I attribute to it, must also be well grounded. This is possible only if there is some law according to which the subjective affections and ideas of the individual are referred to objective things with unfailing and universally valid necessity. Now the universality of the conviction that real objects correspond to our sensations does indeed prove that we are under a psychological obligation to affirm the reality of what is felt. But it is also proved by the fact that our senses often, as it is said, deceive us, and by the conflicting statements of different observers of the same object, that this universal obligation is no guarantee of complete unanimity in each particular case. So that here again we may find, and often do find, that we have to distinguish between that which actually occurs according to psychological laws, and that which is universally valid, and we cannot speak of a sufficient foundation for such judgments unless all subjective differences can be eliminated. This, however, is only possible if we can bring to light universal laws according to which we necessarily refer the subjective sensation to an objective reality, and can test every case by them. Then only can we pass from the judgment, "I am sure that I saw and perceived this or that," to the judgment, "this or that is there, has happened."

3. Once it is recognised that in perception we have at first only subjective occurrences, and that only the presence of the idea is immediately given, its reference to an external thing being a subsequent, though generally unconscious, step, then for every judgment concerning external existence we need a law, according to which the idea must always—at least

under certain conditions—be necessarily referred to some external existing object. Scepticism denies that there is any such necessity, or, at any rate, that it can be known to us. Subjective idealism affirms the necessity, but only in the sense that we necessarily think of what is perceived as a real external object. To it this affirmation of an external existence is a mere act of Thought, and though the necessity thus leads to a second stage in our ideas, it does not lead to an existence independent of ourselves. The reality which we affirm is only a reality of appearances, not of independent things.

We are not concerned here to settle this disputed question. It is sufficient to show that the immediate certainty of our judgments of perception does not rest upon any absolute necessity, until some general law has been shown according to which the fact of perception necessitates the recognition of the existence of an external object.

From a logical point of view it is, moreover, quite indifferent whether this law is represented as assuring us of the actual existence of external things—that is, in the realistic sense; or in the idealistic sense as only necessitating that the idea of real objects should follow upon perception. The logical character of the resulting judgments, their necessity and universal validity, would be the same in either case; the only difference would be in the meaning of the predicate “to be” (in the empirical sense). Only the sceptical view that it is impossible to attain to necessary judgments would exclude them from logical treatment.

Now, the universal law for which we are seeking cannot be to the effect that whenever I perceive anything there exists something, which corresponds to the particular idea developed according to psychological laws from my perception. On the contrary, it has been shown over and over again, and from wholly different points of view, that it is possible even to doubt the existence of the whole external world, and it has been maintained that no logical necessity invalidating that doubt corresponds to the psychological necessity of assuming such a world.¹ If, then, there are, nevertheless, ways and means of attaining to a conviction of external reality, they are not to be found in the simple facts of perception, but in the particular nature of our perceptions.

4. But even if we allow that there are principles which necessitate the reference of our perceptions to something really existing, we have still to

¹ Cf. Baumann's treatment of these propositions in the first chapters of his *Philosophie als Orientierung über die Welt*.

consider the conditions under which these principles are applicable. The question is, what are the presuppositions necessary in order that the subjective fact of perception may warrant an objectively valid judgment? The extent to which judgments of perception may vary with different individuals sufficiently shows that such judgments cannot under all circumstances be universally valid, since they lead to contradictions.

Generally speaking, there are two grounds, to either of which these divergencies amongst individual percipients may be due. The difference may lie in the very beginning of the process, in the empirical data from which the formation of the subject idea and the judgment about it starts; in the affections of our organs of sense, or, more exactly speaking, in the manner in which we become aware of such affections. Or it may lie in the subsequent processes which in ordinary thought take place unconsciously, but which nevertheless do take place, and may be shown—principally by means of the so-called deceptions of the senses—to be analogous to inferences.

5. With reference to the first of these grounds, the certainty with which we ascribe to objects our sensations of colour, temperature, etc., as their attributes, depends upon the conviction that there is an invariable connection between the assumed object and ourselves, a connection such that the same attribute in the object always and invariably corresponds to the same sensation in every subject. Even Bacon, in his firm belief that cellars are colder in summer than in winter, assumed that his sensation of temperature was an unvarying and infallible test of the state of the object: whatever feels cold, is cold, and just as cold as it feels. The contradiction to which this assumption leads, by forcing us to affirm and deny the same thing, brought such judgments of the senses into discredit in very early times. From the very beginning of Greek philosophy sense-perception was, to some extent at least, excluded from the sphere of true knowledge, because of its subjective variability and the extent to which it differed in different individuals. Only since the time of Bacon have men bethought themselves that after all this is the basis upon which the greater part of our knowledge rests, and that all we have to do is to learn the art of using the instrument correctly. But the Logic which followed Plato and Aristotle has perpetuated to the present day the neglect of the conditions upon which these judgments are valid. The concept was thought to be a more than sufficient substitute for an untrustworthy perception, until Kant showed clearly that in mere conceptual knowledge we merely retrace

our steps, and never advance any nearer to the object. But any logic is incomplete which fails to inquire into the conditions of the validity of these judgments, for it is these which both guide us in our formation of concepts and justify its importance.

6. A judgment of perception, then, can lay claim to universal validity only when the sense-affection upon which it depends is the expression of an invariable relation between the assumed object and the subject, when the sensation is the infallible sign of an objective quality. And it can claim this validity only to the extent in which we can assure ourselves that it is invariable, hence that every one has an exactly similar organization and sensibility, or else that variations can be accurately corrected. In our third part we shall have to investigate the means by which we may succeed in finding a basis for our judgments which shall satisfy—at least practically—this necessity for such an absolute similarity or reducibility in our affections as will enable them to be substituted for each other.

7. The idea of the spatial nature of that which is perceived is indissolubly connected with the perceptions of sight and touch; we think of what we see and touch as spatially extended and as having a definite form and size, and we assign to it a definite position in space. Here, again, what we first have is a subjective idea, and the position assigned to things is determined in the first instance by reference to our own bodies as the centres from which our local determinations proceed. It may be questioned how many of our spatial ideas are given from the very first, and are inseparably combined with the sensation itself; that some of our spatial ideas, such as the idea of the corporeal form of objects and their distance from us and from each other, are not originally presented with the sensation, but are the result of combinations which for the most part take place unconsciously, can be indisputably proved.

In the first place, it is clear that if there is to be an objectively valid judgment, not about my idea, but about something which exists in space and has a definite extension and form, there must be absolute certainty that the idea of space is the same for all, and that the spatial interpretations which we give to our sensations are necessarily determined in a given way. Only when this is the case can the judgment concerning objective extension and spatial relations which results from my sensation be objectively necessary, and the spatial object be the same for all. One condition, therefore, of objectively valid judgments of perception is that the idea of space shall be the same for every one, not arbitrary or other-

wise liable to variations, but completely determined, and that every one shall think of space in the same way ; and such judgments cannot attain to certainty unless we recognise these laws which govern the representation of space. It is not merely with the possibility of pure geometry as a science that we are concerned here. In geometry every one has his own space, and different spaces are only congruent, or at least similar ; it makes no difference whereabouts in such a space we draw our lines or construct our figures. Geometrical figures have no position in space. It is different when we assert that something exists in the objective space which is the same for all. When I judge that "this is here," or "that is there," I state something which is to be valid for every one and which determines the position of an object in such a way that the place where it is to be found may be generally known, and its spatial relations be the same for all. Constant practice generally enables us to determine such relations in our immediate surroundings without special discipline ; but this does not make it any the less necessary in strict theory to investigate the conditions and normal laws for an objectively valid determination of the form and place of particular objects. In astronomy we have a proof of how many presuppositions are involved in judgments concerning the position of heavenly bodies which lay claim to objective validity, and how such judgments are really valid only when the presuppositions are acknowledged to be absolutely certain and necessary. But these presuppositions include not merely the universal propositions of geometry, they include also propositions concerning the reference of sensations to a definite place, and these in their turn depend upon optical laws, *e.g.* that the rays of light move in straight lines throughout the same medium, but are refracted upon entering into another. We are not concerned here with the way in which we arrive at our knowledge of these presuppositions ; only so much is clear, that they must ultimately depend upon something which is immediately certain, and which does not derive its necessity from elsewhere, if the particular judgment is to be objectively valid.

8. It is the same with motion. The immediate perception of motion cannot lead to a valid judgment unless we presuppose a local determination which has taken place according to necessary laws. But this is not sufficient. All movement, so far as we can perceive it, is only relative ; that is, a mutual change of position amongst visible objects. But our judgments are meant to bear the objective significance that *A* moves towards *B*, and hence we have need of universal laws according to which

we may interpret relative change of position into actual movement, into change of position in space, and by which we may judge what is to be regarded as in motion and what as at rest. Here again the history of astronomy proves to us that we cannot arrive at objectively valid judgments concerning motion unless we presuppose universal principles according to which the subjective perception of motion is referred to actual motion, and the subjective phenomenon interpreted into an objective event. The difficulty of clearly distinguishing between the concepts of relative and absolute motion is sufficient proof of the labour involved in discovering these ultimate principles.

9. Still more important is the reference of our sensations to definite things. It is true that the form in which we refer the material given by sensation to permanent things is part of the unalterable nature of our thought ; and we could not throw off this psychological constraint even if we would. But just because the thought of a thing is not already included in the affection itself, we can conceive of a difference in the process. This difference, indeed, is hardly apparent when we have to do with motionless and permanent phenomena ; anything which, for our apprehension, is unchanging, which keeps the same position in space, and has fixed limits, is unhesitatingly looked upon as one and the same thing. In the same way it is so universally assumed that two things cannot occupy the same position in space that all our references are founded upon this presupposition ; not even spatial movement, if it is observed as continuous, is by itself sufficient to make us uncertain in our reference.

But wherever we find change of form, magnitude, or sensuous qualities, it becomes a problem as to how the successive stages in the change should be referred to the assumed substance. The necessity appears of generally accepted principles according to which the individual must guide his judgment, if he wishes not merely to state *his* views, but to make objectively valid propositions. When a column of quicksilver expands or contracts, we describe the succession in our perceptions by the proposition "this grows smaller," or "this grows larger." We use the same words to describe the growth of a crystal or the diminution of a piece of ice in the air ; such propositions seem to imply that in both cases it is a definite thing, and, moreover, the *same* thing, which changes its volume. But for the physicist the two propositions have a different meaning. In the first case, he does really consider that it is the same subject which occupies now more, now less, room ; in the second case, the increased crystal or the half-melted ice

is no longer the same thing as before ; something has been added to the original thing or a part has been taken away. For the child water vanishes when it evaporates, and wood when it burns ; for the physicist the same thing remains, it is only in another form. The proposition "this water evaporates" has a quite different meaning as apprehended by the one or the other. Kant has included the principle of the permanence of substance amongst the *a priori* principles of the understanding. This it is not, if we mean that in all our references of sensations to objects this principle guides us by a natural necessity of the understanding ; if it were so, words signifying beginning and ceasing, growth and diminution, could never have arisen and been applied to things in any language. It is only as a condition of consistent experience that a principle is necessary which regulates the manner in which we add substance to accident in thought ; but the principle, as Kant understands it, is impossible until it has been established that weight is to be the measure of the quantity of substance, and this is one of the later results of science. The only truth in Kant's teaching on this point is, that we can have no consistent and necessary judgment concerning the particular unless there is such a principle ; in this sense alone is it originally necessary—it is necessary if there is to be any science of experience. Whether we are to accept it as necessary because *a priori* in the ordinary sense of being independent of all experience and self-evident, or because our experience can only be made thoroughly consistent by means of this principle, is another question.

10. The difficulty of establishing the real identity of the same thing upon the ground of temporally disconnected perceptions is only a special case of the difficulty of applying the concept of the thing as that which is identical with itself in time to particular instances. Here again we need definite rules upon which to base the assertion.

11. Our analysis has already shown that the conditions upon which the validity of any judgment concerning the particular depends are different from, and much more complicated than, those of the validity of merely analytical judgments, based upon a generally accepted system of concepts. It has further shown that the demand for perfectly valid judgments puts an end to the natural immediateness of narrative judgments, and necessitates their mediation in order that they may be true, and that we may be certain of their truth.

Thus from the point of view of the conditions of science, as distinguished from that of the psychological genesis of judgments, Kant was right, after

all, in regarding only conceptual judgments as analytical, all others as synthetical, and in inquiring into the principles of their synthesis *a priori* as the conditions of their objective validity.

12. The necessity of guiding principles is still more evident in judgments of causality than in the inference from phenomenon to substance.

We are so accustomed in our ordinary judgment to apply the idea of efficiency, and we make use of it in simple, every-day matters so entirely without reflection, that judgments which state that a blow has broken a window-pane and that drinking quenches the thirst are formed as if they were immediate; in employing transitive verbs we unconsciously appropriate the idea of efficacy which they contain. The given relation between events is unhesitatingly expressed by verbs and adjectives containing the thought of efficacy, and we therefore believe that we apprehend this efficacy as directly as we do change or movement. But let us reduce the particular verbs which express an effectuation to definite concepts. We shall find that they contain some elements which are of a nature to be intuitively perceived—a movement of the one thing and a subsequent change of the other. But in addition to these, there is an element which is not intuitable, that is, the causal-relation itself, which implies that the subsequent change is really produced by the other, that it does not proceed from the subject in which it takes place, but is forced upon it by the cause. The objective validity of our apprehension of events which we can perceive is to be estimated according to principles already laid down. For the statement that a part of them is caused by something else we need a further fundamental principle, according to which a perceptible event may be known, with an objectively valid and necessary knowledge, to be an instance of causality. Only in this way can a causal judgment concerning the particular attain to objective validity. For here, even more than with the concept of substance, it is obvious that while it is a necessary result of human nature and of the laws according to which our thought develops, that events are causally connected by us, and that we cannot get rid of the need to regard the one as the consequence of the other; yet this does not exclude widely different applications of the same general principle, and wide differences in our way of referring the particular to causal connections. It was the natural desire for causality which drove men to seek the causes of events in the power of spirits or in the position of the stars; but no such proposition has objective validity unless there is a fixed and necessary rule, according to which we may refer events to

causes, and discover what is the cause of a given event ; indeed, it is only on this condition that any causal connection amongst particular events can be asserted with validity. It was such a rule as this that Kant sought in his *a priori* principle. He looked for a condition of scientific experience and of objectively valid causal judgments which should necessitate a particular manner of connecting the subjectively given manifold, and a particular interpretation of that which is empirically co-presented, thus transforming (to use Kant's distinction) a judgment of perception into a judgment of experience.¹ Here again he believes that in the synthetical principle *a priori* that everything which happens presupposes something upon which it follows according to rule, he has set forth this ultimate condition of objective judgments, and that by showing it to be *a priori* he has given the ground of its necessity. But here again it may be asked whether the fundamental principle in this form is really to be acknowledged as necessary and *a priori*, or whether it is merely accepted as necessary because it is the only hypothesis which enables us to shape our experience consistently. Moreover, it may even be questioned whether in this form it is sufficient, or, indeed, at all fitted, to serve as a basis for the objectivity of our causal judgments. So much, however, is certain, that only in proportion as our reference of perceptions to causal relations takes place in accordance with fixed rules can we assert in any particular instance that a phenomenon, *B*, is the effect of another, *A*, and from this it follows that every particular causal judgment must have its ground in a reference to the general principle ; that is, it must be inferred and synthetical. If we consider how difficult it often is to decide what is the cause of a given event, we shall be the more ready to agree with the statement that there is absolutely no causal judgment, the necessity of which is immediately certain.

§ 48.

The final and highest general rules—besides the Principle of Agreement—upon which all other propositions are grounded, are either axioms of

¹ His well-known example (*Prol.*, § 20) is the judgment "when the sun shines on the stone it becomes warm." This is a mere judgment of perception, and however often I may have perceived it or others may have perceived it, it contains no necessity ; it merely happens that the perceptions are generally connected in this way. But when I say "the sun warms the stone," the concept of cause has been added from the understanding to the perception, and this establishes a necessary connection between the concept of sunshine and that of warmth ; the synthetical judgment becomes necessarily and universally valid, hence it is objective, and passes from perception to experience.

the formation of concepts, or postulates concerning the existent. The assumptions made upon the ground of these postulates are ultimately regulated by the law of contradiction.

1. From what has been said it will at least be evident that the purely empirical view, which regards the particular facts of experience taken as objective statements as that which is immediately certain and upon which all other propositions are founded, can never be the ground for a science consisting of universally valid propositions. Since the facts of perception belong to the individual, each man's statements about them are originally valid for him alone. Nor could we get further than this subjective validity unless there were rules according to which a universally valid proposition may be inferred from the subjective fact. Any view which maintains that the facts of perception, as ordinarily understood, are the final certainty, must either lead to the scepticism of Hume, which will not allow us to pass from subjective impressions to any statement concerning "being"; or else—if we are allowed to proceed so far as to state that something is—to the proposition of Protagoras, that what appears to any one *is* for him. In either case there is no possibility of a truth which is valid for all. It is true that particular empirical theories, such as that of Mill, do aim at erecting a science upon this foundation. But such theories always surreptitiously assume propositions having universal validity; sometimes assuming as a matter of course that judgments of perception are the same for every one, and state objective being as well as guarantee real knowledge; sometimes setting forth our inferences from these judgments of perception as self-evident, although without some universally valid assumption there is no manner of justification for them.¹

2. In opposition to this view, I hope that I have shown that perception cannot be a ground for necessary and universally-valid judgment concerning the existent, unless the necessity of our particular judgments is based upon universal principles. Ultimately these principles must somehow be immediately certain; and they cannot derive their certainty from an experience which, in the form of true judgments, is impossible without them. Thus the question arises whether there are any immediately certain propositions of this nature. Such propositions would state the necessity of the processes by which, from the fundamental subjective facts of immediate sensation, we obtain the idea of a world of particular things existing in

¹ We reserve a detailed examination of Mill's theory until we come to investigate the process of induction.

space and time, and the reality of their attributes and actions as of their manifold relations. Their general formula would be : given the conditions of individual perception, to obtain necessary statements concerning the existence of objects, and given statements concerning the existence of these objects to obtain from them other necessary statements.

If, from the fact that I have definite spatial intuitions, it could be inferred in accordance with these principles that a space, such as I imagine it, exists objectively ; if it followed that, because I have a sensation of light at a particular point of this space, there exists a bright object at this point, according to the principle that any felt quality requires a substance in which it is inherent ; if, from the fact that a thing is or changes, it could be inferred that another thing is or changes, the necessity of such propositions being as obvious as the principle of contradiction—then there would be no difficulty in finding a ground even for judgments of perception. For the subjective fact that I have this or that idea must be recognised as immediately certain, and this would be the empirical *datum* from which our judgments concerning the existent would necessarily follow in accordance with such laws.

These propositions would have to be certain *a priori*, in the sense that they would be merely the conscious expression of an invariable and inevitable function of our thought, and accompanied by the conviction that so surely as we are ourselves we are obliged to judge in such a way. They would not proceed from the contents of the object of thought as expressed in the concept, but would ascribe to these contents a predicate, derived not from themselves, but from the particular manner in which they happen to be presented, from the specific character of the perception. To this extent synthetical judgments might be grounded upon them.

Here we may see how from this point of view also the Kantian question, "How are synthetical judgments *a priori* possible?" is of fundamental importance. For it is evident that upon the answer to this question depends the possibility of passing from individual thought as it constantly recurs to universally valid propositions, and from the subjective formation of ideas to judgments concerning the existent.

3. That there are such propositions is recognised whenever it is taught that there are AXIOMS upon which our knowledge of the existent depends. In accordance with Aristotle's teaching,¹ axioms have been distinguished

¹ Treating of those elements of our knowledge for which no further ground can be given, Aristotle distinguishes in the *Anal. post.*, i. 2 and 10, between ἀξιώματα (ἀρχὴ ἢ ἀνάγκη

from definitions and the analytical judgments following from these, on the one hand, and from postulates on the other. When thus distinguished they are looked upon as propositions having a truth and certainty which is immediately evident. Hence their contradictory cannot possibly be thought, and as they are not mere explications of concepts they form the ultimate presuppositions upon which all knowledge must be grounded. Moreover, the name Axiom is not given to particular judgments of immediate certainty—for instance, to the utterances of immediate self-consciousness—but only to universal propositions which express a necessity of extensive application. Aristotle, for instance, in addition to the absolutely first and most universal axiom—the principle of contradiction—recognised special axioms for each department of knowledge, *e.g.* mathematical axioms. Postulates, on the other hand, are propositions which neither admit of further proof or derivation, nor can be accepted as immediately and necessarily certain; their certainty however is assumed, not upon the ground of logical necessity, but upon psychological reasons of a more general kind.

An inquiry as to whether all the propositions which have at various times been employed as axioms have really merited this title could only be carried out by an investigation into the special departments of knowledge which is beyond the scope of general logic. Nevertheless our previous investigations are a sufficient ground upon which to base an important distinction with reference to the import of such propositions, one which has not generally been recognised, though it was indicated by Kant.¹ We mean the distinction between axioms referring to the for-

ἔχειν τὸν ὁτιοῦν μαθησόμενον—δ' ἀνάγκη εἶναι δι' αὐτὸ καὶ δοκεῖν ἀνάγκη), and θέσις (ἢν μὴ ἔστι δεῖξαι, μὴδ' ἀνάγκη ἔχειν τὸν μαθησόμενον τι). He again divides the θέσις into ὑπόθεσις, which states that something is or is not, and ὁρισμὸς, which gives the "what" only, not the "that." While a ὑπόθεσις which contradicts the presuppositions of the learner is αἰρημα.

The last term has never attained to any fixed meaning. The modern use of the term postulate has been determined—though again not absolutely—by Kant, when in the *Kritik d. r. V.* he refers to the mathematical use of the term: "the practical proposition which contains nothing but the synthesis whereby an object is presented to us, and its concept created, is a postulate." Accordingly he calls the principles of modality postulates because they point out the manner in which the concept of things is connected with our faculty of knowledge. But in the *Kritik der praktischen Vernunft* a postulate is a theoretical proposition—though it is not demonstrable as such—inasmuch as it is inseparably dependent upon an *a priori* practical law of unconditional validity. We find the same discrepancy in Kant's *Logic*. I have given a wider meaning to the second definition in the text.

¹ In his distinction between the mathematical and dynamical use of the synthesis of the pure concepts of the understanding.

mation of concepts, and axioms referring to the knowledge of a particular existing thing.

We have laid it down that the possibility of a logically perfect formation of our concepts depends upon our being able to state necessary laws according to which all our ideas are formed. It is certain that logically perfect concepts are no natural product, but have to be obtained by a conscious synthesis; and it is as certain that this synthesis must be governed by rules which are obviously necessary, but which constitute primarily a ground only for the form of our concepts and the mutual reference of their elements, not for the assertion that a particular thing exists. Thus the proposition that we cannot think of any attribute as real without presupposing a thing to which it attaches is a rule which determines the construction of our ideas and the relation between their elements.

All propositions, again, concerning the incompatibility of certain characteristics are axioms for the construction of concepts. It is fixed by the nature of our thought itself that certain determinations cannot be combined in one idea, and this impossibility can be certain for us only in the same way as the principle of agreement is certain. (Essentially different from these are propositions concerning incompatibility which are only empirically inferred; as, for instance, that a gaseous state is incompatible with great specific weight.)

Mathematical axioms are to be classed with these axioms for the construction of concepts (when, that is, they are not merely analytical propositions, such as the principle that two magnitudes which are both equal to a third are equal to one another, which follows analytically from the concept of equality). For inasmuch as all geometrical figures presuppose space and are governed by the nature of our idea of space, such axioms express nothing but the manner of synthesis which is necessitated by our idea of space. It is upon the inevitable rules of this idea that the axiom "two straight lines cannot enclose a space" is based.

From one point of view all these axioms may be treated as analytical propositions; when, that is, we look to the fact that though not derived from the concepts of the grammatical subjects, they are involved in the nature of the ideas presupposed in these subjects (§ 18, 5, p. 110 sq.). That they appear to be synthetical is due only to their being relational judgments, which must have been preceded by a synthesis in the ideas without which there would have been no relation. They are based upon the

fact that the different elements of our ideas are not independent of one another.

Such axioms there are also in reference to all we think of as existent ; they hold good, however, only when it is the concept of being which is in question, and not with reference to any assertion that this or that particular thing is. Spinoza's axiom—*omnia quæ sunt, vel in se vel in alio sunt*—is one of these : and it refers to the fact that nothing but substance with accident can be thought of as existing.

But these axioms are not meant to serve as grounds for the judgment that this or that particular thing is. The one last quoted, for instance, leaves it quite undetermined to what we are to apply the concept of being in itself, and being in something else (independent and dependent being). Other axioms, however, are needed for our judgments concerning the particular existent ; axioms upon which we may base the assertion that a certain particular thing must be thought to exist because we form an idea of it in a certain way, or because another particular thing either is or has been. And it is just this which constitutes their characteristic difference from the former axioms. The axiom of causality, for instance, as formulated in the law of inertia, says nothing about the necessary idea of motion ; what it says is that if a given body actually moves at one moment, it will continue to move in the next in the same direction and with the same velocity, and that if it changes its motion another body is there which has influenced it. Thus the general formula of such axioms is sometimes "if I perceive a particular thing under given conditions, it exists" ; sometimes "if some particular thing is, then some other is also." In this way they regulate the process, by which my ideas of the particular are interpreted as reality.

We need only attend to what always takes place in the formation of our ideas to become conscious of the necessity of the former axioms. The necessity of the latter, just because they refer to the existent, cannot be derived from the necessity of thought so simply, except in so far as we assume as our highest axiom that thought and being correspond.

4. The history of science proves irresistibly how vain is the belief that we can ground the statement that a given thing is, and is this or that, upon simple and immediate axioms, or that the universe of the particular can be derived from them as necessary consequence. Neither the principle "*non datur vacuum*" nor the axiom that "a thing can only take effect where it is," neither the assertion that "like only effects like," nor the statement that "the effect does not continue without the cause," have been of any

avail, and the criterion of inability to think otherwise has been constantly misconstrued as the psychological impossibility due to habit, instead of logical necessity.¹

Even Kant's splendid attempt to exhibit the synthetical judgments *a priori*, which are the ground of all experience, has really only resulted in shewing that such synthetical judgments *a priori* must hold good if experience is to be possible as science. He starts by assuming that there is a science of experience, and working back from this he seeks the conditions of it, guided by the principle that all cognitions must admit of being united in one consciousness. But neither his derivation of the categories from the forms of judgment given in the traditional Logic and supplemented by himself, nor yet the synthetical principles obtained from this basis and their proofs, have succeeded in convincing his readers that they have here propositions which are absolutely necessary and self-evident, whose contradictory cannot possibly be thought, and which are contained *a priori* in our understanding. On the other hand, again, the proof that our sensations as they occur must necessarily submit themselves to the categories and *a priori* principles leaves much room for question.

Schopenhauer abandoned the extensive fortress of the twelve categories in order that he might maintain more strongly the citadel of causality ; but although his simplification of Kant is very instructive, it cannot take the place of the Kantian pure forms of the understanding and synthetical principles *a priori*. Even if he aimed at nothing more than psychological explanation of the process by which every one is forced to objectify his spatial intuitions and think of them as external, the principle of causality is insufficient. We may, indeed, infer from it that I must assume something other than myself as the cause of my sense affections, since I am not conscious of having produced them myself. But it does not follow either that this cause is necessarily in space, nor that it consists in the object of intuition itself considered as an existing thing. No doubt scientific reflection upon our sense perceptions, which begins by assuming that they are occasioned by external objects, finds itself confirmed in this assumption by the fact that it is thus enabled to explain our sensations, and it is for this reason that Schopenhauer's theory has been approved by Helmholtz. But it is after all convincing only after we have already tacitly presupposed the existence of objects, the assumption of which it was intended to explain. When once it is evident that the general principle of causality contains no

¹ Cf. Mill's *Logic*, bk. 2, ch. 7, and bk. 5, ch. 3.

information as to the nature of the cause of a given effect, we see the impossibility of inferring from it the existence of any definite cause.

There are still more serious defects in this principle when regarded as a principle of objective truth. Even if we allow that it holds good as a universal and self-evident axiom, it cannot be the ground of our inference of external objects unless the proposition "I am not aware of having myself caused my affections," is a proof that I am really not their cause. Thus its application involves the axiom that I am not the cause of anything which I do not produce consciously; an axiom which no one will assert to be valid *a priori*. Nor could it be a principle of objective truth unless it were to guarantee that everything thus objectified by the individual were *eo facto* valid. It may be a natural law of our thought; but the conditions under which it may become a normal law are still to be discovered.¹

Thus the principle of causality also fails to enable us to assert that this or that particular thing necessarily corresponds to my perception, and is like my idea of it; for by itself it tells us nothing of the nature of the cause.

It cannot be allowed, then, that the general propositions which guarantee the objective validity of our judgments of perception, are obvious as simple, self-evident truths; nor that we find them in a form which, by itself, makes the reference of our perceptions to an existing thing, and of certain perceptions to a certain thing, *a priori* certain. But it still remains open to us to acknowledge the existence of an external world which is the same for all, as a postulate of our search for science and knowledge which we cannot avoid believing, although we recognise that it is not self-evident.²

¹ I agree with Windelband (p. 76 *loc. cit.*) that it is a first principle of knowledge—more accurately of our attainment of knowledge—which makes us seek a cause for every phenomenon; but then this law is not sufficient to show the sufficient ground for every phenomenon. Such a law, however, would be necessary as a ground for our judgments of perception. Cf. the criticism of this theory of causality in the work of Spir, *Denken und Wirklichkeit*, p. 121 sq.

² Baumann's proof of realism (*Philosophie als Orientierung über die Welt*, p. 248 sq.) comes to much the same as this, so far as I can see.

Zeller's investigation "über die Gründe unseres Glaubens an die Realität der Aussenwelt" (*Vorträge und Abhandlungen, dritte Sammlung*, p. 225 sq.), which is a model of clearness and caution, leads to practically the same result, although he attempts to dispose of the argument that I myself am the only real being which exists by a refutation to which he attributes the importance of a proof. The proof, however, is only derived by him from the fact that such an assumption would render the contents of consciousness inexplicable, and thus it presupposes the necessity not only of explanation but of causal explanation. But this necessity is in the first instance subjective, a necessity imposed by our desire to know. That the conviction of the reality of an external world is justified by the necessity

This postulate being granted, the question arises: What are the universal conditions required by the nature of our perceptions, if they are to be referred to an external existence, and if the judgments resulting from them are to be made thoroughly consistent? To discover these presuppositions is, then, the aim, and not the starting point of science. Nevertheless for our guiding clue in this discovery we must fall back upon a principle whose likeness to the principle of contradiction may impose upon us, but which is really only a particular application of that principle; *i.e.* it is impossible that the same thing should both be and not be, that it should both be *A* and not be *A*. The principle of contradiction as a natural law of our thought states that it is impossible that we should consciously both affirm and deny the same proposition. If we assume that there is a definitely fixed conceptual system which is present without variation to an ideal consciousness, and the same for all thinking beings, then all conceptual judgments are established by the principle of agreement. Hence it follows also from the principle of contradiction that all judgments contradicting these are false, whether they are direct negations, or judgments predicating incompatible characteristics. When this is the sense in which "the same cannot be both *B* and not *B*," we mean by the same, the same concept, the definite contents of my idea.

But when we are forming judgments about the existent, this principle prohibits us in the first place from thinking that the same thing both is and is not. If then we can infer from the assumptions we have made about the existent one conclusion, that a particular object of thought is, and another conclusion that the same particular object of thought is not, the two propositions cannot both be true, and there must be something false in our assumptions. And in the same way it is impossible to think that the same particular *A* both is *B* and is not *B*.

Since it is part of the concept of being, that it is the same for all who think, so that true judgments about the same thing must agree, whoever of explanation and understanding is true only as a postulate. We cannot accept it as an axiom that nothing incomprehensible can exist; indeed a complete understanding of all that is presented to us can never be more than a problem set before us which we shall never completely solve. Though, therefore, I completely agree with him that the theory which regards our ideas as only the products of the conscious subject is untenable, still I regard the final presupposition upon which is based our conviction of the reality of an external world as nothing more than a postulate. This however is quite sufficient to enable me to accept the statement of the problem as formulated on p. 263: the causes of those phenomena of consciousness which we call perceptions must be determined in a way which will correspond to the facts.

forms them, it follows that contradictory judgments, even when inferred by different people on the ground of their perceptions, cannot be true of one and the same existing thing. It is true that this argument is ultimately based upon our concept of being, and that we cannot go beyond this. But there is no other scientific knowledge than the knowledge which tells us that if we would think of anything as existing, we must necessarily think of it in a given way. If we could suppose it possible for contradiction to be repugnant to our thought only, and to be admissible in the existent, all our efforts to know the existent would be rendered fruitless.

In the third part, I hope to show how the nature of the problems to be solved, and the conditions under which we attain our knowledge, have necessarily given rise to that process in our experiential knowledge which is exhibited by the history of the actual development of science. I shall show, that is, that the whole task has consisted in affirming an existent in accordance with the postulate that something is, and upon the ground of our perception, and in making our hypotheses about it in such a way that our statements concerning it may be free from contradiction. The history of science shows us a continual process of re-construction and correction in our ideas of the existent, a process which enters upon a new stage whenever our hypotheses lead to contradictions. There is no other confirmation of our belief that any given thing is, than the complete consistency of all our judgments with reference to the existent; the return of the circle into itself. All the general propositions which we accept with respect to the existent must finally be such that the immediately certain, the subjective fact of perception, which was the starting point of the whole process, may in its turn be a necessary consequence of them. In this way the immediate assumption from which we always set out, that sensible qualities are immediate attributes of the existent, has been rectified; by accepting it we were led to contradictory conclusions. In this way the axioms of physics, the principle of the permanence of substance, etc., have been discovered. Kant, in his Antinomies, chose this same way to show that space and time are only subjective forms of intuition, and that everything therein is only phenomenal; to assume that they are real in the ordinary sense leads in his opinion to contradiction.

The principle of causality, at any rate in the form in which alone it can be applied, enters into this process of building up knowledge from experience. It enters, that is, as the postulate that the existent is knowable as necessary, *i.e.* determined according to universally valid laws. For not even the

firmest conviction that everything has its cause could enable us to assert confidently the existence of a particular thing, if causes worked by chance.¹

¹ In order to avoid repetition I postpone a complete exposition of the principle of causality to Part III.

CHAPTER III.

THE RULES OF INFERENCE AS THE GROUND FOR MEDIATED JUDGMENTS.

It has been shown in the previous section that judgments which we were obliged to regard as immediate from the point of view of natural thought, must nevertheless admit of being presented as necessary consequences of a general law when we have to ask for the ground of their certainty. Analytical judgments must be shown to follow from the principle of agreement ; judgments of perception from the laws, according to which we derive from our subjective affections the conviction that there are material things. And since these general laws can be known only in the form of judgments, the contents of the previous chapter are to a large extent dealt with under this one ; only the highest and ultimate laws, together with the immediate utterances of self-consciousness, are excluded, as incapable of being reduced to more remote principles.¹

§ 49.

The most general formula according to which we derive one judgment from another is the **HYPOTHETICAL SYLLOGISM**. This may be either the simple application of the proposition that the affirmation of the ground involves that of the consequence, the denial of the consequence that of the ground (in which case it is called the Mixed Hypothetical Syllogism) ; or (as the so-called Pure Hypothetical Syllogism) it may be based upon the principle that the consequence of the consequence, is the consequence of the ground.

1. The process of drawing a conclusion or inference takes place psychologically whenever we are led to believe in the truth of a judgment by our belief in the truth of one or more other judgments, and not immediately by the ideas of the subject and predicate connected in it. There are many ways in which this belief may be brought about psychologically (§ 19, 3-4, p. 114 sq.) ; and it often happens that we are not even clearly conscious of

¹ For some appropriate remarks on the relation between judgment and inference, cf. Schuppe's *Erk. Logik*, p. 124 sq.

the process of mediation by which we derive the certainty of one judgment from that of another. This mediation depends frequently upon habits of association and combination which are governed, as a matter of fact, by fixed rules of which we are not expressly conscious. Every expectation of a future event is based upon an inference which extends beyond what is given ; but the ground of our expectation, which is to be found in previous experiences, is not expressly present in consciousness in the form of a universal proposition every time we expect that an unsupported body will fall to the ground, that eating will satisfy hunger, or that what we say will be understood of those who hear. We pass without conscious mediation from the certainty of the given event to the certainty that a future one will also take place.

But logical theory has to enquire as to what are the conditions under which inference is valid. That is to say, since every inference involves the belief that one judgment (the conclusion or inferred proposition) is true, because one or more other judgments (the premises) are true, we have to investigate the logical necessity of this belief that the premises are a sufficient ground for the conclusion.

2. The question as to how one judgment may be grounded upon another, may be regarded from two points of view. We may either start from a given judgment which is accepted as valid, and ask what further judgment can be grounded upon this ; or we may start from a question, from the tentative suggestion of a mediated judgment, and ask "how can this judgment be grounded, and under what conditions? What must be certain in order that it may be valid?"

3. It is evident that, given a valid judgment A , another and a different judgment X cannot be safely grounded upon it unless there is an unconditional and universally valid proposition "If A is true, X is true." All that is expressed by this hypothetical judgment is, that X is the necessary consequence of A , and that any one who accepts A must also accept X . Without such a rule there would be no logical justification for an inference ; if A might be true without X being true, the certainty of the former would be no ground for the certainty of the latter. Thus the objective validity of an inference from A to X must always be dependent upon the validity of this hypothetical rule.

Hence the most general, logical scheme for all inference is the so-called mixed hypothetical syllogism :¹

¹ Cf. Kant's *Logik* (Hartenst., I, p. 453 § 57) : "The general principle upon which

A is true.	If A is true, then X is true.
If A is true, then X is true ;	A is true ;
Therefore X is true.	Therefore X is true.

The order of the premises depends upon what the movement of thought happens to be. If the validity of the judgment A represents the matter-of-fact element in the ground, the datum from which the inference is made, while the hypothetical judgment is the law which contains the necessity, the rule according to which the inference is made, then either the one or the other may come first in the actual course of thought. But in logical terminology the rule according to which we infer is always called the major premise (G. Obersatz), the datum from which we infer the minor premise (G. Untersatz) or assumption.

4. If A presents itself first, the question arises "is there a judgment which tells us that if A is true X is also true?" But if, on the contrary, it is the rule which is first present, the question is whether we can apply the rule: "Is A true, and X because of A ?"

In the latter case the rule is capable of a two-fold application. It may be applied if A is true, *i.e.*, is recognised as certain; but it may also be applied if X is not true, according to the law that the denial of the consequence involves the denial of the ground.

In this way there arises the inference:

If A is true, X is true.
X is not true ;
Therefore A is not true.

5. All the ways in which we can make an inference of a simple statement must admit of being reduced to one of these two forms, which are generally known as the *modus ponens* and *modus tollens* of the mixed hypothetical syllogism,¹ since by inference in this sense we cannot understand anything else but that one judgment necessarily follows from the other.

We may, then, lay it down as certain that the validity of a judgment can depends the validity of all inference made by reason may be definitely expressed in the following formula: that which falls under the condition of a rule falls under the rule itself." This proposition contains just that view of the nature of inference which is given in what follows.

¹ The so-called *modus tollens* may itself be reduced to the *modus ponens*, for we can always infer from the judgment "if A is true, then X is true," the other judgment "if X is not true, A is not true."

never be inferred from one single judgment, but that two premises at least are always essential.

One judgment can be inferred from others only upon condition that one of the premises is an unconditionally valid judgment, the expression of a necessary connection.

It is this premise which really carries us from one certainty to the other, and the ground for our progress is the law that the affirmation of the (hypothetical) ground involves that of the consequence, the denial of the consequence that of the ground.¹

6. The hypothetical judgment which mediates a conclusion, may itself be mediated and inferred: indeed the proposition that X is a necessary consequence of A is known to be necessary if X is a consequence of a consequence of A . If then it is true that

If A is true, M is true;
 If M is true, X is true; then it follows that
 If A is true, X is true.

The principle upon which this syllogism is grounded follows from the concept of consequence; it may be formulated "the consequence of the consequence is the consequence of the ground."²

¹ Another proof that this form of hypothetical inference is the natural and general formula for all inference is its universal appearance in the phrases which we generally employ to state our conclusions. Such conjunctive forms as "since—because—hence—for," etc., are nothing more than verbal abbreviations of the schema, for these particles have a double import; they express the validity of both propositions, antecedent as well as consequent, and they also express the necessity of the connection between them, thus pointing to a hypothetical judgment.

² The principle that denial of the consequence involves denial of the ground may be applied in two ways:

I. If A is true, B is true.
 If C is true, B is not true.
 therefore If A is true, C is not true.
 and If C is true, A is not true.

i.e. if two premises have contradictory consequences, each involves the denial of the other.

II. If A is true, B is true.
 If A is not true, C is true.
 therefore If C is not true, B is true.
 and If B is not true, C is true.

i.e. the consequence of an affirmation and the consequence of its denial are mutually exclusive. Both these formulæ, however, may be reduced to the two given above. Instead of the minor premise in I. we may substitute:

If B is true, C is not true;

This is the so-called pure hypothetical syllogism ; here again we see the necessity of having at least two premises. But what we have said of two members holds good of an unlimited number : the affirmation of the ground involves that of every consequence of its consequence, and thus we are enabled to obtain a complete series of inferences connecting the first ground with the last consequence. This is the hypothetical sorites, and its premises may be arranged in two ways :

- | | |
|---|--|
| I. If <i>A</i> is true, <i>B</i> is true.
If <i>B</i> is true, <i>C</i> is true.
If <i>C</i> is true, <i>D</i> is true.
If <i>A</i> is true, <i>D</i> is true. | II. If <i>C</i> is true, <i>D</i> is true.
If <i>B</i> is true, <i>C</i> is true.
If <i>A</i> is true, <i>B</i> is true.
If <i>A</i> is true, <i>D</i> is true. |
|---|--|

In the first case the premises proceed from one consequence to another still further removed from the original ground (epi-syllogistically) ; in the second case they proceed backwards to anterior grounds (prosyllogistically).

§ 50.

In the mixed hypothetical syllogism the hypothetical rule only enables us to derive one given judgment from one other given judgment. But if the consequence follows merely from the attribution of a given predicate to any subject, then the hypothetical rule may be applied to an indefinite number of judgments. In this case the minor premise serves for the introduction (*πρόσληψις*) of a definite subject, and so leads to the conclusion.

1. The logical theory of inference would be now complete if our only object were to express in a general formula the essential conditions to be fulfilled by all inference in deriving the validity of one judgment from the validity of another.

But this formula of the hypothetical syllogism is deficient in a way which seriously impairs its value. If we could make no inferences except in accordance with it, we should need a special rule every time we de-

and then we get,

- If *A* is true, *B* is true.
If *B* is true, *C* is not true.
If *A* is true, *C* is not true.

that is a simple progress from consequence to consequence. In the same way in II. we may substitute for our major premise :

- If *B* is not true, *A* is not true.
If *A* is not true, *C* is true.
If *B* is not true, *C* is true.

rived one simple judgment from another ; so that we should have as many rules as we had instances of their application, while for the derivation of a hypothetical judgment two others would be necessary. Moreover, before any inference could be made, everything which enables us to proceed from one judgment to another must have been already fully thought out ; so that any real progress, any truly synthetical judgment, would be impossible. Progress to new judgments—which alone in the process of thought is of any value—is always presupposed as having already taken place by the hypothetical syllogism in its simplest form as given above. For the knowledge we seek is just this necessary dependence of one judgment upon another.

2. Any further development of the theory of inference must then start from the question “upon what is this necessity of connexion between *A* and *X* based?” and from the further question as to whether there is no other means of attaining to a hypothetical judgment than by way of the pure hypothetical syllogism, which always presupposes other hypothetical judgments ; whether, that is, we may regard all particular connexions of this sort as ultimate and unanalysable, or whether it is possible to derive the necessity from a smaller number of laws.

In many cases, no doubt, a connexion of this kind, which obtains between a definite antecedent and a definite consequent, and is expressed by a hypothetical judgment is ultimate ; and the necessary sequence is given immediately. Whenever I resolve upon a definite course of action *in the event of certain circumstances*, or make a promise under certain conditions, or enter into some contract, I give rise to a hypothetical judgment which owes its validity to my will ; and the fulfilment of the intention, promise, or contract depends upon the simple, hypothetical syllogism “if *A* is, *B* ought to be, *A* is, therefore *B* ought to be. The connexion is established by my will, and owes its validity to my actual volition. The necessity grounded thereon admits of no further analysis ; the dependence of the one judgment upon the other is determined directly (cf. Appendix B, p. 383).

3. But the law according to which *X* follows from *A* is not necessarily the judgment “if *A* is true, *X* is true.” Spinoza makes the following inference (*Eth.*, I, 11): if anything exists, then there also exists an absolutely infinite Being ; now I certainly exist, therefore an absolutely infinite Being—*i.e.* God—exists. Expressed in a general form : the judgment *C* is *D* (God exists), follows from the judgment *A* is *B* (I exist) not only when we know that if *A* is *B*, *C* is *D*, but also when we know that if any-

thing is *B*, *C* is *D*. That is, the inference holds good if the inferred judgment necessarily follows whenever the predicate belongs to any subject; when it is the consequence not merely of predication concerning a given subject, but of predication concerning any subject whatever with this predicate.

4. Such a law by virtue of its generality covers an unlimited number of particular cases, and it owes its generality to the fact that the consequence depends only upon the predicate, not upon the particular subject to which this predicate may be ascribed.

Thus besides the inference expressed by the hypothetical syllogism we find here a definite subject introduced in place of the indefinite substratum of the predicate, the process called by the Aristotelians *πρόσληψις*.¹ Because the same predicate may be assigned to an indefinite number of particular subjects, the consequence holds good for every particular judgment in which this assignment actually takes place. And as we have seen (§ 31, 8, p. 187, and § 33, 2, p. 197), this is the only form in which the necessity is recognisable as such.

5. The hypothetical judgment may be one which makes one predicate depend upon another predicate of the same subject: if anything is *A*, the same thing is also *B*. Then it would not only be applicable to a plurality of data having the same consequence, but would also include a number of consequences; the introduction of the definite subject would take place both in the antecedent and in the consequent.

$$\begin{array}{c} \text{If anything is } A, \text{ it is } B \\ C \text{ is } A \\ \hline \text{Therefore } C \text{ is } B \end{array}$$

Here we have no longer a simple, hypothetical syllogism. It is mediated by the fact that in the minor premise a definite subject is named to which the predication applies, while at first only the general possibility of a subject was assumed. The hypothetical judgment includes in its formula the several judgments: if *C* is *A*, then *C* is *B*; if *D* is *A*, then *D* is *B*, etc., thus necessitating an indefinite number of particular consequences.

¹ In the inference:

$$\begin{array}{c} \text{καθ' οὗτὸν } B \text{ κατὰ τοῦτον } A \\ B \text{ κατὰ τοῦ } \Gamma \\ A \text{ κατὰ τοῦ } \Gamma \end{array}$$

the minor premise is the *πρόσληψις*. Cf. Prantl, I, 376 sq., and my *Programm*, p. 8.

There is added to the necessity expressed by the major proposition the property of being universally applicable : the rule has become a law.

6. The same is true of hypothetical judgments which attach consequences not to simple predications, but to connections between relations, and thus become more complicated in their expression. The proposition that if two magnitudes are equal to a third they are equal to one another, states a connexion between relations which holds good for any objects whatever which come under these relations. When I infer therefrom that because $A=B$ and $C=B$, therefore $A=C$, I have again introduced into the general formula the definite magnitudes A, B, C , to which the relational-predicate of equality applies. The minor premise does not say that any two magnitudes are equal to a third, but that these definite magnitudes A and C are equal to the third B . In this case the introduction must take place by means of a number of particular judgments, which must be taken together to allow of the application of the antecedent.

7. The proposition that one of the premises must state a necessary connexion seems to be contradicted by many instances from ordinary, and even from scientific, practice.¹ I infer, A is the father of B , B the father of C , therefore A is the grandfather of C ; that Breslau is in Silesia, Silesia in Prussia, therefore Breslau is in Prussia; that $A=B$, $B=C$, therefore $A=C$; that $A>B$, $B>C$, therefore $A>C$; that A is to the right of B , B to the right of C , therefore A is to the right of C , and so on. But though these premises seem by themselves to be a sufficient ground for the conclusion, and there appears to be no universal major premise, it is not really so. No doubt we make our inferences in such cases with the greatest confidence, without being conscious of a universal major premise, or without expressly formulating it; in the last example it would be the proposition, "when A is to the right of B , and B to the right of C , then A is necessarily to the right of C ." But would the syllogism be valid unless either this proposition were true, or the still more general one that whatever lies to the right of a second thing which is to the right of a third, lies also to the right of this third? It is true that because the relations of space or of magnitudes are so immediately intuitable, and because they are so incessantly present to thought, we are saved the necessity of always formulating in words the laws by which they are governed. Nevertheless nothing but the validity of the necessary connexion between the

¹ Cf. F. H. Bradley, in his *Principles of Logic*, London, 1883, p. 227; a work which is eminently original and very instructive in its frequently excellent criticisms.

various relations can support the conclusion ; in mathematics, indeed, the fundamental principle that two magnitudes which are equal to a third are equal to each other, is expressly put before everything else. Logical investigation is not concerned with that which is expressly thought and consciously emphasized in the actual process of inference ; but only with that which must be true if a syllogism is to be valid, if the conclusion is to be a necessary consequence of the premises. The propositions $A=B$, $B=C$, do not really constitute the only premises of the syllogism ; they merely contain the *assumptio*, which here consists of two propositions. We cannot pass from them to the conclusion $A=C$ unless we can see that if $A=B$, and $B=C$, it is necessary also that $A=C$.

But these examples show the importance of that class of connexions which we have just noticed (6), and the frequency of inferences from major premises which state that the two relations in which one object stands to two others necessitate a third between these two others. Many of the propositions upon which all inference is ultimately based will for this reason take the form of hypothetical judgments with antecedents which have two members because they contain a twofold relation. To this class of inference belongs also the principle according to which we infer identity. Identity itself is nothing but a relation between objects of thought. The premises A is identical with B , B is identical with C , yield the conclusion A is identical with C only because the third identity follows from the two first by virtue of the concept of identity ; because, therefore, the law holds good that two objects of thought which are identical with a third are identical with each other.

In the same way when we substitute an equivalent expression for the subject or predicate in any judgment—whether it be a different manner of denoting one and the same individual, or a different expression for a concept—we are only justified in so doing by our knowledge that the same predicate must be affirmed or denied of the same subjects.¹

8. The psychical operation actually taking place in such a process of inference presents different aspects, which have led to different views of the syllogism. On the one hand, it is pointed out that what really takes place in inference is a synthesis of different elements, and that the con-

¹ In such examples as : Aristotle was the philosopher of Stagira.

Aristotle was the tutor of Alexander, therefore, etc., we have directly but the substitution of one expression for another ; only when there is some indirect implication can such inferences as this be significant and rise above the level of mere verbal trivialities.

clusion is an immediate judgment, inasmuch as it only analyses this synthesis. When I infer that because A is to the left of B , and B to the left of C , therefore A is to the left of C , the combination of the two premises suffices to give me the three points A, B, C , in this particular position, and from this it is immediately evident that A is to the left of C . Thus the essential part of the syllogistic process is the combination of different elements in one whole, a construction which yields complete all that is expressed in the conclusion.¹ In the same way suppose we have an individual S , possessing a complex M of tangible and visible attributes, and in this complex a perceptible attribute P ; S and M and P form, as it were, the image of a single thing. Our knowledge that P appertains to S is certain, as evidently, and on the same ground, as that M belongs to it or that P belongs to M . The premises effect a combination, and the intuition of this gives us an immediate knowledge of the fact that the elements connected in the conclusion belong together.² According to the opposite view, inference consists in comparing the two premises and recognising the necessity of attributing a predicate P to the subject S ; only a discernment of this necessity can be the ground for actually thinking the unity SP . According to this view, the person making the inference stands in the same position as some one hearing the judgment SP spoken; he first receives the ideas apart from one another in the two premises, and is then called upon to unify them. In other words, according to the first view, the real inference takes place before the formulation of the conclusion, this being only an analytical expression of knowledge already acquired; while, according to the second view, the inference first creates the conviction that the predicate P must be attributed to the subject S , thus giving rise synthetically to the thought of the unity SP . This latter view is more emphasized in ordinary accounts of the syllogism, in proportion as they aim at making inference mechanical by the rules and figures of the syllogism, and at turning it into a species of calculation. In an algebraical calculation I work only with symbols; not until I have finished do I interpret the equation I have obtained, and then I realize again what I have denoted by symbols; I get my result not before the conclusion, but by means of the conclusion. But even when the process of inference is not one which is at first carried on merely in words or symbols, the result being afterwards realized, it still depends upon the nature of premises and

¹ Cf. Bradley, *Principles of Logic*, p. 235.

² Schuppe, *Erk. Logik*, p. 260.

conclusion whether or not the former will at once blend into one whole, which can then only be expressed analytically. When we have to do with a negative premise, such a synthesis is prohibited by the nature of the negation. But even with positive premises Bradley's theory breaks down as soon as we come to deal with relations which are less immediately and obviously intuitable than the simple spatial relations from which he starts, or with predicates which do not belong to those contents of the concept denoted by the middle term which are invariably present to thought.

But in any case this distinction in the syllogistic operation applies only to the psychological *process*; the question as to whether or not the conclusion follows necessarily from the premises is not affected by it. For when the synthesis actually takes place in such a way that it finds only analytical expression in the conclusion, it is not necessary and unambiguous unless there is some law which prescribes this synthesis and prohibits any other. The synthesis $A-B-C$, results from the premises A to the left of B , B to the left of C , only because it is prescribed by the law of spatial relations. The truth of the conclusion depends upon the truth of the premises, and it makes no difference to this dependence whether in any particular instance there is unconscious obedience to the law in the synthesis, or whether it forms a conscious ground by which the synthesis is guided.

§ 51.

The general hypothetical rule according to which an inference is made is of a *synthetical* nature when it is not already included in the judgment which is taken as a ground, or in its elements, but is added to it from other sources. Such rules may be either axioms, which serve to connect relations, or general propositions, gained by induction from experience, or laws which state some connection established by the will.

Other hypothetical rules are given with the *judgment* itself from which the inference is made, and may be *analytically* developed from it. They may be derived from the form of the judgment, in so far as the act of judgment is governed by universal logical laws, or from the contents of the concepts of which it is composed in so far as these concepts contain universal judgments.

1. Hypothetical propositions which state a universal connection as explained in the preceding paragraph may be derived from very different sources.

In the first place, there are the universal propositions which are the expression of an immediately obvious necessity to be found in the relations of certain objects of thought (synthetical judgments *a priori* in Kant's sense). The most important of these are the mathematical axioms which state how the relations of number, space and time are connected.

A constant experience without any exception may be the ground of our belief in other universal connections. We shall have to enquire in Part III. how it is possible to pass from particular perceptions to judgments having a universal and unconditional validity; here we may be content with the universal conviction that many necessary connections may be inferred from experience. That a body expands when it is heated, that white light is broken up when passed through a refracting medium, are instances of such laws. If the condition is at any time present, we confidently infer that the result mentioned in the law will also appear, and our confidence is ultimately based upon simple facts of perception which have shown us the one event connected with the other.

We find a wide field, again, for our inferences in the application of general laws which have their origin in our will and are meant to regulate that will. In laying down a general rule of conduct, our will determines that there shall be a universally valid connection between certain conditions and certain modes of action. If we will the general law, it is logically necessary that we should will the particular actions prescribed by the law, if our will is to be constant and consistent, and valid for every one who agrees in willing the general law. All penal codes in imposing a penalty of imprisonment for theft, of capital punishment for murder, lay down a series of hypothetical judgments which establish a universal connection between committing the crime and incurring the penalty. These judgments, moreover, may also be regarded as theoretical propositions in so far as they express the general obligation of the judge to give sentence in accordance with the law.

In analytical geometry an equation such as $y^2 = px$ determines the construction of a curve; by means of it to every value of the abscissa is assigned the appropriate value of the ordinate. This relation between x and y has the force of a hypothetical judgment, and may be selected at will to enable us to construct freely any spatial image, and to this extent such a formula is comparable with the positive establishment of a law.

In cases like these we add to the judgment A , which is the ground of

our inference, a general law which is not thought with the judgment, nor analytically contained in it.

2. The case would be different with connections involved in the fact that a certain judgment was uttered or thought ; rules, that is, which could be derived from the judgment itself, and which would tell us upon the ground of universally valid laws, that if this judgment be true, some other must also be true ; rules which could be adduced without applying to any external source.

How can the fact that the judgment *A* is *B* is true yield any further knowledge ? It may do so in two ways. It may be that the particular form of synthesis between the two elements in the judgment *A* is *B*, renders it possible and necessary to apply other forms of connection (in judgments) quite apart from the meaning of *A* and *B* ; that is, there are laws by which all judgment is governed, and according to which any judgment whatever will yield other judgments containing the same elements. But it may also be that the predication of *B* of the subject *A* involves other judgments by reason of the particular meaning attaching to *A* and *B* in this judgment. In the former case the rules will be formal, in the latter, material.

§ 52.

The so-called IMMEDIATE INFERENCES, which are only formal variations of the same judgment, are based upon the universal nature of the judgment, which is the same, however the matter varies. Amongst these we generally find enumerated the inferences of OPPOSITION, of CHANGE OF RELATION, of EQUIPOLLENCE, of SUBALTERNATION, of MODAL CONSEQUENCE, of CONVERSION, and of CONTRAPOSITION.

1. The most obvious inferences of all, and those which can be derived entirely from the meaning of judgment itself, are generally quite omitted. The judgment "*A* is *B*" involves the judgments "it is true that *A* is *B*," and "it is necessary to state that *A* is *B*"; also that "*A* and *B* are compatible."

2. This leads to the INFERENCE OF OPPOSITION,—inference, that is, from the truth of one judgment to the falsity of its contradictory opposite, and, *vice versâ*, from the falsity of one judgment to the truth of its contradictory opposite. This kind of inference is based upon the principle of contradiction and of twofold negation, which simply tells us that the two judgments "*A* is not *B*," and "it is false that *A* is *B*," and the two judgments "*A* is

B” and “it is false that *A* is not *B*,” have just the same meaning. It is the same with respect to hypothetical judgments. The denial of the judgment “if *A* is true, *B* is true,” is equivalent to the judgment “even if *A* is true, it is not necessary that *B* should be true”; if the latter judgment is false, the former is true.

3. When we change the unconditionally universal judgment “all *A*’s are *B*’s” into the hypothetical judgment “if anything is *A*, it is *B*,” we take for our predicate that necessity which, in the unconditionally universal judgment formed the ground of its universality; and the unconditionally universal judgment, when substituted for the hypothetical, expresses universality as the consequence of necessity. In the same way, when a disjunctive judgment is broken up into hypothetical judgments, or when several hypotheticals (if *A* is not *B*, it is *C*; if *A* is not *C*, it is *B*) are combined in one disjunctive (*A* is either *B* or *C*), the meaning of the verbal forms finds a different expression.

4. Other forms generally given are:

(a) INFERENCES OF ÆQUIPOLLENCE. From the judgment *A* is *B* we may infer that *A* is not non-*B*, an inference which the indefiniteness of non-*B* makes worthless. (The inference that snow is white and therefore not red cannot be regarded as purely formal; it presupposes a judgment concerning the matter of the predicate, the judgment that anything white is not red.)

(b) INFERENCE BY SUBALTERNATION. Here it is inferred from the judgment all *A*’s are *B* (or not *B*) that some *A*’s are *B* (or not *B*), and from the falsity of the judgment some *A*’s are *B* (not *B*) the falsity of the judgment all *A*’s are *B* (not *B*). But the true predicate in the universal judgment is “all,” and hence this inference is dependent upon the contents of the predicate, and is only a particular instance of the rule that the smaller number is contained in the greater. The same rule enables us to conclude that where there are three there must be two, etc; so that the inference here is not a formal transposition authorised by the nature of the act of judgment; it is merely a conclusion drawn from the meaning of the predicate. We should have as much justification for saying that the inference that the part must be where the whole is, is immediate.

(c) In inference by what is called MODAL CONSEQUENCE we are said to derive actuality and possibility from necessity, and possibility from actuality; further the negation of actuality and necessity follows from the negation of possibility, the negation of necessity from that of actuality. So far as concerns the act of judgment itself, there is no distinction between

necessity, possibility, and actuality ; if, on the other hand, these terms are used as predicates, having reference to reality, then the inference depends upon their contents, and has no place here.

5. Since the time of Aristotle the CONVERSION of judgments has played the most important part in the doctrine of immediate inferences. By means of this process it is said that from the judgment *A* is *B* we may obtain another having *B* for its subject and *A* for its predicate. We are taught that by conversion :

The universal affirmative judgment, all *A*'s are *B*, will give us some *B*'s are *A* (*conversio per accidens*, the quantity being changed).

The universal negative, no *A* is *B*, gives us no *B* is *A* (*conversio simplex* the quantity being unchanged).

The particular affirmative some *A*'s are *B* gives us some *B*'s are *A* (*conv. simplex*).

While the particular negative some *A*'s are not *B* admits of no conversion.

Taking first the affirmative judgments, we may point out that, if their conversion is to have any meaning, they must be judgments in which the predicate is the generic idea to the subject, both belonging to the same category, so that the predicate may become subject in the same sense as the original subject. They must be, moreover, judgments concerning individual subjects, so that it is no strain to their meaning to interpret them by saying that the subjects named may be included amongst the objects denoted by the predicate-term. They must, that is, be such judgments as "all firs are trees," "no larches are firs," etc. ; where the correctness of the conversion may be proved by enumeration.

Where these conditions are not complied with, the conversion has a forced sound, and the meaning of the judgment is altered. The judgment "all planets move in ellipses" is based upon the category of action ; change it into "some things moving in ellipses are planets," and instead of taking the predicate for subject we have introduced a new subject ; we have connected the concept of thing with the predicate, and, since it is absurd to determine a concept of substance by a temporal *process* our concept is an unnatural creation. Instead of a judgment of action we have obtained a judgment of subsumption. In passing, therefore, from one judgment to another, we are not really independent of the meaning of the terms employed.

The true significance of such conversion is first to tell us that the predi-

cate is compatible with the subject, and next to supplement the universal judgment by showing that though it may be necessary to think of A as B it does not follow that B belongs exclusively to A . It is this warning which is the most important part of the meaning of the conversion, and it coincides with the rule that we may not infer the ground from the consequence. Applied to the hypothetical judgment, it prohibits us from simply transposing the judgment if A is true, then B is true, and inferring that if B is true, then A is true. All that we can infer is that if B is true, A may be true, which corresponds to the particularity of the converted categorical judgment.

The case is different with the conversion of the universal negative judgment. This expresses the fact that exclusion between two concepts is always mutual; that if a subject A excludes a predicate B , it certainly cannot be true that anything possessing B can be A . Or we may avoid the awkwardness of changing adjectival and verbal predicates into substantives by making use of the hypothetical formula, and then, from the judgment

If anything is A it is not B , it follows that

If anything is B , it is not A .

Where the consequence, the negation of B , is denied of any object, there we can no longer use the subject-concept as a name for it.

6. Besides conversion we have also CONTRAPOSITION. Here a new judgment is formed from A is B by taking the so-called contradictory opposite of the predicate as subject, and the original subject as predicate, and altering the quality of the judgment; that is, changing affirmation into negation and *vice versa*. By this means we may obtain

from all A 's are B ,	no non- B is A ,
from no A is B ,	some non- B is A ,
from some A is B ,	nothing,
from some A is not B ,	some non- B is A .

We may leave the reader either to prove these inferences himself or to find the proofs elsewhere. We need not point out that this form is nothing but an artificial perversion, which conceals the good sense really contained in these propositions by introducing the unmanageable non- B , and by arbitrarily constraining the predicate-concepts to take the form of substantives. The results of this process are propositions such as "nothing not having equal diagonals is a rectangle."

The whole meaning of contraposition is clear at once when we use the hypothetical form and retain the predicate as predicate. Instead of all *A*'s are *B* we then have

If anything is *A*, it is *B*, and from this it follows that

If anything is not *B* it is not *A*.

Contraposition in this form is parallel with the conversion of negative judgments, which from the judgment

If anything is *A*, it is not *B*, infers that

If anything is *B*, it is not *A*.

Both these instances of what is called simple conversion and contraposition have a meaning and are useful ; they express the complete meaning of the statement that a predicate must either belong, or not belong, to a subject. The remaining cases yield only particular judgments ; that is, we cannot obtain any definite conclusion, but only the denial that certain concepts are incompatible or necessarily connected.

No *A* is *B*, *i.e.*,

If anything is *A* it is not *B*,

but this does not necessitate the inference that because something is not *B* it is *A*, although it may be *A*.

7. The doctrine of immediate inferences may be still further extended to apply to the disjunctive judgment. If *A* is either *B* or *C*, it is false that it is both *B* and *C*, and false that it is neither *B* nor *C*. If it is false that *A* is either *B* or *C*, then *A* may be both *B* and *C*, or *A* may be neither *B* nor *C*, or *A* may be either *B* or *C* or *D*. Here again the conclusion follows entirely from the meaning of disjunction, and quite independently of the particular elements of the judgment.

8. Finally, by extending the concept of immediate inference beyond its customary sphere, we may also include all those operations by means of which we combine a plurality of particular judgments, and thus form conjunctive or copulative judgments. From *A* is *B* and *A* is *C* it follows that *A* is both *B* and *C* ; from *A* is not *B* and *A* is not *C* it follows that *A* is neither *B* nor *C*. The conjunctive judgment is only the verbal expression of the fact which is involved in our consciousness of the validity of both judgments ; it tells us nothing new as to the matter, and merely serves to make us expressly aware of a connection which was already there. These operations are of great importance to thought, in its arrangement

and combination of particular items of knowledge, and for this reason they are worthy of being mentioned here.

9. The value of this doctrine of the so-called immediate inferences consists, as Mill rightly said, in enabling us to recognise the same judgment in different forms and expressions. The judgments thus inferred from each other may be simple transpositions of a given statement, which enable it to be reduced to a form more convenient for the context to which it refers ; or they may serve to make prominent special aspects of the statement which are not emphasized by the verbal expression ; or, finally, they may serve as precautionary rules, teaching us not to confuse one judgment with another like it, or to find more in it than it really contains.

§ 53.

Given a simple judgment, we may derive from it others having their ground in the contents of its elements. In so doing we must be guided by rules which may be obtained either from an analysis of the predicate-concept, or by reference to the extension of the subject-concept.

1. If it is true that A is B , then it is evident that in affirming B of A we affirm of A everything which we think of as connoted by B . In the same way by affirming B of A we exclude from A everything excluded by the connotation of B .

Suppose that B contains the conceptual characteristics c, d, e , or the derivative determinations f, g, h , and that it excludes the characteristics m, n, o , and the concepts P, Q, R , etc. Then, because we affirm B of A , we must also affirm c, d, e, f, g, h , and deny m, n, o, P, Q, R .

These conceptual relations find their simple expression in the judgments :

If anything is B , it is c, d, e , etc.

If anything is B , it is not m , not n , not P , etc.

In this way, by analysing the concept B , and by enumerating the characteristics incompatible with it, we obtain the rule by which to pass from A is B to another judgment in accordance with the principle *Nota notæ est nota rei, repugnans notæ repugnat rei*. Thus we get the following forms of inference :

1. If anything is B , then it is c, d, e

A is B

Therefore A is c, d, e

2. If anything is B , then it is not P, Q, R
 A is B

Therefore A is not P, Q, R

It is clear that these inferences are valid, whatever A may be—a particular thing or a concept—and whatever the sense in which B is attributed to A . Everything contained in the thought of the concept B is predicated by it; everything excluded by it is denied when it is predicated; the new judgments are necessary consequences of predicating B of A .

It is clear also that if A is a definite particular subject, there is no other way of passing beyond the judgment A is B without having recourse to other propositions.

2. The judgment A is B may be explicative or unconditionally universal—one, that is, in which the denotation of the subject is not used as the name of a certain particular thing, but as the symbol of a concept, so that the judgment A is B has itself the significance “if anything is A , it is B .” In this case there is another way in which we may pass from the judgment A is B to a different one; we may attribute B to everything of which A is predicated, or which is contained in the sphere of A . Our judgment then reverts to the particular kinds of A , or to the individuals comprehended in A , and A is B serves as major premise.

If anything is A it is B
 X, Y, Z are A

 Therefore X, Y, Z are B

In the former kind of inference the content of the original predicate was unfolded, and we proceeded from it to particular determinations or derivative predicates. In the latter kind the extension of the original subject is specified, and the predicate is attributed to the subjects comprehended by the original concept; and in so doing we are guided by the rule known as the *Dictum de omni*, *Quidquid valet de omnibus, valet etiam de singulis*. This rule has reference to the fact that the formula “If anything is A , it is B ” generally appears as the so-called universal proposition, “All A ’s are B .”¹

¹ In his essay, *von der falschen Spitzfindigkeit der vier syllog. Figuren*, Kant has shown briefly and clearly that what is called the *Dictum de omni* follows from the principle “*nota nota est nota rei*.” That is, the particular thing falls under a concept only because this concept is to be found in it as its characteristic. This does not make the two formulæ equivalent, as B. Erdmann maintains (*Philos. Aufsätze zu E. Zeller’s Jubiläum*, p. 202); the first as being original and primary is distinguished from the second, which is derivative and secondary.

Here again the treatment of the negation runs parallel with that of the affirmation. If in place of the affirmative judgment we have the negation A is not N —meaning anything which is A is not N —then the same negation applies to everything which is A .

$$\begin{array}{c} \text{That which is } A \text{ is not } N \\ X, Y, Z \text{ are } A \\ \hline X, Y, Z \text{ are not } N \end{array}$$

(*Dictum de nullo.*)

3. A comparison between these two cases, the analysis of the predicate and the specialization of the subject, shows that notwithstanding the difference between them, they both lead to the same formula.

$$\begin{array}{c} \text{If anything is } A, \text{ it is } B \text{ (it is not } W) \\ S \text{ is } A \\ \hline S \text{ is } B \text{ (} S \text{ is not } W) \end{array}$$

The difference is only in the significance of the predication, and more especially of the minor premise. When this serves to subsume a subject under its genus, the predicate being thus fitted to take its place as subject-concept without changing its meaning, then the function of the inference is to specialise the extension, in the other case the inference serves to unfold the contents. In the first case it is natural to express the major premise (the rule) as a general proposition, but not in the second; and while it is the major premise which stands first in the former, in the latter it is the minor premise.

In the inference: All men are mortal; Caius is a man, therefore Caius is mortal, I pass from my original proposition to the extension of the subject-concept. In the inference:

Caius has fever;
Any one having fever is ill,
Therefore Caius is ill,

I pass from my original predicate "having fever" to the further determination "ill" connoted by it. But "having fever" is not a generic idea to particular individuals. We may take a universal proposition of the ordinary form for our major premise:

All people having fever are ill,
Caius is a person having fever,
Therefore he is ill,

and this inference seems to have the same contents as the one preceding. But the way in which the major premise is expressed is forced, and the minor premise seems to be intended to state a subsumption under a generic concept, while it really denotes a temporal condition.

4. It is evident that the more complicated relational-judgments may be treated in just the same way by this explication of intension and specialization of extension, when they contain analogous predicates or subjects, and this is true even though the respective terms may not appear verbally as the grammatical subject or predicate.

The inference "Gravitation imparts the same velocity to all bodies, therefore a piece of lead and a feather (in a vacuum) fall at the same rate," may be resolved into two inferences. On the one hand, it is a development of the predicate into its consequences; on the other, a specialization of the term "all bodies," which, though not the grammatical subject, really denotes that concerning which the statement is made. It would be superfluous to begin by making this term the grammatical subject by means of a forced transposition. We are as much justified in substituting species for genus as if the major premise were "All *A*'s are *B*," and there is therefore no need for a special principle of substitution in addition to the *dictum de omni* by which to justify such inferences. They differ from others only in the grammatical form of their propositions.

5. When we start from a negative judgment it does not hold good that everything which is necessarily contained in the thought of the negated predicate is also negated. When I deny that this figure is a square, I do not deny that it is rectangular or quadrangular; I only deny that all the attributes are present. Hence no inference is possible by merely analysing the negated predicate. We cannot say:

$$\begin{array}{l} \text{If anything is } B \text{ it is } c, d \\ \quad A \text{ is not } B \\ \hline \text{Therefore it is not } c, d \end{array}$$

Nor does it hold good that anything excluded from the negated predicate may be affirmed; it does not follow that because something is not red it is black. Hence it is no valid inference to say:

$$\begin{array}{l} \text{If anything is } B, \text{ it is not } C \\ \quad A \text{ is not } B \\ \hline \text{Therefore it is } C \end{array}$$

The invalidity of this inference follows obviously from the rule that the negation of the ground does not necessitate the negation of the consequence.

It is the same when we turn to the extension. Here again it does not follow that because A is not B , therefore that which is not A is B ; we are unable for the same reason to infer

$$\frac{\begin{array}{l} \text{If anything is } A, \text{ it is not } B \\ C \text{ is not } A \end{array}}{\text{Therefore it is } B}$$

But when the judgments are such as express conditions having the negated predicate for consequence, or judgments which specialise the extension of the predicate, then we get the following inferences:

$$\frac{\begin{array}{l} A \text{ is not } B \\ \text{If anything is } C, \text{ it is } B \end{array}}{\text{Therefore } A \text{ is not } C} \qquad \frac{\begin{array}{l} A \text{ is not } B \\ C, D, E \text{ is } B \end{array}}{\text{Therefore } A \text{ is not } C, D, E}$$

These may be shown to be applications of the rule that the denial of the consequence involves that of the ground:

$$\frac{\begin{array}{l} \text{That which is } A \text{ is } B \\ C \text{ is not } B \end{array}}{C \text{ is not } A} \qquad \frac{\begin{array}{l} \text{is not } N \\ C \text{ is } N \end{array}}{C \text{ is not } A}$$

6. These are the only ways possible in which we can pass from a simple judgment to another definite judgment by means of the conceptual relations present. In all of them we are led by two principles. The one is that all which is contained in the connotation of a concept must be affirmed of everything of which the concept is affirmed, hence of all the species of the concept and of all the individuals which fall under it. The other, that whatever is excluded from a concept is excluded from everything in the thought of which this concept is contained; hence from its whole extension. We have moreover shown how the *modus ponens* and *modus tollens* of the hypothetical inference reappears in these forms.

7. We obtain the same result if we start differently from the other point (§ 49, 2), and ask whether or not there are any grounds for some synthesis A is B . This question may be solved at once if we can recognise B as contained in A , the judgment thus proving to be analytical. But if not, some mediation is needed to assure us that A is B , and unless we are to

adduce propositions from elsewhere, this mediation can consist only in discovering in A a predicate X from which B necessarily follows. If the two propositions, "If anything is X , it is B ," and " A is X ," are both valid, then we can infer that A is B . It makes no essential difference to this process whether X is the generic idea to A , and B belongs to it, or whether it is another predicative determination of which B forms part of the connotation; all that is affected is the meaning of the minor premise A is X .

In the same way, if we can discover in A a determination Y , of which it is true that if anything is Y , it is not B , the question can be decided in the negative. The inference then is:

$$\begin{array}{c} \text{If anything is } Y, \text{ it is not } B \\ A \text{ is } Y \\ \hline \text{Therefore } A \text{ is not } B \end{array}$$

These two forms represent the shortest and simplest way of arriving at some decision as to a suggested synthesis, and they represent the only way if we assume beforehand that all necessary connections must be derived from the relations which already exist between concepts, hence that they must be ultimately reduced to analytical judgments. It is this which makes the middle concept important in inference as being both predicate of A and subject of a universal affirmative or negative judgment having the predicate B ; it mediates the attribution or negation of the predicate B to the subject A .¹

The negation of the hypothesis A is B may also be brought about in a less direct manner when the judgment, if anything is Y it is not B , is not immediately obtainable. Then the judgment " A is Y " may be supplemented by a second, "that which is B is not Y ; or else we may know that anything which is B is Z , and A is not Z . Then we get—

$$\begin{array}{cc} \begin{array}{c} \text{That which is } B \text{ is not } Y \\ A \text{ is } Y \\ \hline \text{Therefore } A \text{ is not } B. \end{array} & \begin{array}{c} \text{That which is } B \text{ is } Z \\ A \text{ is not } Z \\ \hline \text{Therefore } A \text{ is not } B \end{array} \end{array}$$

The first of these formulæ is really reducible to the preceding negative formula; for from the major premise it follows that that which is Y is not B .

¹ Cf. Kant, *von der falschen Spitzfindigkeit der vier syllogist. Figuren*, § 1.

This kind of mediation is based upon the principle that when a predicate is affirmed of one subject and denied of another, these two subjects cannot be compatible. It is naturally substituted for the preceding method when *A* is a substantival concept, *Y* and *Z* being predicate determinations such as are ill adapted to become subjects. Suppose, *e.g.*, that the question arises whether this stone is a diamond, I know that diamonds do not possess double refraction, and I argue :

That which is a diamond does not possess double refraction.

This stone possesses double refraction ;

Therefore it is not a diamond.

This is more natural than to say, Everything possessing double refraction is not a diamond.

Thus our investigation of the different kinds of mediation by means of which any given question may be decided leads to exactly the same result in all cases. If our inference is to be based upon simple analytical relations and opposition of concepts, then the *modus ponens* and *modus tollens* in their different applications, prove to be the only forms of inference of which we can make use.

§ 54.

The ordinary doctrine of the CATEGORICAL SYLLOGISM, which has grown out of the Aristotelian theory, is based upon the presupposition of the preceding section ; *i.e.*, that our inferences are grounded upon established relations between concepts. The distinction of figures and moods was justified from the point of view of the Aristotelian doctrine ; but, from the point of view of the traditional doctrine, they are superfluous specializations, which may be resolved into the more general formulæ of the preceding paragraph.

1. Both the Aristotelian doctrine of the syllogism and the ordinary teaching which has grown out of it presuppose established relations amongst concepts. Aristotle himself assumes an objective system of concepts which realizes itself in the material world in such a way that the concept manifests itself everywhere as constituting the essence of things and as the cause of their particular determination. Thus all judgments containing true knowledge are for him the expression of necessary relations, and the function of the syllogism is to reveal the whole force and bearing of each particular concept in our knowledge by combining particular

judgments and making them mutually dependent through their conceptual unity. The verbal expression of these conceptual relations is due to the fact that they always appear as the essence of particular things, which are, therefore, when conceptually determined, the real subject of judgment; in this way the relations between concepts manifest themselves in universal or particular, affirmative or negative, judgments. Ordinary logic, on the contrary, is based upon a subjective system of concepts, which is not sought for in the process of knowledge, but is assumed as a preliminary datum.

2. Now the fundamental relation is that of the subordination of concepts. The natural predicate to every concept is the concept next above it; thus if A , B , Γ are three concepts, which are subordinated to each other, their relation is expressed in the two propositions; A κατὰ παντὸς τοῦ B , B κατὰ παντὸς τοῦ Γ ; and from these we obtain, by means of the syllogism, A κατὰ παντὸς τοῦ Γ . It is from this relation of subordination that the terminology is derived, according to which B is called the μέσος ὄρος (terminus medius), A and Γ , the two ἄκρα— A being the μείζων ἄκρον (terminus major), Γ the ἐλάττω ἄκρον (terminus minor). For this reason also the first proposition, which predicates the highest concept (the predicate of the conclusion) of the middle concept, has been called the major proposition; the second, which predicates the middle concept of the lowest (the subject of the conclusion), the minor proposition; the result of the syllogism is the conclusion.

Taking the usual symbols, P for the highest concept, M for the middle concept, and S for the lowest, then the form in which the nature of the syllogism is most directly and immediately obvious is the familiar—

$$\begin{array}{c} \text{Omne } M \text{ est } P \\ \text{Omne } S \text{ est } M \\ \hline \text{Ergo omne } S \text{ est } P. \end{array}$$

3. Aristotle next proceeds to introduce the distinction between negative and affirmative judgments, and he shows that if the major proposition is negative, it is still possible to have an inference with a negative conclusion.

$$\begin{array}{c} \text{No } M \text{ is } P \\ \text{All } S \text{ is } M \\ \hline \text{Therefore no } S \text{ is } P. \end{array}$$

If, on the other hand, the major proposition were affirmative, and the minor negative, no inference could be drawn, "for nothing would follow as necessary from the validity of the premises." When P is true of all M , while M is not true of any S , then P may be true of all S , or it may not (living being—man—horse; living being—man—stone). Nor can anything be inferred when both premises are negative.

When the distinction between universal and particular judgments is introduced, it follows for similar reasons that the major premise cannot be particular, though the minor premise may be a particular affirmative judgment.

We may then infer—

All M is P	No M is P
Some S is M	Some S is M
Therefore some S is P	Some S is not P

These are the four *τρόποι* or *modi* of the syllogism, which result from two premises, in the first of which the middle concept is subject, while in the second it is predicate. They are the four perfect syllogisms (*συλλογισμοὶ τέλειοι*), and in them the four kinds of judgment are deduced from premises which contain the middle concept in the way we have stated.

4. But it may also be that the middle concept is predicate in both premises, or subject in both premises. In the former case we have the second figure, in the latter the third (*δευτέρον* and *τρίτον σχῆμα*). In these figures the syllogisms possible are—

In the second figure :

Mood 1.	Mood 2.
No P is M	All P is M
All S is M	No S is M
<hr/> No S is P	<hr/> No S is P
Mood 3.	Mood 4.
No P is M	All P is M
Some S is M	Some S is not M
<hr/> Some S is not P	<hr/> Some S is not P

In the third figure :

Mood 1.	Mood 2.	Mood 3.
All M is P	No M is P	Some M is P
All M is S	All M is S	All M is S
<hr/> Some S is P	<hr/> Some S is not P	<hr/> Some S is P
Mood 4.	Mood 5.	Mood 6.
All M is P	Some M is not P	No M is P
Some M is S	All M is S	Some M is S
<hr/> Some S is P	<hr/> Some S is not P	<hr/> Some S is not P

The syllogisms of these two figures are not recognised by Aristotle as perfect, and he reduces them to the first figure either by conversion of the judgments or by indirect proof. He then proceeds to investigate in a similar manner the various inferences from premises which are judgments of necessity and possibility.

5. Notwithstanding many attacks, this Aristotelian doctrine has always reasserted itself as the essential part of all scholastic logic, although its original meaning and the significance which Aristotle attached to it has in most cases been lost sight of. This misunderstanding is obvious from the introduction of the so-called fourth figure;¹ and more particularly from the fact that instead of recognising the necessity of conceptual relations to be at the root of all inference, it became customary to look upon the premises as mere statements as to how concepts are related in their exten-

¹ It was discovered, that is, that Aristotle had overlooked one way of arranging the middle concept, that in which it is predicate of the major premise and subject of the minor. By introducing this it was found that five additional moods were possible :

1.	2.	3.
All P 's are M	All P 's are M	Some P is M
All M 's are S	No M is S	All M is S
<hr/> Some S 's are P	<hr/> No S is P	<hr/> Some S is P
4.	5.	
No P is M	No P is M	
All M is S	Some M is S	
<hr/> Some S is not P	<hr/> Some S is not P	

It is evident that any one forcing the concepts into this unnatural position must have forgotten all the fundamental presuppositions of the Aristotelian theory; the need of supplementing the Aristotelian doctrine could only have been felt in a treatment which dealt with the external form alone.

sions. It was for this reason that the cogency of the premises as proof was in the first instance based upon the relations between numerical expressions, as if the object of inference were to find one or more particular things in a given number, and its problem to think of the objects falling under a concept all at once, and then to find out what is included amongst them and what is not. The same misunderstanding has led to the favourite method of proving the particular figures of the syllogism by means of a purely intuitive comparison of the spheres of the particular concepts; as if the function of all judgments were to place the subject within the sphere of the predicate concept, and to exhibit it as a part of a greater number of objects bearing the same name, and not rather to state what it is and what it does. This mistake is in a large measure due to the irrational manner in which the particular judgment is usually treated. Thus the doctrine of the syllogism degenerated into a sort of calculating machine, by means of which any one who would take the trouble to remember the 19 moods by aid of the *versus memoriales*, and to practise them upon meaningless examples, could learn everything from the external forms of judgment, from the position of subject and predicate, without need of further reflection.

6. Leaving the presuppositions of the doctrine for the present untouched, what strikes us first is that in the fig. 1 the distinction between the third and first, and between the fourth and second, moods, is quite unimportant. No change whatever is made in the course of thought because the minor proposition is particular in the third and fourth moods. Since the "some *S*" of the minor proposition and of the conclusion must always mean the same *S*, and since the predicate is attributed to them because of a conceptual determination which is common to both, the meaning of the inference is exactly the same as when the subject is universal. It is the value of the result with respect to the determination of the concept *S* which differs, not the operation of inference; and it was in reference to this difference of value that Aristotle, who always kept the conclusion in view as the principal object, made his distinction. So far as concerns the form of deduction we have, strictly speaking, only two kinds of inference :

All <i>M</i> 's are <i>P</i>	No <i>M</i> is <i>P</i>
(All, some, one) <i>S</i> are <i>M</i>	(All, some, one) <i>S</i> are <i>M</i>
<hr/>	<hr/>
(All, some, one) <i>S</i> are <i>P</i>	(All, some, one) <i>S</i> are not <i>P</i>

In the first case the middle concept adds a predicate to its subject ; in the second case it excludes one.

In the same way the moods of the fig. 2 may be reduced to two kinds of inference ; when a predicate M , which is excluded from another concept P , is contained in any subject S , then P is itself excluded from the subject ; and when a concept M , which comprehends another concept P , is excluded from a subject, then P is excluded from the subject ; it makes no difference whether the S is expressed as universal or as particular. Thus we have :

No P is M	All P is M
(All, some, one) S are M	(All, some, one) S are not M
<hr/>	<hr/>
(All, some, one) S are not P	(All, some, one) S are not P

Now, the indispensable rule according to which we infer, when reduced to its most appropriate expression, runs for the first figure as follows :

If anything is B , it is A (moods 1 and 3).

If anything is B , it is not X (moods 2 and 4).

In both cases the assumption takes the form :

Certain subjects C are B ,

and it follows :

Therefore they are A , therefore they are not X .

But the second figure must also be based upon these same rules, for there is no other method of inference from simple conceptual relations. In the second figure, however, we infer from the fact that the consequence is wanting ; that is, we infer from the invalidity of the consequence to the invalidity of the ground.

If anything is B it is A ;

Now C (all C , some C) is not A ,

Therefore it is not B (moods 2 and 4).

If anything is B it is not X ;

Now C (all C , some C) is X ,

Therefore it is not C (moods 1 and 3).

Thus both the connexion between the first and second figures, and their difference become evident when we see that in the first figure we infer from the validity of the ground to the validity of an affirmative or negative consequence, while in the second we infer from the invalidity of

the affirmative or negative consequence to the invalidity of the ground.¹ The first two of the Aristotelian figures, therefore, coincide exactly with the results of § 53.

All the moods of the first and second figures can thus be exhibited in a single formula in such a way as to show both the grounds of inference and how they differ.

Major premise :

If anything is *B*, it is *A*—it is not *X*.

Minor premise and conclusion of fig. 1 :

C (all, some, one *C*) is *B*,

Therefore *C* (all, some, one *C*) is *A*—is not *X*.

Minor premise and conclusion of fig. 2.

C (all, some, one *C*) is not *A*—is *X*,

Therefore *C* (all, some, one *C*) is not *B*.

7. The particular judgments of fig. 3. differ essentially in meaning from the particular judgments of figs. 1. and 2. In the latter the term which is particular stands from the first as subject, and the fact that it is particular is unimportant, may indeed be due merely to the verbal expression ; the subjects of the minor premise and of the conclusion are the same. But in the third figure the particular term is subject only in the conclusion, and possesses therefore all the indefiniteness of the particular. It is equivalent to a mere judgment of possibility, and in the third figure there can be no such thing as a necessary consequence in the ordinary sense. In the first, third and fourth moods alike, the essential point is that two predicates belong to the same subject, for in the third and fourth the burden of inference rests only upon that part of all *M* which is identical with some *M*. But all that follows from this is that the two predicates are compatible, *i.e.*, do not exclude each other. In the same way the common characteristic of the second, fifth and sixth moods is, that one predicate *P* is absent from a subject to which the other predicate belongs, and from this it follows that they are not

¹ This gives us also a solution for such difficulties as that two negative premises may yield a conclusion notwithstanding Aristotle's rule ; *e.g.*, that which is not *M* is not *P*, *S* is not *M*, whence it follows that *S* is not *P*.

The conclusion is doubtless true, but it is false that it follows from two negative premises in the Aristotelian sense. The proposition "that which is not *M* is not *P*," is negative merely in form ; really it is equivalent to all *P*'s are *M* ; and the connexion between the negatives is based upon the positive relation between the predicates. Objections such as these arise only where the method of treatment is quite superficial.

necessarily connected. Strictly speaking, then, the rule according to which the inference takes place, and upon which the deduction of the conclusion from the premises is based, finds no expression in the premises. The major premise suppressed in the affirmative moods is: "If two predicates belong to the same subject, they are compatible, they do not necessarily exclude each other"; the two premises taken *together* form the minor for this suppressed major premise. In the moods with negative conclusions, again, the major premise is: "If one of two predicates belongs to a subject, but not the other, then they are not necessarily connected"; here again the two premises together represent the minor to the above-stated rule.

That which is inferred, then, in this figure is the definite *denial of a necessity*, either of a necessary exclusion or of a necessary connexion. The weakness of the third figure is that it cannot prove necessity, but can only deny it, and this weakness finds its expression in the fact that the conclusion is particular.

From this point of view, as Lotze shows (p. 113), two negative premises may yield a similar conclusion as to the negation of a necessity. If M is not P and M is not S , it follows that the negation of P does not necessarily involve the affirmation of S , nor the negation of S the affirmation of P ; that which is not P is not necessarily S , and *vice versâ*. That which is denied is therefore the connexion which would be expressed by the disjunctive judgment " M is either P or S ." The two premises may be combined in the judgment: " M is neither P nor S ," which is the negation of the disjunction " M is either P or S ." Aristotle's exclusion of these inferences may be explained by the fact that their results cannot be expressed by any of the kinds of judgment which he had in view; for if we follow the ordinary formula, the conclusion must run "some not S is not P ." But this tells us nothing about the relation between the concepts S and P , neither whether they exclude each other wholly or partially, nor whether they wholly or partially coincide. Thus the rule *ex mere negativis nihil sequitur* in its original signification is not affected, although the proposition that everything must be either X or Y may be refuted by an instance in which Z is neither X nor Y .¹

¹ On Schuppe's attempt further to extend the number of inferences which can be drawn from the combinations of premises rejected by Aristotle, see Appendix C.

§ 55.

Where the categorical syllogism presupposes an analytical judgment concerning concepts as its major premise, it cannot serve as a basis for thought in its perpetual progress to fresh knowledge. Its sole function is to keep before us the established relations between our concepts in every application of them. The categorical syllogisms gain a higher significance only when we make them subservient to the formation of our concepts, as was done by Aristotle; or when their major premises are not merely judgments concerning concepts, but synthetical propositions in the Kantian sense.

1. The whole syllogistic process becomes of doubtful value if, instead of regarding it as a means to the construction of concepts by a Socratic *ἐπαγωγή*, we look upon it in the light of the scholastic logic. If, that is, we accept it as based upon a finished system of concepts incapable of further growth, and upon the analytical judgments yielded by such a system.

That is to say, if the normal syllogism consist of three concepts, *S*, *M*, *P*, in simple subordination to each other, then the presupposed relations of these concepts involve the conclusion *S* is *P* as directly as they involve the minor premise *S* is *M*, or the major premise *M* is *P*; *P* forms a part of the intension of the concept *S* just as much as any other of its characteristics or combinations of characteristics. But if *S* represent a particular thing, we cannot be certain of its subordination to *M* until we have enumerated all its characteristics (§ 47, 1, p. 306), thus including those which constitute *P*; indeed, we must know that *S* is *P* before we can say that it is *M*. In the proposition "the square is a quadrangular figure," the predicate is certainly not more remote than in the proposition "the square is a parallelogram," and to infer that "the square is a parallelogram and therefore a quadrangular figure" is quite superfluous. The proposition "this figure is a parallelogram" presupposes that "this figure is quadrangular"; a particular figure cannot be recognised as a parallelogram until it is known to be quadrangular; hence to infer "it is a parallelogram and therefore quadrangular" is not merely superfluous but perverse. Again, what do we gain by this process of continuous ascent to higher concepts? If our object is to extend our knowledge by means of judgments, we are moving in the wrong direction; our predicates become poorer and less significant, we learn less about our subjects, and

lose instead of gaining by our progress. If I know that a square is a parallelogram, I know much more than I can learn by erecting a ladder of inferences which end by teaching me that it is spatial or divisible, or that it is a being of some kind. To this last predicate all inferences must finally attain which mount the conceptual pyramid by successive steps (from species to genus).

2. The nature of the syllogistic process, as represented in the scholastic logic, appears most clearly from the success achieved by the theory that syllogistic inference is really only concerned with the substitution of one term for another. Beneke¹ tells us that in syllogistic inference we replace one of the elements of a given judgment by another, being led to do so by a second judgment, which tells us of a relation between the first term and the new one. The substitution may take place whenever the new element in no way surpasses the old (keeps within the limits of the old). Either the substituted element may be the same expressed differently, or it may be a part of that for which it is substituted. In the inference "some quadrangular figures are not parallelograms; all rhombs are parallelograms, therefore some quadrangular figures are not rhombs," I have substituted rhombs for parallelograms, the part for the whole. In the inference "some parallelograms are oblique-angled; all parallelograms are quadrangles, therefore some quadrangles are oblique-angled," I have substituted another expression (some quadrangles), for the same subject (some parallelograms). In the first instance the new element (rhombs) is a part of the extension of the old (parallelogram); in the second instance the substituted element (quadrangle) is a part of the intension of the old (parallelogram), and enables us to think of the same thing under another expression.

From this theory Beneke derived all possible kinds of inference, and the conclusion he then came to was that such inferences in no way extend or enrich our thought. The part must always be contained in the whole, and in substituting the former for the latter I lose rather than gain in the ideas which form the material for thought.

It is only in inferences with a negative conclusion that any progress is made; for when we think of a concept, we do not think of everything

¹*System der Logik*, I, p. 217. Cf. Ueberweg, *Logik*, § 120. This substitution differs from that which we have mentioned above (§ 50) as Introduction of the Minor (*πρόσληψις*). There the process is one which supplies a definite subject where one was wanting, here we substitute one definite concept for another in which it is contained.

which it is not, and the syllogism thus serves to introduce new differentiations. But every concept as such belongs to us only in so far as it is member of a system and disjunct from its co-ordinate members. Hence the proximate and most important negations are certainly included in the thought of the concept itself, and nothing is to be gained by adducing others which are remoter and less relevant. If I know that man is an animal, that is sufficient to distinguish him from other existing things which are most nearly allied ; I need no syllogism to assure me that he is not a metal nor a geometrical figure.

From this point of view, then, the syllogism can at most serve to warn us, when we connect no definite concepts with our words, of all that is involved in a statement by reminding us of what is really said by our predicate. By recalling all that is involved in our statements, it would lead us to analyse them ; it would thus interpret a proposition when we failed to understand it, but would not help us forward when we did understand. It would be an aid in teaching, or a polemical weapon, but not an organon of knowledge. To demand, therefore, that the syllogism should proceed entirely according to the so-called principle of identity, as, for instance, Leibnitz emphatically did, is to render the syllogism worthless.

3. The importance of the syllogism, or more exactly of the form in which the syllogism is ordinarily presented, was attacked from another point of view by J. S. Mill.¹ In the inference—

All men are mortal ;
Socrates is a man,
Therefore Socrates is mortal,

the conclusion seems to be drawn from the major premise. But in reality the major premise presupposes the conclusion, for before I can know that all men are mortal I must know already that Socrates is mortal ; as long as there is any uncertainty about this proposition, the proposition "all men are mortal" is not yet certain.² Every such inference, therefore, contains a *petitio principii*, it presupposes what it is meant to prove. It is no

¹ *Logic*, book ii., ch. 3, § 2.

² In the same way the minor premise also presupposes the conclusion, as Lotze (*Logik*, 2. ed., p. 122) has shown most completely with reference to conceptual judgments of subsumption. Where would be the truth of the minor premise, "Socrates is a man," if it were doubtful whether or not the attribute mortality belonged to him as well as the other attributes of men, for the major premise adduces mortality as a general characteristic common to all men.

solution of the difficulty to say that the conclusion is not stated explicitly and directly in the premises. We are not, of course, called upon, whenever we state a universal proposition, to think of all the particular instances to which it applies ; nevertheless our statement involves the validity of the proposition for all particular instances, and it is a statement for which we have no ground unless we are already certain of all particular instances.

Must we then say that the syllogism is absolutely useless and without significance ? Mill endeavours to avoid this conclusion by means of a distinction. The real ground for my statement that any man now living is mortal cannot be the general proposition "all men are mortal" ; this presupposes, if valid, that I have some means of knowing that those who are now living are mortal. But we may find the ground in our previous experience of a number of particular cases. Because a number of men have died we infer that those who are now living will also die. Thus we really infer from particular cases to other particular cases. It would seem that the general proposition is quite superfluous, and that we go out of our way in passing through it.

Nevertheless it has a meaning. It is evident that the particular instances known to us cannot enable us to infer a new instance with confidence, unless they provided a sufficient ground for the general proposition also. This general proposition is an abbreviated formula for all that we hold ourselves justified in inferring upon the testimony of experience ; in it is contained the only real inference, and what follows is merely a "deciphering our notes" which we have made to remind us that experience justifies us in passing to further instances. We may have forgotten these particular instances, and remember only that the general proposition was grounded upon them. We then turn to the general proposition and interpret it ; we do not infer *from* this abbreviation of the results of our experience, but *according to* it. In the same way the application of a law or of a general rule accepted on some authority is an interpretation ; we interpret that which the legislator or authority meant to say.

Thus, though we do not naturally pass through the general proposition in inference, we add greatly to the security of our process by so doing. For the experience which justifies the inference to one case must be such as is sufficient to support the general proposition ; and a consciousness of the general proposition is most valuable in enabling us to avoid hasty and ill-grounded inference. It obliges us to consider more carefully the adequacy of our experience ; and if there happen to have been

any contradictory experiences, our attempt to frame a universal judgment recalls them to mind.

These criticisms of Mill's are most instructive, for while disclosing a weak point in the syllogism, they, nevertheless, are an involuntary confirmation of its true and fundamental importance. The weak point disclosed lies in the meaning ordinarily attached to the proposition "All *A*'s are *B*," which is taken to be a mere summation of particular judgments in an abbreviated expression, an enumeration of particular instances. If this were so, then, of course, the certainty of the judgment with regard to the sum-total would depend upon its certainty in regard to the particular items. But the universal major premise should not be understood as the statement of this *numerical generality*; it is the statement of the *necessity* of connecting the predicate with the subject. This necessity cannot be attained, even by a complete enumeration; indeed it can never be known at all in a directly empirical manner. To investigate the conditions under which we may infer from particular experiences to a necessary law upon which they rest, is the main object of a theory of induction; and we hope to show that such an inference can never be possible unless we assume that some unconditionally valid principles are to be found. Mill's position is justified to the extent that the universal major premise is drawn ultimately from particular data, which are the real proof for those judgments which have reference to the empirical; but it is false that the major premise might be dispensed with in inference. Only by proving necessity can the particular data be proof of any particular case. His argument rests upon a confusion between the two processes of describing the psychological process of inference and laying down logical laws for it. There is no doubt that we frequently infer from one particular case to another, but the question is whether we are justified in so doing. This question is decided by the validity of the general proposition, which does not merely—as Mill represents—yield a collateral security, but is the one thing which makes the inference legitimate. Mill himself allows that the inference from particular cases to a new instance is not justified unless the general proposition may be inferred; and in reality the truth of the universal major premise is the condition of the truth of the conclusion. Thus the conclusion does, after all, depend upon the major premise, and cannot be proved without it.

But all that is maintained by the Aristotelian doctrine of the syllogism is nothing more than this: that a satisfactory and scientifically valid inference is possible only in the syllogistic forms, and on condition of a universal

major premise. Aristotle himself teaches that we arrive at universal major premises by means of induction. But then his induction is certainly not based upon the purely empirical ground of a collection of instances, which, as it can yield no necessity, would make logic impossible ; but upon the presupposition that particular phenomena are governed by conceptual necessity, which must therefore be knowable from particular instances.

Thus the attack fails to touch the absolute validity of the syllogistic rules wherever a reliable inference is to be made from one judgment to another. It is only when the attempt is made to base the syllogism entirely upon the so-called principle of identity—when therefore the premises are purely analytical propositions—that the syllogistic process seems to be without value.

4. So far is this from being Aristotle's view that he regards the syllogism as the preliminary means of attaining that which is presupposed in the scholastic logic—definition. His premises are mainly empirical judgments concerning that which is given in intuition, and the syllogism is the means by which he arranges these items of knowledge in such a way as to exhibit their mutual dependence. In this way the material dependence of the conceptual determinations realized in being, the true causal relation is exhibited ; and thus an exhaustive definition is framed, which expresses a dependence of the special determinations upon the general corresponding to these conceptual relations. It is for this reason that the middle concept is said to correspond to the cause, and that the premises are selected and arranged in such a way as to represent the real dependence amongst things.

Although the syllogism as thus applied is most closely connected with the Aristotelian metaphysics, yet the logical laws are not limited to this special application ; the end in view merely determines the particular way in which they are formulated. The traditional logic has forgotten the end, but has retained the formulation to which it led, and which is apparent in the exclusively categorical treatment, and more especially in the coordination of the particular with the universal judgment. It is no wonder that the formulæ of logic no longer correspond to the altered problems of science.

5. Mathematics, which throughout makes use of the syllogism and owes its scientific certainty to this form, is often cited as an answer to all attacks upon the theory of the syllogism. This is quite justifiable so far as showing that all mathematical propositions, with the exception of axioms and

definitions, are proved by syllogisms, or at any rate on the principles by which the syllogistic forms are determined. But to overlook the wide difference between mathematical propositions and the examples of the scholastic logic, with their analytical judgments, is a great mistake. Do we find such inferences as "the square is a parallelogram, therefore it is a quadrangular figure," "the circle is a curve of the second degree, therefore it is a conic section," in geometry? does it ever concern itself with such trifling subsumptions? This is all included in the definition of the particular object, and the syllogism is not there for the purpose of repeating this definition; the real task of geometry is to develop the laws of the relations which certain conditions bring about between particular objects, such as lines and angles, the relations, that is, of equality, inequality, etc. From the point of view of the concept these relations are predicates which are added to it from without; they are not contained in the definition, and cannot be learned from it; indeed they do not exist until the particular objects enter into spatial relations. In the concept of the triangle—that is, in its definition, we find absolutely nothing about the fact that its angles are equal to two right angles, the idea of two right angles is not contained in the idea of the triangle. The judgment is based firstly upon the addition of the angles, and secondly upon their comparison with two adjacent angles, that is, it is based upon relations which have to be created. It forms no part of the concept of the right-angled triangle that the square of the hypotenuse is equal to the sum of the squares of the two remaining sides; for in the concept of the triangle I think of nothing more nor less than a plane surface, bounded by three intersecting straight lines, and in this there is nothing to oblige me to construct squares upon the sides and then compare them. It is not until I have done so by means of inventive construction that I can investigate the relations in which these squares stand to each other.

Geometry, then, always goes beyond mere conceptual judgments in seeking its propositions, and by combining necessary relations taken from some other source with that which is given in the definition, it obtains predicates which are not contained in the definition. But just for this reason we cannot, as a rule, regard its major premises as judgments of subsumption, and though its syllogisms may appear to be framed in the scholastic form *Barbara*, they are not so in reality. Ueberweg,¹ for instance, gives

¹ *System der Logik*, edn. 3, p. 304; edn. 5, p. 360. We find a similar example in Wundt, *Logik*, I, p. 297. Cf. my "*Logische Fragen*," *Vierteljahrschr. f. wiss. Phil.*, iv. p. 478.

as an example of this figure : All triangles with proportional sides are triangles with respectively equal angles—all triangles with respectively equal angles are similar figures—therefore all triangles with proportional sides are similar figures ; and this inference is to all appearance the same as : All negroes are men, all men are mortal, therefore all negroes are mortal. In reality, however, the difference is very wide. There is no specific concept formed from the concept of triangle by means of the differentia “proportional sides,” nor any general concept “similar figure” to which the specific concept might be subordinated by means of the middle concept “triangle with respectively equal angles.” The inference does not proceed by means of such subordination at all, but solely by means of relations which form no part of the concept triangle. Where two or more triangles are given having proportional sides, it follows that the other relation—equality of angles—will also be present ; and because equality of angles involves the similarity of the triangles it follows that the relation of similarity is given together with the relation of proportionality of sides. It is only by a crude inaccuracy of expression that these propositions can take the form of a proposition concerning “all triangles” of a particular construction, as if the predicate might be true of each particular triangle in itself. The inference, correctly expressed, would run :

If two or more triangles have proportional sides, they have equal angles.

If two or more triangles have equal angles, they are similar. Therefore, if two or more triangles have proportional sides, they are similar.

It is evident that from the nature of the case the propositions must be expressed hypothetically, if what they mean is that *one* relation between different things necessitates *another*.

It is significant that the guiding principle of mathematical inferences is the proposition that two magnitudes which are equal to a third are equal to each other ; that is, it is a proposition concerning necessary connection between relations. Equally significant is it that mathematical reasoning frequently proceeds by substituting one magnitude for another to which it is equal. Processes such as these find no place in the ordinary forms of the syllogism, though by means of these general laws they can always be exhibited as strictly syllogistic.

6. What we have said of geometry applies also to other branches of science. What we have to establish by inferences are elements not yet contained in the concept, not given analytically ; and these are, on the one hand, relations, on the other everything which depends upon varying and

changeable events, more especially therefore all causal relations. The inference of the judge does not proceed by subordinating particular offences. When once the case before him is subsumed and recognised as murder he does not infer analytically "therefore it is a crime, and therefore an infringement of the law," but he proceeds at once to the conclusion commanded by the synthetical rule of the law—"therefore it is to be punished by death." Capital punishment is not analytically contained in the concept of murder, it is synthetically connected by the will of the legislator with that particular crime. The physician who has diagnosed an illness as typhus does not go on to infer that it is therefore an infectious disease, but he infers that therefore the treatment must be this or that; the remedies which counteract typhus are not analytically contained in its concept, but are synthetically demanded by the rules of experience. When the physicist knows that a body has fallen for four seconds it would avail him nothing to analyse the concept of the fall; but if he introduces the particular value into the formula $S = \frac{1}{2}gt^2$ he knows that the distance of the fall is 15×16 feet.

Here again Kant's teaching becomes important. His question "How are synthetical judgments *a priori*, i.e. unconditionally and universally valid synthetical judgments, possible?" is for the syllogism also the vital question, without which it is but a barren process.

7. From what has been said we may understand how important for inference are all those general principles which have reference to necessary connections between relations, and by means of which we obtain relational judgments. Such are the principles that two concepts or objects which are identical with a third are identical with each other; that two magnitudes which are equal to a third are equal to one another; that if equals are added to equals the results will be equal; as well as those principles which govern spatial relations. It is, after all, a question of secondary importance whether these principles are themselves looked upon as analytical because derived from the concepts of identity, equality, etc.; or whether they are accepted as synthetical judgments *a priori*. The important point is that because they refer to relations they enable us to pass beyond the merely analytical judgments which alone are treated of by the ordinary logic.

8. For this reason the scholastic categorical syllogisms are much too narrow, and ill-adapted to serve as general formulæ of easy application. For judgments of subsumption, and judgments which state simple predi-

cates of a subject, they are the natural expression; so soon as we have to do with more complicated relations, such as the dependence of a predicate upon several conditions, they become inapplicable. Here the hypothetical form with its consequent *πρόσληψις* becomes the natural expression; as comprehending all universal categorical judgments, it is the formula which naturally presents itself for use, more especially because in it necessity and not generality appears as the true basis of inference. We need but glance at any mathematical or physical textbook to assure ourselves that by far the greater number of propositions which are used as major premises do not take the form of universal categorical judgments, but are hypothetical in expression or at any rate in nature. Propositions such as "two circles which intersect have no common centre," are hypothetical in nature; the relative proposition states the condition upon which the predicate is denied. And in the same way the first axioms are of the nature of hypothetical judgments. The proposition "two straight lines cannot enclose a space" means "no matter where or how I draw two straight lines, the *two together* will not include a space"; it says nothing about two straight lines in the sense of stating an attribute which is common to both. The proposition, "everything which happens has a cause" presupposes by the predicate of the first clause that something actually happens; it does not develop the concept of happening, but states the connection of every particular event with something else which exists. It is the same with the formulæ of analytical mechanics; these and others of the same description are hypothetical judgments, and inferences are made in accordance with them by substituting definite values for the general symbols.¹

¹ Some writers—e.g. Wundt. (*Logik*, I, p. 291)—include amongst the forms of inference a special "inference of identity," and even of "equation"; but I consider this superfluous. We might as well give a special place and name to the inference $a > b, b > c, \text{ therefore } a > c$. The peculiarity of these forms does not lie in the manner of inference itself, but in the particular principle upon which they are based, though this, no doubt, is frequently not expressly formulated because it is taken for granted. *A* the same as *B*, *B* the same as *C*, therefore *A* the same as *C*, is nothing more than an application of a principle referring to the particular predicate of identity. The only possible reason for laying special emphasis upon this inference is that in inaccurate language the judgment *A* is *B* frequently means identity without expressly saying so.

On the whole it seems to me that there is little to be gained by specializing the doctrine of inference any further. On the contrary, I prefer to emphasize the common element in all kinds of inference instead of multiplying distinctions of *form* which in fact consist merely in the different *contents* of the premises which determine the conclusion; to exhaust these would be impossible. It is true that Wundt attempts to give a general formula for all inference. "When different judgments are related to each other by concepts which are common to both, then the other concepts in the judgments which are not com-

§ 56.

The syllogism which has a CONJUNCTIVE JUDGMENT for its major premise serves to subsume the particular under established concepts by means of their definitions.

1. The syllogism finds a special function in subsuming the particular under established concepts, and in so doing it falls into certain forms adapted to this purpose.

There is no other way of finding out whether any thing A falls under a concept B than by showing that it contains all the characteristics of B ; if these are to be found in it without exception, then it falls under the concept B . Here, then, the middle concept is not a single predicate, but a series of predicates which are combined in a conjunctive judgment, but which assume the function of a single concept just because they belong together.

One characteristic alone is sufficient to show that a thing A does not fall under a concept B if it belong to A and is excluded from B ; the subsumption is then rejected by a syllogism of the second figure; *i.e.*, *modo tollente*.

Thus we get the forms having for major premise a definition: P is a , b , c , or the reverse

That which is a , b , c is P ;

S is a , b , c ,

Therefore S is P .

The form of exclusion is:

P is a , b , c ;

S is not a ;

S is not P .

and this coincides with the syllogisms of the second figure *modo tollente*.

mon to both are also related, and this relation finds its expression in a new judgment. But even if we disregard the vagueness of this formula, the proposition is false, because too wide; how are the concepts S and P related when S is not M and M is P ? Lotze again, remarks that the most important inferences would be such as would not merely predicate a general P of S through the mediation of M , but would show how the particular modifications of the M which belonged to the S necessitated a particular form of P . This is quite true so far as it goes, but it involves no special mode of inference; it merely involves principles different from those which are ordinarily noticed in the categorical syllogism—principles which state the law according to which every modification of M brings with it a modification of P , but from which our conclusions must always be drawn according to the simple rules of hypothetical inference. Let S be a figure, M an ellipse, and P eccentric. Then $E = \sqrt{a^2 - b^2}$: a states the law according to which every modification of the relation of the axes involves a modification of the eccentricity; but the equation is by nature a hypothetical judgment, from which we may draw conclusions by substituting definite values.

§ 57.

Inference from a DIVISIVE JUDGMENT, which some logicians have called inductive inference, cannot lead to an unconditionally universal judgment when the division is only *empirical*. If it is *logical*, the inference is superfluous, unless, indeed, it is introduced incidentally, say, as a link in a chain of inferences.

1. An attempt has been made to add to the syllogistic forms the so-called inductive syllogism, which resembles the inference from definition, in that the middle concept is not simple in form. When, that is, we have a complete division of a concept *A* into the species *M*, *N*, *O*, or a complete enumeration of the individuals which fall under it, and a common predicate belonging to all the species or individuals, then we get the syllogism :

A is partly *M*, partly *N*, and partly *O*;

M and *N* and *O* are all *P*,

Therefore *A* is *P*.

2. But in this formula there lurks an ambiguity, which may be cleared away by our distinction between empirical and logical extension.

Let us begin by considering an example, that, for instance, given by Apelt :¹

Major premise : The solar system consists of the sun, and the planets Mercury, Venus, Earth, Mars, etc.

Minor premises : Mercury moves round the sun from west to east, Venus moves round the sun from west to east, etc.

Conclusion : All planets move round the sun from west to east.

Here the major premise states the extension of the concept planet ; the conclusion affirms of all planets a predicate which, according to the minor premises, belongs to each one by itself.

But what is gained by this? Not an unconditionally universal judgment, attributing to the concept planet the necessity of moving from west to east ; but merely an empirically universal judgment which combines the particular subjects of the minor premises under one name, after it has been established by the major premise that those mentioned are all the planets—of course, only so far as we know at present. The word planet does not serve as the symbol of a definite concept, but only as the common name of a definite number of particular things. Hence the inference in which one judgment is grounded upon others, refers only to our right to substitute a

¹ *Theorie der Induction*, p. 17.

common denotation for the proper names, and the expression "all" for the sum of the particular planets.¹ It is, however, in no way proved that everything which is a planet must *necessarily* move from west to east. The judgment which merely contains an empirical enumeration of the planets cannot tell us whether they owe their proper movement to the attributes on account of which they fall under the concept of planet, or to some other cause, which, so far as concerns the concept in question, is accidental. If it could, then it must also follow that because all the kings of Prussia have been called Frederick and William, therefore all kings of Prussia must of necessity be so called.

3. It is just the same when, instead of naming the individuals, we name the empirically known species of a genus. At one time, when none but the older metals were known, it was a valid inference to say :

The metals are gold, silver, iron, etc.,

Gold, silver, iron, etc., are heavier than water

Therefore all metals are heavier than water.

By "all metals" is meant those which were known and were so called because of their common properties. But it does not follow that these common properties necessitate a specific weight greater than that of water, and this conclusion has been refuted by the discovery of kalium.

To call such an inference inductive is a fundamental error, since the essence of inductive inference consists in passing from empirical data to an unconditionally universal judgment. For such an inference it is necessary to show that those attributes upon which the common denotation is grounded necessitate also the new predicate.

4. If again the judgment should rest upon a *logical* division such as

¹ Given in full the premises would be :

Mercury, Venus, Earth, etc., move round the sun from west to east.

Mercury, Venus, Earth, etc., are planets.

Therefore so many planets move round the sun from west to east.

The number of these planets is equal to the number of all the planets.

Therefore all the planets move from west to east round the sun.

The nature of the inference is just the same as when I infer from the premises :

$M_1 M_2 M_3$ have attribute P

$M_1 M_2 M_3$ are three M 's,

that therefore three M 's have attribute P .

Thus the inference by which we obtain a judgment of empirical universality is based upon counting, upon the expression of a given plurality by a definite numerical concept ; my right to substitute "three M 's" for $M_1 M_2 M_3$ is based upon the identity of the number. Cf. above § 52, 4.

would guarantee the absolute completeness of all members of the division, the inference would be a superfluous circumlocution. For if one predicate necessarily belongs to all the species of a genus, this predicate must have a ground in something common to them all; *i.e.*, in their generic concept, from which it may be inferred:

Parallelograms are squares, oblongs, rhombs or rhomboids.

Squares, oblongs, rhombs and rhomboids have diagonals which bisect each other.

Therefore all parallelograms have diagonals which bisect each other.

Such an inference as this is an instance of unnecessary circumlocution, for the predicate might have been derived directly from the determinations which constitute the concept of the parallelogram.

There are cases, however, in which we naturally pass through a complete enumeration of particular cases on our way to the universal proposition. The proof that the angle at the centre of the circle is double the angle at the circumference on the same arc, begins by showing that the apex of the angle at the circumference falls either upon one side produced of the angle at the centre, or upon a chord falling within or without the angle at the centre. In any one of these three cases it may be shown that the angle at the centre is double the angle at the circumference; therefore it is universally true that when an angle at the centre of a circle and an angle at the circumference stand upon the same arc the former is double the latter. Here again the proof is drawn from conditions which are common to each case; but in each case the conditions are subsumed under different major premises, and we arrive at the truth of the minor premises in different ways. It is, however, clear that this can happen only when the minor premises are inferred, never when they are immediately certain.

5. In the second figure such a division seems to give rise to an inference in a somewhat different way. If it is true that—

A is *B*, *C* or *D*,

S is neither *B* nor *C* nor *D*,

then it follows that *S* is not *A*.

That which falls under none of the species of a genus cannot fall under the genus itself. But here again we may say that if the division is empirical, the inference is invalid, for the empirical extension is no guarantee that the generic attributes are not to be found outside of the known species. If, on the other hand, the division is logical, then the charac-

teristic which excludes *S* from all species must be incompatible with the genus, and it is unnecessary to go through the division.

§ 58.

The so-called DISJUNCTIVE SYLLOGISM has no special principle of its own; it ought not therefore to be given as a particular kind of syllogism.

1. In addition to the hypothetical and categorical syllogisms ordinary logic gives also disjunctive syllogisms. These have a disjunctive judgment for their major premise, and it is upon the relation of terms expressed in the disjunction that the inference depends. If, for instance, the disjunction is of two terms, and *A* is either *B* or *C*, then by attributing one predicate we exclude the other, while the denial of one predicate necessitates the affirmation of the other.

I. Modus ponendo tollens :

A is either *B* or *C* ;
But *A* is *B* (or *C*),
Therefore *A* is not *C* (not *B*).

II. Modus tollendo ponens :

A is either *B* or *C* ;
But *A* is not *B* (not *C*),
Therefore *A* is *C* (or *B*).

Where the disjunction contains more terms the first modus leads to a conjunctive negative judgment; the second gives a simple affirmative only when the minor proposition is a conjunctive judgment denying all the terms but one; where this is not the case the conclusion only limits the disjunction to fewer terms.

I. *A* is either *B* or *C* or *D* ;
A is *B*,
Therefore it is neither *C* nor *D*.

II. (a) *A* is either *B* or *C* or *D* ;
A is neither *B* nor *C*,
Therefore *A* is *D*.

(b) *A* is either *B* or *C* or *D* ;
A is not *B*,
Therefore it is either *C* or *D*.

The major premise as here formulated, however, does not yield the most

general formula of the disjunctive syllogism, it is only a special instance of the major premise :

Either the judgment B is true, or the judgment C ;
 B is true, therefore C is not,
 B is not true, therefore C is, etc.

2. There is no ground here for a special syllogistic form, with a principle of its own. All that the disjunctive judgment tells us is, in the first place, that its terms are mutually exclusive, hence that the affirmation of one involves the exclusion of the other ; that is, the *modus ponendo tollens* is an inference from the hypothetical judgment contained in the disjunction “if A is B , it is not C (neither C nor D)” ; in the second place, that the negation of all the terms but one necessitates the affirmation of this one ; that is, the *modus tollendo ponens* is an inference from the hypothetical judgment “if A is not B , it is C (when the disjunction has more terms—if A is neither B nor D).” Thus the principle of the inference differs in no way from that of the hypothetical syllogism. The disjunctive judgment owes its importance to the fact that it expresses this twofold necessity, but the distinction between the disjunctive and hypothetical syllogisms is based upon the grammatical form alone.

3. In the actual use of disjunctive inference the major premises are frequently—in meaning, at least—hypothetical judgments having a disjunctive consequent ; and from these the inference is drawn by means of a *πρόσληψις* :

If anything is A , it is either B or C ;
 S is A , and also B ,
Therefore it is not C .
 S is A , but not B ,
Therefore it is C .

Such inferences serve for the progressive subsumption of an object under more and more definite concepts.

4. Further, the inference which, from the denial of all the terms of a disjunction, infers the denial of the antecedent presupposition common to all is akin to the form of inference given in § 57, 4.

If A is true, then either B or C is true ;
But neither B nor C is true,
Therefore A is not true.

Or by making use of a *πρόσληψις* :

If anything is P , it is either M or W ;
 S is neither M nor W ,
 Therefore S is not P .

In the categorical form :

A is either B or C ;
 S is neither B nor C ,
 Therefore S is not A .

Here we have the so-called dilemma, trilemma, etc. Here again the inference is based upon the general principle that the denial of the consequence involves the denial of the ground; the only difference is that here the consequence is not simple, but consists of a number of mutually exclusive possibilities.

§ 59.

The rules of the syllogism apply in just the same way even when the premises are not stated as valid judgments, but only as assumed hypotheses. They then give rise to a hypothetical judgment, which exhibits the conclusion as the necessary consequence of the premises.

In this way the principles that the ground involves the consequence, and the denial of the consequence the denial of the ground, are applied to the relation between the truth of the conclusion and the truth of the premises, as well as the propositions that the negation of the ground does not necessitate that of the consequence, nor the affirmation of the consequence that of the ground.

1. There would be no object in a detailed investigation of the different combinations which may be added to the list of inferences by means of verbal abbreviations, or the introduction of copulative, conjunctive and disjunctive propositions. The inference is always brought about by the same means; its fundamental condition is a major premise which in some form or another includes a necessary consequence, and constrains us to affirm a proposition in case another proposition is true. To this is added the minor premise which indicates an instance to which the major premise is applicable. This may be done either directly, as in the mixed hypothetical syllogism, or by applying a general rule to a special case included in it by means of a judgment which shows that the general rule of the major premise is applicable to a certain subject. For this reason we shall not here investigate the sorites, which are nothing more than repeated applications of the rules of the syllogism, expressed in an abbreviated form.

2. In every syllogism it is asserted that the validity of the conclusion follows from the validity of the premises; and where the rules of the syllogism are observed this is true even when the premises are only hypothetically assumed. The simple hypothetical syllogism then reduces itself to its major premise. Others, which contain more than the simple assumption of the hypothesis, may be exhibited in hypothetical judgments of the form "if A is true and B is true, then C is true" (if all men are mortal, and Caius is a man, then Caius is mortal); judgments which emphasize only the relation of consequence, apart from the validity of the premises. Most hypothetical judgments are really based upon syllogistic relations such as these; when one premise is taken for granted and not expressly stated, they then appear as hypothetical judgments with a simple antecedent.¹

3. From this it follows that the principles concerning the relation between ground and consequence may be applied to the relation between conclusion and premises when these are regarded as nothing more than hypotheses.

Not only, then, may we say that if the premises are true the conclusion is necessarily true, but also that if the conclusion is false then the ground from which it necessarily follows must be false. But since this ground consists of two premises, we can infer from the falsity of the conclusion only the falsity of one at least of the premises—either of major or minor premise.

But it does not follow that if the premises are false, the conclusion must also be false; nor does it follow that if the conclusion is true the premises must also be true. Indeed it may happen that a true conclusion follows with syllogistic necessity from false premises.

More especially, therefore, it does not follow that if one premise and the conclusion are true, the other premise must also be true. Hence, the fact that a proposition known to be true can be represented as the syllogistic consequence of two premises, one of which is also known to be true, does not enable us to infer that the other premise is also true.

¹ Cf. my *Programm*, p. 40, and the examples there given. The necessity expressed in the hypothetical judgment "if Caius is a man, he is mortal" is grounded upon the suppressed proposition that all men are mortal. The judgment "if the earth moves round the sun, the fixed stars have an annual parallax" presupposes a whole series of inferences in which the premises are geometrical propositions of absolute certainty, and in no way partaking of the hypothetical nature. But besides these hypothetical judgments there are others of which the necessity is immediately known.

APPENDICES

APPENDIX A.

THIS view of the negation, and of its relation to the positive statement that a predicate *P* belongs to a subject *S*, is at variance—though in different ways—with the view of Lotze, Brentano, Bergmann, Windelband (in the *Strassburger Abhandlungen*, 1884, p. 167 sq.). These writers all agree in co-ordinating affirmation and negation ; they teach that the thought which predicates *P* of *S* is at first undecided, and that according to the attitude of mind with which we approach this thought we state it either as valid or as invalid. But Lotze (edn. 2, p. 61) regards the thought of the relation between *P* and *S* as the essential part of the judgment, and represents the affirmation and negation of this thought as two opposed minor judgments which predicate validity or invalidity of it ; while the other logicians take the opposite course, and consider the essence of the judgment to lie in this decision as to validity or invalidity ; the subject of the decision, they say, is not a judgment, but a combination of ideas, or—like Bergmann—simply an idea.

They have been led to this sharp distinction between the act of affirmation or negation, and the subject which is affirmed or denied, by the fact that in the former a psychical function becomes active which differs essentially from the mere formation of ideas, or of combinations of ideas, a function more nearly akin to practical action than to ideation.

Brentano (*Psychologie*, 1, p. 266 sq.) was the first to give a decided statement to this distinction, and he was followed by Bergmann (*Reine Logik*, 1, p. 46), who calls judgment a critical attitude with respect to an idea, a reflection upon its validity. He adds : “ decision as to the validity of an idea—that which judgment adds to mere ideation—is something more than a merely theoretical relation ; it is not only a function of the intelligence as opposed to the will, it is a manifestation of the spirit, in which its practical nature, the faculty of willing, participates.”

Windelband takes the same view, and since his treatment of the subject is the most thorough and the most carefully reasoned, it will suffice to ex-

amine his arguments. As regards Bergmann and Brentano, I may refer to the *Vierteljahrschr. f. wiss. Phil.*, v. 97 sq., and my *Impersonalien*, p. 58. Windelband distinguishes (*Präludien*, p. 28 sq.) between judgments simply (*Urtheile*) and (*Beurtheilungen*) critical judgments. The former express that two ideal contents belong together, the latter the attitude of the critical consciousness towards the object thought of. In a judgment we always state that a certain idea (the subject of the judgment) is added in thought to another idea (the predicate of the judgment), the relation between them differing according to the different forms of the judgment. In a criticism, on the other hand, the critical predicate is attributed to something which is assumed to be completely known, or of which we have a complete idea (the subject of the critical proposition). This predicate in no way extends our knowledge of its subject, but expresses the feeling of approbation or disapprobation with which the critical consciousness regards the object in thought (a thing is white—a thing is pleasant or unpleasant, an idea is true or false, an action is good or bad, a landscape is beautiful or unpleasing, etc.). Moreover, none of these critical predications have any meaning except in so far as the object thought of corresponds or not to an ideal with reference to which it is estimated by the critical consciousness. Critical predicates contain a reference to a consciousness which sets before itself certain ideals.

A particular application of this theory is to be found in reference to our desire for knowledge. In so far as our thought aims at knowledge—*i.e.* at truth—all our judgments are subject to criticism, which expresses either the validity or the invalidity of the combination of ideas which has taken place in the judgment. The purely theoretical judgment is to be found only in the so-called problematical judgment, which combines certain ideas but says nothing as to the truth of the combination. Whenever a judgment is affirmed or denied, there is added to the theoretical function that of a criticism as to its truth . . . All the propositions of knowledge are combinations of ideas whose value for truth has been decided by affirmation or negation.

Every critical judgment (he continues, p. 34) is the reaction of a willing and feeling individual against a certain ideal content. . . . But the point of view from which the criticism is made, is expressed by the contradictories pleasant and unpleasant, true and false, good and bad, beautiful and ugly. The first pair is peculiar to the individual, the others all contain a claim to universal validity. The same view is to be

found in the "*Beiträge zur Lehre vom negativen Urtheil*" (*Strassburger Abhn.*, p. 170), where it is said that the negation is a practical judgment, a *criticism*, that it expresses not merely a relation between ideas, but a disapproving attitude of consciousness towards the attempt to relate them; it is in this that the rejection consists. For this reason Windelband will not follow Brentano in placing judgment, as a special class of psychical activity, between the theoretical formation of ideas and the practical activities of love and hatred. He prefers to rank the logical estimation of ideas on the practical side of psychical life, and holds that value from the point of view of truth must be co-ordinated with other values.

I cannot completely agree with these views, much as there is of truth in them. I have myself (*Introdn.*, §§ 1-4) laid stress upon the fact that logic as such—as a critical and normative science—starts from an *ideal*, the ideal of truth; that it presupposes the desire to think truly, and estimates every actual judgment by this final ideal; and that it seeks to distinguish between the mental operations which are conducive to this aim and those which thwart it. The logical treatment of these operations, as distinguished from the psychological, is based entirely upon the consciousness of the ideal, and I agree also with the further argument of the *Präludien* (p. 43) that logic starts from the ideal of a normal consciousness (cf. § 32, 7 and vol. ii. §§ 61, 62). But not even the logician can infer from this that in particular instances of affirming or denying he assumes a practical attitude towards the judgment because he estimates the particular combination of ideas by the universal ideal of truth; or that his activity is a reaction of feeling or will, and not theoretical at all. When I make it my aim to keep myself in good health, no doubt I do so by means of my will and by reason of some feeling; and if on this account I give up some bad habit or refuse the temptation to excess, there is no doubt that my will is active in giving up the habit or rejecting the temptation; it determines my behaviour with a view to my aim, and my No is a practical "I will not." But after all this will is based upon the purely theoretical knowledge that the habit is injurious, the temptation fraught with danger. My will and feeling have no direct share in this knowledge as to what is conducive to my health or the reverse, for this depends upon our experience of the nature of things, not upon willing or feeling. Nor does it follow that because I desire to know the truth, therefore my critical judgment of a proposition is an act of will. The distinction between a purely objective judgment, and "criticism" with reference to some aim is important enough so far as

regards their contents ; but after all such criticisms are all of them judgments, which may be true or false, and differ only in that they are judgments about the relation of the object to me and to my aim, not judgments about the object by itself. This relation, moreover, simply exists and is recognised ; it is not approved or disapproved. "Sunshine is pleasant" is a criticism of sunshine in relation to my feeling ; but the criticism itself, which the proposition expresses, is neither feeling nor willing, but the simple recognition of the fact that sunshine excites a certain feeling in me. The reaction of man as feeling is his enjoyment of warmth ; the proposition in which he gives expression to this enjoyment is a function of his thought. He has formed the general ideas of pleasant and unpleasant from his experience of contrary feelings, and by means of these ideas—which are not themselves feelings—he expresses the relation which actually exists between himself and certain things. It is the same with good and bad, beautiful and ugly. The judgments in which these are predicated differ from others only in the nature of the predicates, not in the function of judgment itself ; and these predicates express a relation between an object and myself, my will and feeling, which reappears in particular instances.

But the predicates true and false contain not even such direct reference to will and feeling, as is contained in the pairs which Windelband coordinates with them. True and false as general ideas do not denote a relation to the practical side of our life ; it depends neither upon our feeling nor upon our will what is true or false, as it depends upon them what is beautiful and what is good. For true and false are not predicates of any objects of which I form an idea or about which I think ; nor are they—as Windelband somewhat inaccurately says—predicates of concepts. They are predicates of judgments formed by us. As he says more accurately elsewhere, they refer to combinations of ideas, not, that is, to ideas already combined, to completed combinations, as if it should be said that green tree or black horse were true or false ; it is the act itself of combining the ideas, the act through which the consciousness of unity arises, to which the antithesis applies. Thus it is not ideas of any sort of objects which we criticise by the predicates true and false, but the judging activity itself.

Now it is perfectly true that where we actually find these predicates, where the question arises as to the truth or falsehood of a judgment either tentative or completed, there is in the background an ideal which, if not

distinctly conceived, is at least vaguely striven after ; this is the ideal of knowledge, for the antithesis has no significance when we are speaking of arbitrary fiction, and merely trifling with thoughts. It is this ideal which affords us a standard by which to estimate the statements which suggest themselves to us or the assertions of other people ; we regard some as conforming to the ideal, others as contradictory to it. In one sense we may indeed say that this involves an approval and disapproval ; for in proportion as the ideal is clearly conceived and earnestly striven after, the agreement of a given judgment with it is certain to arouse a feeling of pleasure, its disagreement a feeling of pain (though in the narrower and stricter sense approval and disapproval could extend only to actions regarded as voluntary ; we disapprove of error when it is a fault, such as the consequence of inattention). Nevertheless, it is presupposed by this approval and disapproval that the relation of a tentative or completed judgment to the ideal of truth should have been recognised as completely objective. We disapprove of the false because it is false, but it is not false because we disapprove of it. Our feeling must be grounded upon the theoretical knowledge that a judgment is true or false, just as we must know that the means will lead to our end before we choose them.

But from the logical point of view, where we estimate every judgment by the standard of truth, the question as to truth or falsehood extends to negations as well as affirmations. We say of negations also that they are true and false, and this alone is sufficient reason for denying that the antithesis of approval and disapproval is to be unhesitatingly set down as coinciding with that of affirmative and negative judgments ; nor can we find in it any ground for the co-ordination of affirmation and negation.

As a matter of fact both positive and negative judgments are estimated according to a logical standard, and we must therefore distinguish between this logical treatment, which criticises, according to its ideal, the movements of thought as they actually occur, and the psychological treatment. The latter inquires as to what takes place in our actual thought, how the negation arises in its course, and how it is possible that the universal ideal of truth, which is the ground of approval or disapproval, should arise at all. Here I may briefly state my view of the question, which in all essential points agrees with Windelband's. I start from the simplest immediate acts of judgment which arise from intuition ; in these we have the connection between the ideas and the consciousness of its validity, given together without any reflection, nor can it be said that we

are in any way conscious of an ideal. These are acts of judgment which take place quite unintentionally and with all the confidence of a process which is natural and necessary, such as the recognition of objects around us, the judgments that this is here and that is there ; such judgments are accompanied by the immediate certainty that they are evident. If we were psychologically unable to combine ideas in any other ways than this, we should never think of inquiring as to truth and falsehood. But our thought does extend beyond that which is given. Mediated by recollections and associations, judgments arise which, when first formed, are also regarded as expressions of that which actually is ; when, for instance, we expect to find a familiar object in its familiar place, or assume that a flower must have a scent. But some of these conjectures prove to be at variance with the immediately certain ; when we fail to find what we expected, we become conscious of the distinction between the mere idea and the actual thing ; that of which we are immediately certain differs from what we had anticipated in our judgment. Here the negation appears, and annuls the conjecture by denying its validity. This introduces a new attitude of mind in that the subjective combination is separated from the consciousness of certainty ; it is compared with a certainty and recognised as different. In this way there arises the conception of invalidity. But this attitude of mind presupposes not only the subjective combination, but also a tendency to maintain its validity. The negation—as Fichte says—is conditioned in matter, and unconditioned in form only, just as the conception of difference (the importance of which for the negation is rightly emphasized by Schuppe) presupposes the idea of distinguishable objects, but is not given with it, the general idea arising only after reflection upon particular acts of distinction. Thus the negation is doubly dependent upon the positive judgment. It presupposes as its object a thought which was expected to be valid, and it rejects an attempted statement. The ground of its rejection, again, is ultimately something positive, an object given to intuition and recognised as different from my idea—*verum sui index et falsi*. It is only through experience like this that the ideal of truth becomes conscious ; we cannot feel the value of truth until the untrue has drawn our attention to it. We must have experienced both the immediate and evident certainty of immediate judgments, and the difference between subjective combinations and immediate certainty, before we can form the idea of truth.

The negation, then, is not co-ordinate with the positive judgment, but

presupposes it both in the synthesis between subject and predicate and in the certainty of this synthesis, and this relation finds a clear expression in language. If the view that affirmation and negation are two co-ordinate modes of regarding a problematical synthesis SP were correct, it would be strange that while the negation finds special expression in language, the affirmation, as a rule, has none (Bergmann and Windelband expressly recognise this). Only when a statement is threatened with denial do we find the ἤ μὴν, indeed, verily, etc.

Windelband rightly points out (*Strassb. Abh.*, p. 170) that if we carried our reflexion further we might go on indefinitely with the predicates valid and invalid. A is B —it is true that A is B —it is true that A is B is a true proposition, etc.; A is not B —it is true that A is not B —it is false that A is not B is a false proposition—it is false that A is B is a true proposition, etc. But this is no difficulty for our view; on the contrary, it confirms it by showing that the judgment “the proposition is true, the proposition is false” differs in nothing but its predicate from any other judgment. The same never-ending reflection takes place with respect to our self-consciousness: *qui scit, eo ipso scit se scire . . . et sic in infinitum* (Spinoza, *Eth.*, ii. 21)—though indeed it is an abstract possibility only. In reality we always come to some point where there is a certainty which reflection fails to separate from that to which it refers, and which is not therefore specially emphasized. Thus the objection proves what it was meant to refute; it proves that there is no judgment which is not ultimately based upon that immediate judgment in which we can no longer separate the connection of ideas from “approbation” or “confirmation.”

APPENDIX B.

THIS view that the hypothetical judgment states the consequent as *necessary consequence* of the antecedent seems contrary to the way in which logic and grammar generally speak of the antecedent as *presupposition* or condition of the consequent. For if we understand condition in its ordinary sense as meaning the *conditio sine qua non*, as that which must be realized before something else happens or is valid, then it seems to imply that the negation of the antecedent involves that of the consequent, and that the consequent ceases to be valid when the antecedent is not true. But this is just what necessary consequence does not involve; the

consequence may be there even when the ground is wanting, unless, indeed, it is the sole ground, and all agree that the invalidity of the antecedent does not involve that of the consequent. The proposition that if a triangle is equilateral it is acute angled does not state that the attribute equilateral is a condition of the attribute acute angled in the sense that no triangle which is not equilateral could be acute-angled. On the other hand, that which is merely a condition of an event does not of itself suffice to bring about that event; for even if we take condition to mean an integral part of the whole cause, it is still only a part, while in the hypothetical judgment the antecedent is not merely one of the conditions of the consequent, it is in itself sufficient to make the consequent necessary. The contradiction disappears if we distinguish between the subjective conditions of the statement and the matter of the statement. The subjective condition of the affirmation of the consequent is its certainty, and the judgment states that in my present stage of thought the certainty of the consequent is dependent upon that of the antecedent. Only in so far as the antecedent is valid am I able or willing to affirm anything about the subject of the consequent; unless the antecedent is valid I will affirm nothing; unless the condition is fulfilled I will not answer for the result—*e.g.* if you run hard, you will catch him. But this does not mean that objectively quick running is the *conditio sine quâ non* of catching him, for he may stand still, etc. On the other hand, I cannot guarantee that the consequent will be true when the condition contained in the antecedent is fulfilled unless the former is the necessary consequence of the latter.

Bergmann (*Reine Logik*, I, § 19, p. 202 sq.) considers the essential feature of the hypothetical judgment to be that it makes a decision about the thesis depend upon a decision about the hypothesis. He distinguishes between two meanings of the hypothetical judgment according as it indicates only a relative decision, a decision with a reservation (as Wolff teaches), or is intended to emphasize the connection between the validity of the hypothesis and that of the thesis. As examples of the first meaning, he gives the propositions: if it is fine, I will visit you to-morrow—Rome was governed first by kings, if Livy is a reliable authority—a completely general theory of equations will never be found if we may judge from the results of past attempts.

There is no doubt that in such judgments we wish to lay special emphasis on the point that the thesis is not unconditionally asserted, but

only with the reservation that the hypothesis must be true. But does this invalidate the proposition that every hypothetical judgment affirms the necessary connection between the validity of hypothesis and the validity of thesis, that by this alone does it deserve the name of judgment, and that it is this connection which is expressed by the verbal form "if—then"? (I have never said—as Bergmann asserts, pp. 204, 208—that in framing the judgment we are concerned only with the connexion between hypothesis and thesis, nor that the proposition "if *A* is true, *B* is true" is an inadequate expression for "*B* is the necessary consequence of *A*"). In the last two examples the statement of the connexion is at once obvious: If Livy is a reliable authority, that which he tells us is true; he tells us that Rome was first ruled by kings, therefore Rome was really first ruled by kings—the truth of the thesis follows with logical necessity from the truth of the hypothesis. In the last example, again: If we are justified in inferring from past results to future, then future failure must be inferred from actual past failure; here again there is logical necessity of the consequent.

The first example contains an apparent exception: to-morrow I shall visit you if it is fine. This seems to be really no more than a conditional statement in which necessary connexion is absent. How can fine weather have the visit for its necessary consequent? But upon more careful examination even this example proves to be no exception. What it expresses is an intention, a resolution which is formed as well as a promise which is given. This resolution, in itself and as such, is not conditional; I am now bound by my promise, for it is just this dependence of an action upon the appearance of a given fact which is contained in my will; what I will at this moment is that the consequence shall actually take place together with the condition; by my will I establish a connexion, and then by virtue of this willed connexion I affirm that the resolution will be carried out so soon as the condition is present. This statement finds its ground in my will, which cannot be self-contradictory. It is the same with all promises, threats, and contracts which are conditional upon certain events; by my will I determine that a future action shall be the unfailing consequence of the realized condition. Here again what I state is the connexion between condition and consequence (cf. what is said in the work by Enneccerus: *Rechtsgeschäft, Bedingung und Anfangstermin*, 1888, p. 16, 175 sq.), but this connexion, in that it is willed, differs from a real connexion which exists independently of my will and which is

expressed by a purely theoretical statement. As such a statement the proposition "if it is fine, I will visit you to-morrow" is false, for there is no objectively necessary connexion as in the proposition "if it is fine, these buds will open to-morrow"; it is objectively valid only because it is assumed that my will is able to realize the connexion willed, and that it will remain constant, hence that I shall not be untrue to my promise. We still hold therefore that statements containing a conditional "if—then" are judgments only in so far as they state a necessary connexion.

APPENDIX C.

THE number of conclusions which can be drawn from those combinations of premises which are rejected by Aristotle is extended much further by Schuppe (*Erkenntnissth. Logik*, p. 128 sq.).

Even when the two premises a is not b , and b is not c , express mere difference they yet tell us, he says, that a and c coincide in the one point of not being b , and under certain circumstances this may be a most important discovery forming the only connexion between a and c . Still this discovery may be expressed by merely repeating the two premises together in the proposition "neither a nor c are b "; so that Schuppe acknowledges that in this case again it is true that *ex mere negativis nihil sequitur*.

But he holds that in judgments of necessary connexion this principle is obviously false. For his first proof of this he selects the instance explained on p. 456 of an inference having for its major premise the proposition "that which is not M is not P ."

From "no M is P " and " S is not M " we infer that we can certainly find no ground in M for denying P of S , "a result which may sometimes be very important when we are contending with vague ideas which deny P of S because of its similarity with M , though without any definite statement, a result therefore which may lead us to recognise that S is P ." Schuppe omits to illustrate his principles with examples; we shall find it easier to test them if we take one which corresponds to the above words. "No fish has warm blood—the whale is not a fish." From this he would say we can infer that we must not be led by the similarity of the whale to fish to deny that it has warm blood. But do we really draw this conclusion from *both* premises? If we are in danger of concluding that

because the whale is like a fish it has cold blood, it will be the minor premise alone which will check us, not both together. That is, when we have the premises—

That which is fish is not warm-blooded ;
The whale is not a fish,

it is the minor premise alone which prevents the possibility of subsumption ; the possibility of any conclusion whatever is prohibited by the rule that we cannot infer from the failure of the hypothesis to the failure of the thesis. So that here again Schuppe merely tells us in other words that from premises of this kind no conclusion can be drawn. For these premises give us absolutely no information as to whether P belongs to S or not.

“From no P is M and S is not M ” (I change the letters which Schuppe has misplaced here) “we make the similar inference that P is not to be denied of S because of a vague idea that S is M ; therefore, that in this respect at least the possibility of SP must be maintained.” Here again the false conclusion which is feared is prevented by the minor premise S not M alone. S is M would of course yield the conclusion that S is not P ; but from S not M we get nothing definite, neither that S is P nor that S is not P . M is not P , but M is not S either, proves that S and P may both be absent from the same subject (Lotze’s inference which we mentioned above).

He questions in the same way the rule *ex mere particularibus nihil sequitur*. “Some only, but at least some, M ’s are P , and some only, but at least some, S ’s are M , where we do not yet know what M ’s are P and what S ’s are M , assures us of the possibility of at least one case in which S is P .” But an assured possibility (in distinction from one which is quite vague and grounded only upon complete ignorance) would seem to be found only where it is known that S and P do not exclude each other; can we then infer from “some men are blind and some seeing creatures are men,” the assured possibility that in some particular case a seeing creature may be blind? “Such premises,” he goes on to say, “often lead to the valuable knowledge: (1) that P belongs to or is dependent upon the specific or individual differences of M , though it is not necessitated nor excluded by the conceptual content of M . (2) That M belongs to the specific or individual differences of S , though it is neither necessitated nor excluded by the conceptual content S . (3) that whether

S appears with or without P this is in no way connected with the presence or absence of M ." Propositions 1 and 2, however, are not inferred from both premises together; they merely present an interpretation of one of them. No doubt this one, as formulated above, "Some only, but at least some, M 's are P " consists—as Schuppe himself shows later on—in two judgments, "Some M 's are P , some M 's are not P ," and it is only from these two propositions taken together that we can infer that P is "neither necessitated nor excluded by the concept M ." These two judgments form the application of the major premise "that which is connected with the predicate M at one time but not at another is neither necessitated nor excluded by it"; according to this familiar rule we can neither say that all M 's are P nor that no M is P .

But the third of these propositions, the only one which can be inferred from both premises, is false. Instead of some M 's are P , and some S 's are M , let us take, for example, "some regular figures are rectangular, some quadrangles are regular figures." We cannot say that whether the quadrangle appears as rectangular or not this has nothing to do with the absence or presence of the attribute regular; the regular quadrangle is of necessity rectangular. Again, "some crystals have double refraction, some minerals are crystals"—must we infer from this that when we find a mineral having double refraction this has nothing to do with the fact of its being or not being a crystal?

On p. 133 he continues: "It is certain that the propositions, some M 's are P , some M 's are S , do not connect the predicates P and S with the some M 's because of the attribute M ; if they did, then the predicates must belong to all M 's; they connect them with specific or individual differences which fall under the concept M , and thus we are assured of the conclusion that in any particular M the attributes P and S are neither necessitated nor excluded by the characteristic M ." But this is only true if it is also true that some M 's are not P , and some M 's are not S . So that what we do is first to infer—as above—from the first pair of premises (M_1P , M_0P) that P is neither necessitated nor excluded by M , from the second (M_1S , M_0S) that S is neither necessitated nor excluded by M , and then to sum up the two judgments in one " P and S are neither necessitated nor excluded by the characteristic M ." But this process is not one in which we gain a definite relation between S and P by eliminating M ; no such relation can be determined in any way, not even negatively, from these premises.

"From all M 's are P , no S is M , it follows that if S , or one S , is P , it certainly is not so by virtue of M ." It is enough to know that S is not M in order to know that S can have no other predicate by virtue of M .

"The statement that in the form PM and SM both premises must not be affirmative is fundamentally false" (p. 137). Here, he holds, we may safely infer partial identity—sometimes even affinity. The nature of M may be such as to make this partial identity as valuable a result as the partial difference which is also inferred in the second figure. We have spoken on p. 85 of this term "partial identity"; if we let it pass, we find in the long run that all things are partially identical, and the conclusion is therefore without value. If, again, it depends upon the nature of M whether or not it has any value, we must regard the judgment, Both P and S are M , as a simple summation in which the middle concept is not eliminated. That it may be useful to know this as a preliminary step towards the operations of classification, I do not of course dispute; but we must not call it an inference unless we are to call every combination of two judgments into a conjunctive or copulative proposition an inference.

We may make the same answer to Wundt when he brings forward an "inference of comparison" (*Logik*, i. p. 324) which is, he says, sometimes an inference of agreement, sometimes of difference. In the first case we have " A has the characteristic M , B has the characteristic M , therefore A and B agree in one characteristic"; in the second " A has the characteristic M , B has not the characteristic M , therefore A and B have a distinguishing characteristic." But what ground is there for the "therefore" of the first inference? Evidently it must be the major premise that if two objects or concepts have the same characteristic they agree in one characteristic—a mere tautology from which we can learn nothing whatever as to any other relations between the objects or concepts compared, nothing as to whether they are identical or opposed, or anything else. In the second inference, again, the "therefore" presupposes, according to Wundt's rendering, nothing more than the major premise, "if one of two objects has a characteristic which the other has not, then they have a distinguishing characteristic." Here indeed very little would be told us by the inference itself, for it would contain no more than the two premises themselves tell us in another form. But from the fact that two objects have a distinguishing characteristic we may further infer that they

differ as wholes, that they cannot be identical, and, when we are dealing with concepts, that they cannot be predicated of each other in any sense. We have therefore at least a negative determination of their relation, and as the recognition of difference and opposition is one of the fundamental operations of our thought this result is certainly important. It is not merely a question of "partial difference," as Schuppe represents it, like that of "partial identity"; from partial identity nothing can be inferred, but from "partial difference" it follows (p. 138) that *S* and *P* are not identical as wholes, that they cannot be predicated of the same thing nor of each other. It is of course presupposed that one of the premises contains a necessity, and that we do not compare merely casual states. It certainly does not follow that because the stove in my room yesterday was hot, while the stove in my room to-day is cold, therefore the second stove is not the same as the first; a conclusion can be drawn only when the major premise tells us that a predicate must be necessarily affirmed or denied of a subject, so that the absence or presence of this predicate involves that of the subject itself.—From what we have said it would appear therefore that Aristotle was perfectly right in recognising only a negative result in the second figure, saying of two positive premises: οὐκ ἔσται συλλογισμός.

I cannot therefore allow that this criticism of the traditional doctrine is justified. Nevertheless Schuppe is right in emphasizing the view that in drawing an inference from two premises we frequently obtain our conclusion, not from the premises by themselves, but from a major premise which alone enables us to get any result from combining the contents of one premise with the contents of the other. Here we must distinguish between two cases: either one of the premises is itself a hypothetical judgment, in which case we merely need the general principle that the affirmation of the ground involves that of the consequence, the denial of the consequence that of the ground; or else the two premises need a more special principle of which they are the application, and by means of which they yield the conclusion according to the general principle of inference. Inferences of the first and second figure which presuppose a universal major premise belong to the first class; inferences of the third figure to the second.

B. Erdmann (*Philos. Aufsätze zum Doctorjubiläum E. Zellers*, p. 201) maintains that by introducing definitions we may get universal affirmative conclusions in the second and third figures; e.g., all mammals have lacteal

glands, all whales have lacteal glands, therefore all whales are mammals. But he overlooks the fact that the major premise does not tell us that it is a definition, and the conclusion does not follow from what it does tell us, For the inference to be valid it must stand "*only* mammals have lacteal glands"; *i.e.*, that which has lacteal glands is a mammal; then, however, our inference is in the first figure.

END OF VOL. I.

